COMPUTER SCIENCE TRIPOS Part IB

Tuesday 4 June 1996 1.30 to 4.30

Paper 4

Answer **five** *questions.*
Submit the answers in five **separate** *bundles each with its own cover sheet.*
Write on **one** *side of the paper only.*

1 Concurrent Systems

Give a detailed criticism of the following options for supporting interactions between processes in separate address spaces:

(*a*) pipes

(*b*) asynchronous message passing

(*c*) synchronous message passing

[20 marks]

2 Further Modula-3

What are `UNSAFE` interfaces and implementations in Modula-3 and what are they used for? [5 marks]

Write brief notes on the following Modula-3 keywords:

```
UNTRACED
ADDRESS
ADR
LOOPHOLE
DISPOSE
```

[3 marks each]

3 Continuous Mathematics

Suppose we wish to compute a predicted value for some continuous, differentiable function $f(t)$ at future time $t = b$, based only upon knowledge of its value now at time $t = a$, which is $f(a)$, and its first three derivatives now, namely $f'(a)$, $f''(a)$, and $f'''(a)$. How would you compute such a prediction for the future value $f(b)$? Give a formula or explain the sorts of terms that it would include and why they appear. [6 marks]

In practical computing problems with numerical data that are discretely sampled at periodic points in time, derivatives defined in terms of continuous limits must be replaced by discrete estimates. Provide finite-difference expressions for each of the three derivatives $f'(t)$, $f''(t)$, and $f'''(t)$ on the discrete set of samples

$$\ldots, f(t_i), f(t_{i+1}), f(t_{i+2}), f(t_{i+3}), \ldots$$

assuming that in this sequence the samples are separated by a constant interval $\Delta t = t_{i+1} - t_i$. [8 marks]

Real-world signals are generally accompanied by noise. Using the Differentiation Theorem of the Fourier Transform, comment upon how noise is amplified as a function of its spectral composition when computing first-order, second-order, and third-order derivatives of the signal. [6 marks]

4 Compiler Construction

Describe a structure that could be used to represent the abstract syntax tree of the following λ-expression:

$$(\lambda a.(\lambda f.fa)(\lambda n.n+1))3$$

Outline the definition of an evaluator function `eval(expr, env)` that could be used to evaluate a given expression `expr` represented in this way in the context of an environment given by `env`. Pay particular attention to the treatment of bound variables and the mechanism you use for function calls. [7 marks]

Is it possible with your implementation to give it a λ-expression which would cause `eval` to recurse to an unlimited depth? If so, give such a λ-expression; if not, explain why. [3 marks]

Is it possible with your implementation to give it a λ-expression that causes an environment chain of unlimited length to be created during the evaluation? If so, give a λ-expression that would cause this; if not, explain why it is not possible. [3 marks]

In an erroneous implementation of `eval` the value of

$$(\lambda x.body)$$

is an object that does not incorporate the contextual environment, and whose call evaluates *body* in an environment derived from the environment of the call. Explain why such an implementation is wrong, giving an example λ-expression that would yield different results when evaluated by this implementation and yours. [7 marks]

5 Data Structures and Algorithms

Show why comparison-based sorting of n items cannot take much less than $n \log n$ comparisons, being clear about your assumptions. Why can it take *any* less than $n \log n$? [10 marks]

If 1024 numbers are drawn randomly in the range 0–127 and sorted by binary insertion, about how many compares would you expect? A fairly rough estimate will do if your reasoning is clear. [10 marks]

6 Structured Hardware Design

Describe logic synthesis by writing short notes on *each* of the following topics:

(*a*) input language constructs

(*b*) output format

(*c*) use of libraries of subcircuits

(*d*) technology independence

(*e*) guiding metrics and constraints

(*f*) synthesis for multiple instances

(*g*) disadvantages compared with alternative approaches

[20 marks]

7 Operating System Functions

What is meant by the term *demand paging* in a virtual memory management system, and how is it implemented? [5 marks]

Briefly describe five techniques which the operating system and/or hardware can implement to improve the efficiency of demand paging. [5 marks]

What is the *working set* of a program, and how can an operating system use it in the management of virtual memory? [3 marks]

Describe the clock (second chance) algorithm for selecting a VM page for replacement when a page fault occurs. How is the performance of this algorithm affected by the memory size of the computer system, and how may this be avoided? [7 marks]

8 Computation Theory

A *bag* B of natural numbers is a total function $f_B : \mathbb{N} \to \mathbb{N}$ giving for each natural number x the count $f_B(x)$ of occurrences of x in B. If each $f_B(x) = 0$ or 1, then f_B is the characteristic function χ_s of a set S: every set can thus be regarded as a bag.

(a) A bag B is *recursive* if the function f_B is computable. Suppose that the sequence of bags $\{B_n \mid n \in \mathbb{N}\}$ is recursively enumerated by the computable function $e(n, x) = f_n(x)$, which gives the count of x in each bag B_n. Show that there is a recursive set S that is different from each bag B_n. [7 marks]

Hence prove that the set of all recursive bags cannot be recursively enumerated. [3 marks]

(b) A bag B is *finite* if there is $X \in \mathbb{N}$ such that $f_B(x) = 0$ for all $x \geqslant X$. Show that the set of all finite bags is recursively enumerable. [10 marks]

9 Numerical Analysis I

Let x^* be the floating-point representation of a number x. Define the *absolute error* and *relative error* in representing x by x^*. How are these errors related? [3 marks]

Let x_1, x_2 be two numbers. Find expressions for

(a) the *absolute error* in representing $x_1 + x_2$

(b) the *relative error* in representing $x_1.x_2$ (where "." denotes multiplication) [4 marks]

Assume that the numbers 1 and 2 are represented exactly. Find an expression for the absolute error in calculating $2x + 1$. [2 marks]

In an iterative calculation the number y is an improved value of x, derived from the assignments

$$p := x/2 + 1$$
$$q := x - 2$$
$$y := p + 1/q$$

If ε_x is the absolute error in representing x, find an expression for the *absolute error* ε_y in representing y. [6 marks]

What is the approximate *relative error* δ_y in representing y when $x = 2.01$? [5 marks]

10 Graphics

Describe a quad-tree encoding method for greyscale images. [6 marks]

Given the following greyscale image, draw a diagram showing how it would be encoded using your method from the previous part.

33	39	43	72
34	54	64	81
42	54	71	83
60	64	77	89

[4 marks]

An image processing package allows the user to design 3×3 convolution filters. Design 3×3 filters to perform the following tasks:

(*a*) blurring [2 marks]

(*b*) edge detection of vertical edges [2 marks]

Choose one of the two filters (*a*) or (*b*) from the previous part. Explain how it works, using the following image as an example (you may round off any calculated values to the nearest integer).

100	100	100	0	0	0
100	100	100	0	0	0
100	100	100	0	0	0
100	100	100	100	100	100
100	100	100	100	100	100
100	100	100	100	100	100

[6 marks]