1996 Paper 9 Question 13

Types

Briefly explain what is meant by *capture-avoiding substitution.* [3 marks]

What is a *principal typing* and why is it useful? [5 marks]

Suppose that a constant *fix* is added to the expressions of System F, with the typing rule

$$\Gamma \vdash \mathit{fix} \in \mathit{All}(X)\,(X \to X) \to X \qquad \text{(T-F}\textsc{ix}\text{)}$$

and principal evaluation rule:

$$\frac{f\ (\mathit{fix}\ X\ f) \Downarrow r}{\mathit{fix}\ X\ f \Downarrow r} \qquad \text{(E-F}\textsc{ix}\text{)}$$

Also, suppose we are given a built-in type operator *List* and the following expression constants:

$$\begin{array}{rcl} \mathit{nil} & \in & \mathit{All}(X)\ (\mathit{List}\ X) \\ \mathit{cons} & \in & \mathit{All}(X)\ X \to (\mathit{List}\ X) \to (\mathit{List}\ X) \\ \mathit{car} & \in & \mathit{All}(X)\ (\mathit{List}\ X) \to X \\ \mathit{cdr} & \in & \mathit{All}(X)\ (\mathit{List}\ X) \to (\mathit{List}\ X) \\ \mathit{null} & \in & \mathit{All}(X)\ (\mathit{List}\ X) \to \mathit{Bool} \end{array}$$

Use these primitives to write a polymorphic function *fold* of type

$$\mathit{fold} \in \mathit{All}(X)\ \mathit{All}(Y)\ (X \to Y \to Y) \to Y \to (\mathit{List}\ X) \to Y$$

that "folds a function across a list." For example, applying *fold* to +, 0, and a list of numbers should return the sum of the list. [8 marks]

Which of the following existential packages is most useful, and why?

$$[\mathit{Int}, \{x = 5,\ f = \mathit{fun}(i \in \mathit{Int})\ i + 1\}] \in \mathit{Some}(X)\ \{x \in X,\ f \in X \to X\}$$
$$[\mathit{Int}, \{x = 5,\ f = \mathit{fun}(i \in \mathit{Int})\ i + 1\}] \in \mathit{Some}(X)\ \{x \in \mathit{Int},\ f \in X \to \mathit{Int}\}$$
$$[\mathit{Int}, \{x = 5,\ f = \mathit{fun}(i \in \mathit{Int})\ i + 1\}] \in \mathit{Some}(X)\ \{x \in X,\ f \in X \to \mathit{Int}\}$$
$$[\mathit{Int}, \{x = 5,\ f = \mathit{fun}(i \in \mathit{Int})\ i + 1\}] \in \mathit{Some}(X)\ \{x \in \mathit{Int},\ f \in \mathit{Int} \to \mathit{Int}\}$$

[4 marks]