1993 Paper 2 Question 4

X and Y are independent random variables having Poisson distributions with parameters α and β respectively. By using probability generating functions, or otherwise, prove that $X + Y$ has a Poisson distribution and give its parameter.

Find the conditional distribution for X given that $X + Y = n$, and give its mean and variance. Explain your result in words.