

# COMPUTER SCIENCE TRIPOS Part IA

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Wednesday 7 June 1995 9 to 12

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Paper 2

*Answer the question in Section A, **one** question from each of Sections B and C, and **two** questions from Section D.*

*Submit the answers in five separate bundles, each with its own cover sheet. On each cover sheet, write the numbers of all attempted questions, and circle the number of the question attached.*

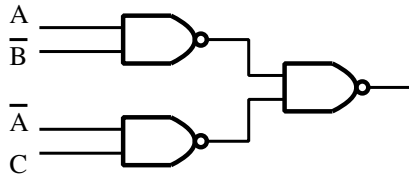
*Write on **one** side of the paper only.*

## SECTION A

**Twenty-part question** (*One mark per part*)

- 1  $A$  and  $B$  are finite sets with  $|A| = m$ , and  $|B| = n$ . How many partial functions are there from  $A$  to  $B$ ? Why?
- 2 How many different ways are there to make up a basket of 15 fruit from apples, pears, oranges and bananas?
- 3 Explain, without writing a program, Euclid's algorithm for greatest common divisor.
- 4 Simplify  $(A \vee B) \wedge (\bar{A} \vee B) \wedge (A \vee \bar{B})$ .
- 5 Given a distribution  $P(X = r) = \frac{\lambda^r}{r!} e^{-\lambda}$ , what is the expectation  $E(X)$ ?
- 6 What is the probability that the scores on two dice sum to 8?
- 7 Following the ML definition `fun tw f x = f(f(x))`, what is the type of `tw`?

- 8 Can the following circuit exhibit a static hazard and if so when?



- 9 What is a tri-state buffer, how does it differ from more ordinary logic gates and how is it used?
- 10 Given a **CARDINAL** variable  $n$ , write Modula-3 code to declare and initialise a variable  $p$  to point to an array of  $n$  **REALS**.
- 11 Write a Modula-3 type definition for a linked list of **INTEGERS**.
- 12 Following the ML definition `fun f g [y,z] = g(y+1, z)` what is the type of  $f$ ?
- 13 What three new offences were created under the Computer Misuse Act of 1990?
- 14 Systems engineers use the terms “availability” and “reliability”. Explain the distinction they make between them.
- 15 List *four* essential differences between a software engineering project and a civil engineering project such as building a skyscraper.
- 16 Verify  $(A \cup B) \setminus (A \cap B) = (A \cap \overline{B}) \cup (\overline{A} \cap B)$ .
- 17  $A, B, C$  are arbitrary events which occur with probability  $P(A)$ ,  $P(B)$ , and  $P(C)$ . Write an expression for the event that only one of them occurs. You do not need to express its probability.
- 18 If the probability of having a boy is  $\frac{1}{2}$  what is the probability that a family of 4 children has more boys than girls? (Ignore the possibility of multiple births.)
- 19 Give a finite *deterministic* automaton with alphabet of input symbols  $\{a, b\}$  that accepts the language denoted by the regular expression  $a^*$ .
- 20 If  $L$  is a regular language over an alphabet  $\Sigma$ , explain why the complement  $\{w \in \Sigma^* | w \notin L\}$  is also a regular language.

## SECTION B

## 21 Digital Electronics and Computer Design

Explain the terms “minterm” and “prime implicant”.

A number of Boolean functions conform to the map shown below:

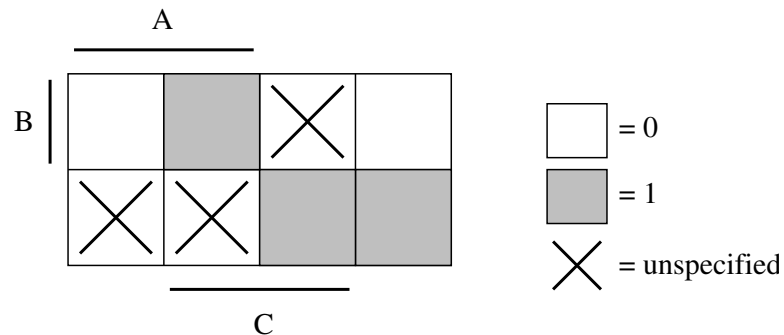


Exhibit maps and minimum “sum of product” expressions for the following particular functions:

- (a)  $F1$ , being a function of two variables;
- (b)  $F2$ , having four minterms;
- (c)  $F3$ , having two prime implicants;
- (d)  $F4$ , having a prime implicant that can not appear in the minimum “sum of products” representation.

[20 marks]

## 22 Digital Electronics and Computer Design

A sequential circuit has been built, and it behaves slightly erratically. When switched on it produces on its three output wires one of the following patterns:

000		010
011 ←	or	101 ←
110		111
100		010
001		
000		

The second pattern is not intended to arise. Deduce the circuit details and propose a modification that ensures that in due course the circuit will always settle into the cycle shown in the first pattern.

[20 marks]

## SECTION C

### 23 Probability

Two gamblers play a game which involves tossing a fair coin. After  $t_2$  tosses the first gambler has scored  $k$  wins. If there is no record of the sequence of tosses, what probability distribution describes the situation after  $t_1$  tosses ( $t_1 < t_2$ )?

If the game is tied after eight tosses, show that the probability that it was tied after four is  $\frac{18}{35}$ .

[20 marks]

## 24 Probability

Find the probability generating function of the discrete uniform random variable  $X$  distributed on

$$\{-a, -a + 1, -a + 2, \dots, b - 1, b\}$$

A *tail generating function*  $T(s)$  for a discrete random variable  $X$  is defined as

$$T(s) = \sum_{n=-\infty}^{\infty} P(X > n) s^n$$

Give the relation between a probability generating function and the tail generating function for  $X > n$ , and hence obtain the tail generating function in this case.

[20 marks]

## SECTION D

### 25 Professional Practice and Ethics

How is private ownership of software justified? What laws are used to protect software ownership? Are there any workable alternatives to private ownership of software?

[20 marks]

### 26 Software Engineering

Describe the process of specifying a major piece of software: the main documents produced, their immediate purpose and their ongoing role in the software life cycle.

Describe the role that formal methods can play at each stage of the software life cycle. Explain any disadvantages of the uses of formal methods that you have discussed.

[20 marks]

## 27 Regular Languages and Finite Automata

Prove or disprove each of the following statements, stating clearly any well known results that you use.

- (a) The set of strings over the alphabet  $\{0, 1\}$  that contain exactly twice as many occurrences of 0 as of 1 is a regular language;
- (b) Let  $L$  be a regular language over an alphabet  $\Sigma$ . Then the language consisting of those  $u \in \Sigma^*$  such that there is some  $v \in \Sigma^*$  with  $uv \in L$ , is also a regular language;
- (b) Any finite subset of  $\{a, b\}^*$  is a regular language;
- (d) For any regular expressions  $\mathbf{r}$  and  $\mathbf{s}$ , the regular expressions  $(\mathbf{r}^*\mathbf{s}^*)^*$  and  $(\mathbf{r}|\mathbf{s})^*$  always denote the same language.

[20 marks]

## 28 Unix Case Study

What is an *i-node* and what information is contained in it?

Describe how named files are mapped to i-nodes.

How is the information associating disc blocks with i-nodes represented?

What restrictions are placed on name to i-node links to simplify file system recovery?

[20 marks]