1993 Paper 1 Question 5

Give an ML definition of the function `map3` which has the property that

$$map3\ f\ [x_1, x_2, \ldots, x_n] = [f\ 0\ x_1\ x_2,\ f\ x_1\ x_2\ x_3, \ldots,\ f\ x_{n-1}\ x_n\ 0]$$

and deduce the type of `map3`. The function `map3iter` is defined as follows:

```
fun map3iter _ (0::_) = 0
 |  map3iter g   x    = 1 + map3iter g (map3 g x);
```

Deduce the type of `map3iter` and explain in words what the function does. Illustrate your answer by considering the call

```
map3iter g [1, 1, 1, 1, 1, 1];
```

in an environment in which `g` is defined as follows:

```
fun g 0 1 _ = 2
 |  g 1 1 _ = 1
 |  g 2 1 _ = 2
 |  g _ 2 0 = 0
 |  g _ n _ = n;
```