1996 Paper 6 Question 9

Foundations of Functional Programming

A new form of abstraction on combinators, $\lambda' x$, is proposed. It is to have the same defining equations as $\lambda^T x$, augmented with equations for the new combinators $\mathbf{B}'$, $\mathbf{C}'$ and $\mathbf{S}'$. The new equations should be applied instead of the existing ones if possible:

$$\begin{aligned}
\lambda' x.O\,P\,Q &\equiv \mathbf{B}'\,O\,P\,(\lambda' x.Q) && x \text{ not free in } O \text{ or } P\\
\lambda' x.O\,P\,Q &\equiv \mathbf{C}'\,O\,(\lambda' x.P)\,Q && x \text{ not free in } O \text{ or } Q\\
\lambda' x.O\,P\,Q &\equiv \mathbf{S}'\,O\,(\lambda' x.P)\,(\lambda' x.Q) && x \text{ not free in } O
\end{aligned}$$

The reduction rules for the new combinators are

$$\begin{aligned}
\mathbf{B}'\,O\,P\,Q\,R &\rightarrow_w O\,P\,(Q\,R)\\
\mathbf{C}'\,O\,P\,Q\,R &\rightarrow_w O\,(P\,R)\,Q\\
\mathbf{S}'\,O\,P\,Q\,R &\rightarrow_w O\,(P\,R)\,(Q\,R)
\end{aligned}$$

Here O, P, Q and R stand for combinatory terms.

Compare $\lambda' x$ with $\lambda^T x$ by applying both abstraction methods to the λ-term $\lambda x\,y\,z.z\,y\,x$. [4 marks]

Give graph reduction rules for the new combinators. [2 marks]

Prove $(\lambda' x.P)Q \twoheadrightarrow_w P[Q/x]$ by induction. (You need to discuss only the new combinators.) [6 marks]

The size of the result of translating $\lambda' x_1 \ldots x_n.P$ is linear in n. Give a convincing argument that this is true. [8 marks]