1994 Paper 5 Question 10

Foundations of Functional Programming

Describe precisely the meaning and main properties of the equality $M = N$, where M and N are terms of the λ-calculus. [5 marks]

In the following, consider an encoding of lists $[a_1, a_2, \ldots, a_m]$ as the λ-term

$$\lambda f x. f a_1 (f a_2 \ldots (f a_m x) \ldots).$$

Answers should include a brief justification. You may assume λ-encodings of the booleans and ordered pairs.

Define the λ-term **cons** such that

$$\mathbf{cons}\, a\, [a_1, \ldots, a_m] = [a, a_1, \ldots, a_m]$$ [2 marks]

Define the λ-term **null** such that

$$\mathbf{null}\, [a_1, \ldots, a_m] \;=\; \begin{cases} \mathbf{true} & (\text{if } m = 0) \\ \mathbf{false} & (\text{if } m > 0) \end{cases}$$ [3 marks]

Define the λ-term **append** such that

$$\mathbf{append}\, [a_1, \ldots, a_m][b_1, \ldots, b_n] = [a_1, \ldots, a_m, b_1, \ldots, b_n]$$ [3 marks]

Define the λ-terms **hd** and **tl** such that, if $m > 0$,

$$\begin{aligned} \mathbf{hd}\, [a_1, \ldots, a_m] &= a_1 \\ \mathbf{tl}\, [a_1, \ldots, a_m] &= [a_2, \ldots, a_m] \end{aligned}$$ [7 marks]