2001 Paper 1 Question 6

Foundations of Computer Science

This question has been translated from Standard ML to OCaml

To represent the power series $\sum_{i=0}^{\infty} a_i\, x^i$ in a computer amounts to representing the coefficients a_0, a_1, a_2, ... One possible representation is by a function of type `int->real` that returns the coefficient a_i given i as an argument. An alternative representation is the following type:

```
type power = Cons of float * (unit -> power)
```

(*a*) Demonstrate the two representations by using each of them to implement these two power series:

(*i*) The constant power series c, with $a_0 = c$ and $a_i = 0$ for $i > 0$. [3 marks]

(*ii*) The Taylor series $\sum_{i=0}^{\infty} x^i/i!$ for the exponential function. [4 marks]

(*b*) Also implement (using both representations) each of the following operations on power series:

(*i*) Product with a scalar, given by $c \cdot \left(\sum_{i=0}^{\infty} a_i\, x^i\right) = \sum_{i=0}^{\infty} (ca_i)\, x^i$. [3 marks]

(*ii*) Sum, given by $\left(\sum_{i=0}^{\infty} a_i\, x^i\right) + \left(\sum_{i=0}^{\infty} b_i\, x^i\right) = \sum_{i=0}^{\infty} (a_i + b_i)\, x^i$. [4 marks]

(*iii*) The product $\left(\sum_{i=0}^{\infty} a_i\, x^i\right) \times \left(\sum_{i=0}^{\infty} b_i\, x^i\right)$, where the ith coefficient of the result is $a_0\, b_i + a_1\, b_{i-1} + \cdots + a_i\, b_0$. [6 marks]

You may assume the OCaml function `float_of_int` of type `int -> float` that maps an integer to the equivalent real number.