

## 1993 Paper 7 Question 5

### Specification and Verification of Hardware

You are given components MUX, REG and COMP whose behaviour is defined by

$$\text{MUX}(sw, in1, in2, out) = (\forall t. out\ t = (sw\ t \rightarrow in1\ t \mid in2\ t))$$

$$\text{REG}\ v\ (in, out) = (out\ 0 = v) \wedge (\forall t. out(t+1) = in\ t)$$

$$\text{COMP}(in1, in2, out) = (\forall t. out\ t = (in1\ t < in2\ t))$$

Use these to implement a device MAX that satisfies the specification

$$\text{MAX}(in, out) \Rightarrow (\forall t. out\ t = \textit{Max}\ in\ t)$$

where the function *Max* is defined by

$$\begin{aligned} &(\textit{Max}\ in\ 0 = in\ 0) \wedge \\ &(\textit{Max}\ in\ (n+1) = (\textit{Max}\ in\ n < in(n+1) \rightarrow in(n+1) \mid \textit{Max}\ in\ n)) \end{aligned}$$

[10 marks]

Prove that your implementation meets its specification.

[10 marks]