# 1993 Paper 7 Question 7

## Numerical Analysis II

State a recurrence formula suitable for evaluating the sequence of Chebyshev polynomials $\{T_n(x)\}$ for an argument $x$. What are the starting values? [2 marks]

The error in Lagrange interpolation can be expressed in the form

$$f(x) - L_{n-1}(x) = \frac{f^{(n)}(\zeta)}{n!} \prod_{j=1}^{n} (x - x_j)$$

for a suitable function $f(x)$. Suggest a choice of the interpolation points $\{x_j\}$ which tends to minimise this error over the interval $[-1, 1]$. [3 marks]

Hence justify and explain the method of *economisation of a power series*. [5 marks]

In what sense is an economised power series a *best approximation*? [2 marks]

Suppose $P_n(x)$ is a polynomial formed by truncating a power series after the term in $x^n$. Perform an economisation of the truncated power series

$$\cosh x \simeq P_4(x) = 1 + \tfrac{x^2}{2!} + \tfrac{x^4}{4!}$$ [5 marks]

Given that the maximum error in $P_4(x)$ over $[-1, 1]$ is approximately 0.0014, compare the error in your economised polynomial with the error in $P_2(x)$. [3 marks]