1994 Paper 9 Question 13

Types

Describe the types and terms of the second-order lambda calculus ($\lambda 2$) and define the type assignment relation for $\lambda 2$. [5 marks]

Are the following expressions typeable in $\lambda 2$? Justify your answer in each case.

$$
\begin{aligned}
L &= \Lambda\alpha.\Lambda\beta.\lambda x : \alpha.\Lambda\gamma.\lambda f : \alpha \to \gamma.\lambda g : \beta \to \gamma.fx \\
R &= \Lambda\alpha.\Lambda\beta.\lambda y : \beta.\Lambda\gamma.\lambda f : \alpha \to \gamma.\lambda g : \beta \to \gamma.gy \\
S &= \Lambda\alpha.\lambda x : \alpha.x(x_\alpha)
\end{aligned}
$$

[5 marks]

Find a type σ that makes the following expression typeable in $\lambda 2$ and give, with justification, the type of C.

$$C = \Lambda\alpha.\Lambda\beta.\Lambda\gamma.\lambda f : \alpha \to \gamma.\lambda g : \beta \to \gamma.\lambda z : \sigma.(z_\gamma f)g$$

[5 marks]

Explain what is meant by *β-reduction* and *β-normal form* for $\lambda 2$ terms. Calculate the β-normal form of the term $C_{\alpha\beta\gamma} fg(L_{\alpha\beta} M)$ where C and L are as above, α, β, γ are type variables, f, g are identifiers, and M is a term in β-normal form with no free identifiers or free type variables. [5 marks]