Compiler Construction

Lecture 8: CPS & defunctionalization

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Continuation-**P**assing **S**tyle

Programs in **continuation-passing style** have some useful properties:

Evaluation order is explicit

Every call is a tail call

$$\mathsf{f}\ (\mathsf{g}\ \mathsf{x}) \rightsquigarrow \mathsf{g}\ \mathsf{x}\ (\mathsf{fun}\ \mathsf{y} \to \mathsf{f}\ \mathsf{y}\ \mathsf{k})$$

Every intermediate result is named

Every *continuation* is reified

CPS conversion of `fib`


```
let rec fib m =
 if m = 0 then 1
 else if m = 1 then 1
 else fib (m−1) + fib (m−2)
```

let-bind function calls

```
let rec fib m =
 if m = 0 then 1
 else if m = 1 then 1
 else let a = fib (m−1) in
      let b = fib (m−2) in
      a+b
```

CPS convert

```
let rec fib_cps m k =
 if m = 0 then k 1
 else if m = 1 then k 1
 else fib_cps (m−1) (fun a →
      fib_cps (m−2) (fun b →
      k (a+b)))
```

CPS conversion of `fib`: details

1. Add a continuation parameter `k` to each function
2. Apply `k` to values returned by the function
3. Replace each application `let` binding with a continuation argument

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let rec fib m =
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```

Use the identity continuation

`fib_cps` has the type `int → (int → int) → int`.

To recover a function of type `int →int`, pass the identity continuation `fun x →x`:

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 if m = 0 then k 1
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 else fib_cps (m−1) (fun a →
      fib_cps (m−2) (fun b →
      k (a+b)))

let fib_1 x = fib_cps x (fun x → x)
```

Now `fib_1` can be used like `fib`:

```
List.map fib_1 [0; 1; 2; 3; 4; 5; 6; 7; 8; 9; 10]
⇝ [1; 1; 2; 3; 5; 8; 13; 21; 34; 55; 89]
```

Correctness of CPS conversion for `fib`

Claim

For all $m \geq 0$,
  for all `k : int →int`,
    `fib_cps m k = k (fib m)`.

Proof

By strong induction on `m`.

```
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 if m = 0 then 1
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NB: We approximate OCaml functions by mathematical functions, ignoring side effects etc.

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fib_cps (m+1) k

≡ (expand fib_cps) ...

if m+1 = 1 then k 1 else fib_cps ((m+1)−1) (fun a → fib_cps ((m+1)−2) (fun b → k (a+b)))

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Claim

For all $m \geq 0$,
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By strong induction on m.

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if m+1 = 1 then k 1 else fib_cps ((m+1)−1) (fun a → fib_cps ((m+1)−2) (fun b → k (a+b)))

≡ (arithmetic) ...

if m+1 = 1 then k 1 else fib_cps m (fun a → fib_cps (m−1) (fun b → k (a+b)))

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if m+1 = 1 then k 1 else fib_cps m (fun a → fib_cps (m−1) (fun b → k (a+b)))

≡ (inductive assumption for $m-1$ and k = (fun b → k (a+b))) ...

if m+1 = 1 then k 1 else fib_cps m (fun a → (fun b → k (a+b)) (fib (m−1)))

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For all $m \geq 0$,
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Correctness of CPS conversion for `fib`

Claim

For all `m` ≥ 0,
  for all `k : int →int`,
    `fib_cps m k = k (fib m)`.

Proof

By strong induction on `m`.

Base case (`m = 0`): `fib_cps 0 k = k 1 = k (fib 0)`.

Inductive step:
Assume for all `n` $\leq$ `m`, `k (fib n) = fib_cps n k`.
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if m+1 = 1 then k 1 else fib_cps m (fun a → (fun b → k (a+b)) (fib (m−1)))

≡ (inductive assumption for *m* and `k = (fun a → (fun b → k (a+b)) (fib (m-1))))` ...

if m+1 = 1 then k 1 else (fun a → (fun b → k (a+b)) (fib (m−1))) (fib m)

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Base case (`m = 0`): `fib_cps 0 k = k 1 = k (fib 0)`.

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We want to show: `fib_cps (m+1) k = k (fib (m+1))`.

if m+1 = 1 then k 1 else (fun a → (fun b → k (a+b)) (fib (m−1))) (fib m)

≡ (beta reduction ×2) ...

if m+1 = 1 then k 1 else k (fib m + fib (m−1))

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For all $m \geq 0$,
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For all $m \geq 0$,
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By strong induction on m.

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if m+1 = 1 then k 1 else k (fib m + fib (m−1))

≡ (if e1 then k e2 else k e3 ≡ k (if e1 then e2 else e3)) ...

k (if m+1 = 1 then 1 else fib m + fib (m−1))

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k (if m+1 = 1 then 1 else fib m + fib (m−1))

≡ (definition of `fib`) ...

k (fib (m+1))

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k (fib (m+1))

QED

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Defunctionalization

Defunctionalized programs have some useful properties:

No higher-order functions

All values are data

All control-flow is first order

Every function is named

Defunctionalization: example

1. Add a constructor to `fn` for each `fun`
2. Replace each `fun` with its constructor
3. Add a case to `apply` for each `fun`
4. Replace each application `p x` with `apply p x`

```
let rec filter p l =
  match l with
  | [] → []
  | x :: xs →
      if p x
      then x :: filter p xs
      else filter p xs

let f l y =
    filter (fun x → x < 3) l
  @ filter (fun x → x > y) l
```

```
type fn = Lt_three
        | Gt of int

let apply fn x =
  match fn, x with
  | Lt_three, x → x < 3
  | Gt y, x → x > y

let rec filter p l =
  match l with
  | [] → []
  | x :: xs →
      if apply p x
      then x :: filter p xs
      else filter p xs

let f l y =
    filter Lt_three l
  @ filter (Gt y) l
```

Defunctionalization: example

1. Add a constructor to `fn` for each `fun`
2. Replace each `fun` with its constructor
3. Add a case to `apply` for each `fun`
4. Replace each application `p x` with `apply p x`

```
let rec filter p l =
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let f l y =
    filter (fun x → x < 3) l
  @ filter (fun x → x > y) l
```

```
type fn = Lt_three
        | Gt of int

let apply fn x =
  match fn, x with
  | Lt_three, x → x < 3
  | Gt y, x → x > y

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```

Combining CPS & defunctionalization

Defunctionalizing fib_cps

```
let rec fib_cps m k =
 if m = 0 then k 1
 else if m = 1 then k 1
 else fib_cps (m-1) (fun a → (* K1 *)
      fib_cps (m-2) (fun b → (* K2 *)
      k (a+b)))

let fib_1 x = fib_cps x (fun x → x) (* ID *)
```

To defunctionalize `fib_cps`, define a constructor for each `fun`:

```
type cont = ID | K1 of int * cont | K2 of int * cont
```

Constructor arguments are free variables, and we treat `k2` as free in `k1`:

```
let k2 = fun a b → k (a+b)
let k1 = fun a → fib_cps (m-2) (k2 a)
```

Defunctionalized `fib_cps`

Now define an apply function of type `cont →int →int`

```
type cont = ID | K1 of int * cont | K2 of int * cont

let rec apply_cont k v = match k, v with
| ID, a → a
| K1 (m, k), a → fib_cps_defun (m−2) (K2 (a, k))
| K2 (a, k), b → apply_cont k (a+b)
```

and call `apply_cont` at every application of a continuation:

```
and fib_cps_defun m k =
  if m = 0 then apply_cont k 1
  else if m = 1 then apply_cont k 1
  else fib_cps_defun (m −1) (K1 (m, k))

let fib_2 m = fib_cps_defun m ID
```

Correctness of `fib_cps` defunctionalization

Claim

Let <c> represent a continuation c : int → int constructed by fib_cps. Then

$$\text{apply\_cont } \langle c \rangle \text{ m} \quad = \quad \text{c m}$$

and

$$\text{fib\_cps n c} \quad = \quad \text{fib\_cps\_defun n } \langle c \rangle$$

(**Proof** left as an exercise)

Observation: continuations have list (stack) structure

```
type int_list =
    NIL
  | CONS of int * int_list
```

```
type cont =
    ID                  (* 'Nil' *)
  | K1 of int * cont    (* 'Cons' *)
  | K2 of int * cont    (* 'Cons' *)
```

Idea: replace cont with standard lists:

```
type tag = SUB2 of int   (* K1: k (a+b) *)
         | PLUS of int   (* K2: fib_cps (m-2) (k2 a) *)

type tag_list_cont = tag list
```

fib_cps_defun revisited, using lists for continuations


```
type tag = SUB2 of int | PLUS of int
type tag_list_cont = tag list

let rec apply_tag_list_cont k v = match k, v with
  | [], a → a
  | SUB2 m :: k, a → fib_cps_defun_tags (m−2) (PLUS a :: k)
  | PLUS a :: k, b → apply_tag_list_cont k (a+b)

and fib_cps_defun_tags m k =
  if m = 0 then apply_tag_list_cont k 1
  else if m = 1 then apply_tag_list_cont k 1
  else fib_cps_defun_tags (m−1) (SUB2 m :: k)

let fib_3 m = fib_cps_defun_tags m []
```

Mutual recursion

Mutual recursion $\rightsquigarrow$ single recursion

Mutual recursion can be eliminated using **indexing**.

Given a set of mutually-recursive functions:

```
let rec is_even n = n = 0 || is_odd (n - 1)
and is_odd n = n <> 0 && is_even (n - 1)
```

define an index datatype with one constructor for each function:

```
type eo = Even | Odd
```

and define a function that maps an index argument to a corresponding body:

```
let rec is f n =
  match f with
  | Even → n = 0 || is Odd (n - 1)
  | Odd → n <> 0 && is Even (n - 1)
```

Mutual recursion $\rightsquigarrow$ single recursion for `fib`


```
type state_type =
| FIB (* for right-hand-sides starting with fib_ *)
| APP (* for right-hand-sides starting with apply_ *)

type state = (state_type * int * tag_list_cont) → int

(* eval acts as either apply_tag_list_cont or fib_cps_defun_tags *)
let rec eval = function
| FIB, 0, k → eval (APP, 1, k)
| FIB, 1, k → eval (APP, 1, k)
| FIB, m, k → eval (FIB, m-1, SUB2 m :: k)
| APP, a, SUB2 m :: k → eval (FIB, m-2, PLUS a :: k)
| APP, b, PLUS a :: k → eval (APP, a+b, k)
| APP, a, [] → a

let fib_4 m = eval (FIB, m, [])
```

Eliminate tail recursion to obtain *The Fibonacci Machine*

CPS

D17n

CPS + D17n

Mutual recursion

```
(* step : state → state *)
let step = function
| FIB, 0, k → (APP, 1, k)
| FIB, 1, k → (APP, 1, k)
| FIB, m, k → (FIB, m−1, SUB2 m :: k)
| APP, a, SUB2 m :: k → (FIB, m−2, PLUS a :: k)
| APP, b, PLUS a :: k → (APP, a+b, k)
| _ → failwith "step : runtime error!"

let rec driver state = function (* clearly tail recursive! *)
| APP, a, [] → a
| state → driver (step state)

(* fib_5 : int → int *)
let fib_5 m = driver (FIB, m, [])
```

(This version makes the tail-recursive structure very explicit.)

Tracing of fib_5 6

```
1   FIB   6   []
```

Tracing of fib_5 6

```
1   FIB   6   []
2   FIB   5   [SUB2 6]
```

Tracing of fib_5 6

```
1   FIB   6   []
2   FIB   5   [SUB2 6]
3   FIB   4   [SUB2 6,SUB2 5]
```

Tracing of fib_5 6


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1   FIB   6   []
2   FIB   5   [SUB2 6]
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```

Tracing of fib_5 6

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1   FIB   6   []
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3   FIB   4   [SUB2 6,SUB2 5]
4   FIB   3   [SUB2 6,SUB2 5,SUB2 4]
5   FIB   2   [SUB2 6,SUB2 5,SUB2 4,SUB2 3]
```

Tracing of fib_5 6


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```

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4   FIB   3   [SUB2 6,SUB2 5,SUB2 4]
5   FIB   2   [SUB2 6,SUB2 5,SUB2 4,SUB2 3]
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7   APP   1   [SUB2 6,SUB2 5,SUB2 4,SUB2 3,SUB2 2]
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Tracing of fib_5 6

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1   FIB   6   []
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6   FIB   1   [SUB2 6,SUB2 5,SUB2 4,SUB2 3,SUB2 2]
7   APP   1   [SUB2 6,SUB2 5,SUB2 4,SUB2 3,SUB2 2]
8   FIB   0   [SUB2 6,SUB2 5,SUB2 4,SUB2 3,PLUS 1]
```

Tracing of fib_5 6


```
1   FIB   6   []
2   FIB   5   [SUB2 6]
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 9   APP   1   [SUB2 6,SUB2 5,SUB2 4,SUB2 3,PLUS 1]
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```

Tracing of fib_5 6


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Tracing of fib_5 6


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```

Tracing of fib_5 6


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12   APP   1   [SUB2 6,SUB2 5,SUB2 4,PLUS 2]
13   APP   3   [SUB2 6,SUB2 5,SUB2 4]
```

Tracing of fib_5 6


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Tracing of fib_5 6


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Tracing of fib_5 6


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```

Tracing of fib_5 6


```
 1   FIB   6   []
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 3   FIB   4   [SUB2 6,SUB2 5]
 4   FIB   3   [SUB2 6,SUB2 5,SUB2 4]
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 9   APP   1   [SUB2 6,SUB2 5,SUB2 4,SUB2 3,PLUS 1]
10   APP   2   [SUB2 6,SUB2 5,SUB2 4,SUB2 3]
11   FIB   1   [SUB2 6,SUB2 5,SUB2 4,PLUS 2]
12   APP   1   [SUB2 6,SUB2 5,SUB2 4,PLUS 2]
13   APP   3   [SUB2 6,SUB2 5,SUB2 4]
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15   FIB   1   [SUB2 6,SUB2 5,PLUS 3,SUB2 2]
16   APP   1   [SUB2 6,SUB2 5,PLUS 3,SUB2 2]
17   FIB   0   [SUB2 6,SUB2 5,PLUS 3,PLUS 1]
18   APP   1   [SUB2 6,SUB2 5,PLUS 3,PLUS 1]
19   APP   2   [SUB2 6,SUB2 5,PLUS 3]
```

Tracing of fib_5 6


```
 1   FIB   6   []
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16   APP   1   [SUB2 6,SUB2 5,PLUS 3,SUB2 2]
17   FIB   0   [SUB2 6,SUB2 5,PLUS 3,PLUS 1]
18   APP   1   [SUB2 6,SUB2 5,PLUS 3,PLUS 1]
19   APP   2   [SUB2 6,SUB2 5,PLUS 3]
20   APP   5   [SUB2 6,SUB2 5]
```

Tracing of fib_5 6


```
 1   FIB   6   []
 2   FIB   5   [SUB2 6]
 3   FIB   4   [SUB2 6,SUB2 5]
 4   FIB   3   [SUB2 6,SUB2 5,SUB2 4]
 5   FIB   2   [SUB2 6,SUB2 5,SUB2 4,SUB2 3]
 6   FIB   1   [SUB2 6,SUB2 5,SUB2 4,SUB2 3,SUB2 2]
 7   APP   1   [SUB2 6,SUB2 5,SUB2 4,SUB2 3,SUB2 2]
 8   FIB   0   [SUB2 6,SUB2 5,SUB2 4,SUB2 3,PLUS 1]
 9   APP   1   [SUB2 6,SUB2 5,SUB2 4,SUB2 3,PLUS 1]
10   APP   2   [SUB2 6,SUB2 5,SUB2 4,SUB2 3]
11   FIB   1   [SUB2 6,SUB2 5,SUB2 4,PLUS 2]
12   APP   1   [SUB2 6,SUB2 5,SUB2 4,PLUS 2]
13   APP   3   [SUB2 6,SUB2 5,SUB2 4]
14   FIB   2   [SUB2 6,SUB2 5,PLUS 3]
15   FIB   1   [SUB2 6,SUB2 5,PLUS 3,SUB2 2]
16   APP   1   [SUB2 6,SUB2 5,PLUS 3,SUB2 2]
17   FIB   0   [SUB2 6,SUB2 5,PLUS 3,PLUS 1]
18   APP   1   [SUB2 6,SUB2 5,PLUS 3,PLUS 1]
19   APP   2   [SUB2 6,SUB2 5,PLUS 3]
20   APP   5   [SUB2 6,SUB2 5]
21   FIB   3   [SUB2 6,PLUS 5]
```

Tracing of fib_5 6


```
 1   FIB   6   []
 2   FIB   5   [SUB2 6]
 3   FIB   4   [SUB2 6,SUB2 5]
 4   FIB   3   [SUB2 6,SUB2 5,SUB2 4]
 5   FIB   2   [SUB2 6,SUB2 5,SUB2 4,SUB2 3]
 6   FIB   1   [SUB2 6,SUB2 5,SUB2 4,SUB2 3,SUB2 2]
 7   APP   1   [SUB2 6,SUB2 5,SUB2 4,SUB2 3,SUB2 2]
 8   FIB   0   [SUB2 6,SUB2 5,SUB2 4,SUB2 3,PLUS 1]
 9   APP   1   [SUB2 6,SUB2 5,SUB2 4,SUB2 3,PLUS 1]
10   APP   2   [SUB2 6,SUB2 5,SUB2 4,SUB2 3]
11   FIB   1   [SUB2 6,SUB2 5,SUB2 4,PLUS 2]
12   APP   1   [SUB2 6,SUB2 5,SUB2 4,PLUS 2]
13   APP   3   [SUB2 6,SUB2 5,SUB2 4]
14   FIB   2   [SUB2 6,SUB2 5,PLUS 3]
15   FIB   1   [SUB2 6,SUB2 5,PLUS 3,SUB2 2]
16   APP   1   [SUB2 6,SUB2 5,PLUS 3,SUB2 2]
17   FIB   0   [SUB2 6,SUB2 5,PLUS 3,PLUS 1]
18   APP   1   [SUB2 6,SUB2 5,PLUS 3,PLUS 1]
19   APP   2   [SUB2 6,SUB2 5,PLUS 3]
20   APP   5   [SUB2 6,SUB2 5]
21   FIB   3   [SUB2 6,PLUS 5]
22   FIB   2   [SUB2 6,PLUS 5,SUB2 3]
```

Tracing of fib_5 6


```
 1   FIB   6   []
 2   FIB   5   [SUB2 6]
 3   FIB   4   [SUB2 6,SUB2 5]
 4   FIB   3   [SUB2 6,SUB2 5,SUB2 4]
 5   FIB   2   [SUB2 6,SUB2 5,SUB2 4,SUB2 3]
 6   FIB   1   [SUB2 6,SUB2 5,SUB2 4,SUB2 3,SUB2 2]
 7   APP   1   [SUB2 6,SUB2 5,SUB2 4,SUB2 3,SUB2 2]
 8   FIB   0   [SUB2 6,SUB2 5,SUB2 4,SUB2 3,PLUS 1]
 9   APP   1   [SUB2 6,SUB2 5,SUB2 4,SUB2 3,PLUS 1]
10   APP   2   [SUB2 6,SUB2 5,SUB2 4,SUB2 3]
11   FIB   1   [SUB2 6,SUB2 5,SUB2 4,PLUS 2]
12   APP   1   [SUB2 6,SUB2 5,SUB2 4,PLUS 2]
13   APP   3   [SUB2 6,SUB2 5,SUB2 4]
14   FIB   2   [SUB2 6,SUB2 5,PLUS 3]
15   FIB   1   [SUB2 6,SUB2 5,PLUS 3,SUB2 2]
16   APP   1   [SUB2 6,SUB2 5,PLUS 3,SUB2 2]
17   FIB   0   [SUB2 6,SUB2 5,PLUS 3,PLUS 1]
18   APP   1   [SUB2 6,SUB2 5,PLUS 3,PLUS 1]
19   APP   2   [SUB2 6,SUB2 5,PLUS 3]
20   APP   5   [SUB2 6,SUB2 5]
21   FIB   3   [SUB2 6,PLUS 5]
22   FIB   2   [SUB2 6,PLUS 5,SUB2 3]
23   FIB   1   [SUB2 6,PLUS 5,SUB2 3,SUB2 2]
```

Tracing of fib_5 6


```
 1   FIB   6   []
 2   FIB   5   [SUB2 6]
 3   FIB   4   [SUB2 6,SUB2 5]
 4   FIB   3   [SUB2 6,SUB2 5,SUB2 4]
 5   FIB   2   [SUB2 6,SUB2 5,SUB2 4,SUB2 3]
 6   FIB   1   [SUB2 6,SUB2 5,SUB2 4,SUB2 3,SUB2 2]
 7   APP   1   [SUB2 6,SUB2 5,SUB2 4,SUB2 3,SUB2 2]
 8   FIB   0   [SUB2 6,SUB2 5,SUB2 4,SUB2 3,PLUS 1]
 9   APP   1   [SUB2 6,SUB2 5,SUB2 4,SUB2 3,PLUS 1]
10   APP   2   [SUB2 6,SUB2 5,SUB2 4,SUB2 3]
11   FIB   1   [SUB2 6,SUB2 5,SUB2 4,PLUS 2]
12   APP   1   [SUB2 6,SUB2 5,SUB2 4,PLUS 2]
13   APP   3   [SUB2 6,SUB2 5,SUB2 4]
14   FIB   2   [SUB2 6,SUB2 5,PLUS 3]
15   FIB   1   [SUB2 6,SUB2 5,PLUS 3,SUB2 2]
16   APP   1   [SUB2 6,SUB2 5,PLUS 3,SUB2 2]
17   FIB   0   [SUB2 6,SUB2 5,PLUS 3,PLUS 1]
18   APP   1   [SUB2 6,SUB2 5,PLUS 3,PLUS 1]
19   APP   2   [SUB2 6,SUB2 5,PLUS 3]
20   APP   5   [SUB2 6,SUB2 5]
21   FIB   3   [SUB2 6,PLUS 5]
22   FIB   2   [SUB2 6,PLUS 5,SUB2 3]
23   FIB   1   [SUB2 6,PLUS 5,SUB2 3,SUB2 2]
24   APP   1   [SUB2 6,PLUS 5,SUB2 3,SUB2 2]
```

Tracing of fib_5 6


```
 1   FIB   6   []
 2   FIB   5   [SUB2 6]
 3   FIB   4   [SUB2 6,SUB2 5]
 4   FIB   3   [SUB2 6,SUB2 5,SUB2 4]
 5   FIB   2   [SUB2 6,SUB2 5,SUB2 4,SUB2 3]
 6   FIB   1   [SUB2 6,SUB2 5,SUB2 4,SUB2 3,SUB2 2]
 7   APP   1   [SUB2 6,SUB2 5,SUB2 4,SUB2 3,SUB2 2]
 8   FIB   0   [SUB2 6,SUB2 5,SUB2 4,SUB2 3,PLUS 1]
 9   APP   1   [SUB2 6,SUB2 5,SUB2 4,SUB2 3,PLUS 1]
10   APP   2   [SUB2 6,SUB2 5,SUB2 4,SUB2 3]
11   FIB   1   [SUB2 6,SUB2 5,SUB2 4,PLUS 2]
12   APP   1   [SUB2 6,SUB2 5,SUB2 4,PLUS 2]
13   APP   3   [SUB2 6,SUB2 5,SUB2 4]
14   FIB   2   [SUB2 6,SUB2 5,PLUS 3]
15   FIB   1   [SUB2 6,SUB2 5,PLUS 3,SUB2 2]
16   APP   1   [SUB2 6,SUB2 5,PLUS 3,SUB2 2]
17   FIB   0   [SUB2 6,SUB2 5,PLUS 3,PLUS 1]
18   APP   1   [SUB2 6,SUB2 5,PLUS 3,PLUS 1]
19   APP   2   [SUB2 6,SUB2 5,PLUS 3]
20   APP   5   [SUB2 6,SUB2 5]
21   FIB   3   [SUB2 6,PLUS 5]
22   FIB   2   [SUB2 6,PLUS 5,SUB2 3]
23   FIB   1   [SUB2 6,PLUS 5,SUB2 3,SUB2 2]
24   APP   1   [SUB2 6,PLUS 5,SUB2 3,SUB2 2]
25   FIB   0   [SUB2 6,PLUS 5,SUB2 3,PLUS 1]
```

Tracing of fib_5 6


```
 1   FIB   6   []                                          26   APP   1   [SUB2 6,PLUS 5,SUB2 3,PLUS 1]
 2   FIB   5   [SUB2 6]
 3   FIB   4   [SUB2 6,SUB2 5]
 4   FIB   3   [SUB2 6,SUB2 5,SUB2 4]
 5   FIB   2   [SUB2 6,SUB2 5,SUB2 4,SUB2 3]
 6   FIB   1   [SUB2 6,SUB2 5,SUB2 4,SUB2 3,SUB2 2]
 7   APP   1   [SUB2 6,SUB2 5,SUB2 4,SUB2 3,SUB2 2]
 8   FIB   0   [SUB2 6,SUB2 5,SUB2 4,SUB2 3,PLUS 1]
 9   APP   1   [SUB2 6,SUB2 5,SUB2 4,SUB2 3,PLUS 1]
10   APP   2   [SUB2 6,SUB2 5,SUB2 4,SUB2 3]
11   FIB   1   [SUB2 6,SUB2 5,SUB2 4,PLUS 2]
12   APP   1   [SUB2 6,SUB2 5,SUB2 4,PLUS 2]
13   APP   3   [SUB2 6,SUB2 5,SUB2 4]
14   FIB   2   [SUB2 6,SUB2 5,PLUS 3]
15   FIB   1   [SUB2 6,SUB2 5,PLUS 3,SUB2 2]
16   APP   1   [SUB2 6,SUB2 5,PLUS 3,SUB2 2]
17   FIB   0   [SUB2 6,SUB2 5,PLUS 3,PLUS 1]
18   APP   1   [SUB2 6,SUB2 5,PLUS 3,PLUS 1]
19   APP   2   [SUB2 6,SUB2 5,PLUS 3]
20   APP   5   [SUB2 6,SUB2 5]
21   FIB   3   [SUB2 6,PLUS 5]
22   FIB   2   [SUB2 6,PLUS 5,SUB2 3]
23   FIB   1   [SUB2 6,PLUS 5,SUB2 3,SUB2 2]
24   APP   1   [SUB2 6,PLUS 5,SUB2 3,SUB2 2]
25   FIB   0   [SUB2 6,PLUS 5,SUB2 3,PLUS 1]
```

Tracing of fib_5 6


```
 1   FIB   6   []
 2   FIB   5   [SUB2 6]
 3   FIB   4   [SUB2 6,SUB2 5]
 4   FIB   3   [SUB2 6,SUB2 5,SUB2 4]
 5   FIB   2   [SUB2 6,SUB2 5,SUB2 4,SUB2 3]
 6   FIB   1   [SUB2 6,SUB2 5,SUB2 4,SUB2 3,SUB2 2]
 7   APP   1   [SUB2 6,SUB2 5,SUB2 4,SUB2 3,SUB2 2]
 8   FIB   0   [SUB2 6,SUB2 5,SUB2 4,SUB2 3,PLUS 1]
 9   APP   1   [SUB2 6,SUB2 5,SUB2 4,SUB2 3,PLUS 1]
10   APP   2   [SUB2 6,SUB2 5,SUB2 4,SUB2 3]
11   FIB   1   [SUB2 6,SUB2 5,SUB2 4,PLUS 2]
12   APP   1   [SUB2 6,SUB2 5,SUB2 4,PLUS 2]
13   APP   3   [SUB2 6,SUB2 5,SUB2 4]
14   FIB   2   [SUB2 6,SUB2 5,PLUS 3]
15   FIB   1   [SUB2 6,SUB2 5,PLUS 3,SUB2 2]
16   APP   1   [SUB2 6,SUB2 5,PLUS 3,SUB2 2]
17   FIB   0   [SUB2 6,SUB2 5,PLUS 3,PLUS 1]
18   APP   1   [SUB2 6,SUB2 5,PLUS 3,PLUS 1]
19   APP   2   [SUB2 6,SUB2 5,PLUS 3]
20   APP   5   [SUB2 6,SUB2 5]
21   FIB   3   [SUB2 6,PLUS 5]
22   FIB   2   [SUB2 6,PLUS 5,SUB2 3]
23   FIB   1   [SUB2 6,PLUS 5,SUB2 3,SUB2 2]
24   APP   1   [SUB2 6,PLUS 5,SUB2 3,SUB2 2]
25   FIB   0   [SUB2 6,PLUS 5,SUB2 3,PLUS 1]
26   APP   1   [SUB2 6,PLUS 5,SUB2 3,PLUS 1]
27   APP   2   [SUB2 6,PLUS 5,SUB2 3]
```

Tracing of fib_5 6


```
 1   FIB   6   []
 2   FIB   5   [SUB2 6]
 3   FIB   4   [SUB2 6,SUB2 5]
 4   FIB   3   [SUB2 6,SUB2 5,SUB2 4]
 5   FIB   2   [SUB2 6,SUB2 5,SUB2 4,SUB2 3]
 6   FIB   1   [SUB2 6,SUB2 5,SUB2 4,SUB2 3,SUB2 2]
 7   APP   1   [SUB2 6,SUB2 5,SUB2 4,SUB2 3,SUB2 2]
 8   FIB   0   [SUB2 6,SUB2 5,SUB2 4,SUB2 3,PLUS 1]
 9   APP   1   [SUB2 6,SUB2 5,SUB2 4,SUB2 3,PLUS 1]
10   APP   2   [SUB2 6,SUB2 5,SUB2 4,SUB2 3]
11   FIB   1   [SUB2 6,SUB2 5,SUB2 4,PLUS 2]
12   APP   1   [SUB2 6,SUB2 5,SUB2 4,PLUS 2]
13   APP   3   [SUB2 6,SUB2 5,SUB2 4]
14   FIB   2   [SUB2 6,SUB2 5,PLUS 3]
15   FIB   1   [SUB2 6,SUB2 5,PLUS 3,SUB2 2]
16   APP   1   [SUB2 6,SUB2 5,PLUS 3,SUB2 2]
17   FIB   0   [SUB2 6,SUB2 5,PLUS 3,PLUS 1]
18   APP   1   [SUB2 6,SUB2 5,PLUS 3,PLUS 1]
19   APP   2   [SUB2 6,SUB2 5,PLUS 3]
20   APP   5   [SUB2 6,SUB2 5]
21   FIB   3   [SUB2 6,PLUS 5]
22   FIB   2   [SUB2 6,PLUS 5,SUB2 3]
23   FIB   1   [SUB2 6,PLUS 5,SUB2 3,SUB2 2]
24   APP   1   [SUB2 6,PLUS 5,SUB2 3,SUB2 2]
25   FIB   0   [SUB2 6,PLUS 5,SUB2 3,PLUS 1]
26   APP   1   [SUB2 6,PLUS 5,SUB2 3,PLUS 1]
27   APP   2   [SUB2 6,PLUS 5,SUB2 3]
28   FIB   1   [SUB2 6,PLUS 5,PLUS 2]
```

Tracing of fib_5 6

1	FIB	6	[]
2	FIB	5	[SUB2 6]
3	FIB	4	[SUB2 6,SUB2 5]
4	FIB	3	[SUB2 6,SUB2 5,SUB2 4]
5	FIB	2	[SUB2 6,SUB2 5,SUB2 4,SUB2 3]
6	FIB	1	[SUB2 6,SUB2 5,SUB2 4,SUB2 3,SUB2 2]
7	APP	1	[SUB2 6,SUB2 5,SUB2 4,SUB2 3,SUB2 2]
8	FIB	0	[SUB2 6,SUB2 5,SUB2 4,SUB2 3,PLUS 1]
9	APP	1	[SUB2 6,SUB2 5,SUB2 4,SUB2 3,PLUS 1]
10	APP	2	[SUB2 6,SUB2 5,SUB2 4,SUB2 3]
11	FIB	1	[SUB2 6,SUB2 5,SUB2 4,PLUS 2]
12	APP	1	[SUB2 6,SUB2 5,SUB2 4,PLUS 2]
13	APP	3	[SUB2 6,SUB2 5,SUB2 4]
14	FIB	2	[SUB2 6,SUB2 5,PLUS 3]
15	FIB	1	[SUB2 6,SUB2 5,PLUS 3,SUB2 2]
16	APP	1	[SUB2 6,SUB2 5,PLUS 3,SUB2 2]
17	FIB	0	[SUB2 6,SUB2 5,PLUS 3,PLUS 1]
18	APP	1	[SUB2 6,SUB2 5,PLUS 3,PLUS 1]
19	APP	2	[SUB2 6,SUB2 5,PLUS 3]
20	APP	5	[SUB2 6,SUB2 5]
21	FIB	3	[SUB2 6,PLUS 5]
22	FIB	2	[SUB2 6,PLUS 5,SUB2 3]
23	FIB	1	[SUB2 6,PLUS 5,SUB2 3,SUB2 2]
24	APP	1	[SUB2 6,PLUS 5,SUB2 3,SUB2 2]
25	FIB	0	[SUB2 6,PLUS 5,SUB2 3,PLUS 1]
26	APP	1	[SUB2 6,PLUS 5,SUB2 3,PLUS 1]
27	APP	2	[SUB2 6,PLUS 5,SUB2 3]
28	FIB	1	[SUB2 6,PLUS 5,PLUS 2]
29	APP	1	[SUB2 6,PLUS 5,PLUS 2]

Tracing of fib_5 6

1	FIB	6	[]
2	FIB	5	[SUB2 6]
3	FIB	4	[SUB2 6,SUB2 5]
4	FIB	3	[SUB2 6,SUB2 5,SUB2 4]
5	FIB	2	[SUB2 6,SUB2 5,SUB2 4,SUB2 3]
6	FIB	1	[SUB2 6,SUB2 5,SUB2 4,SUB2 3,SUB2 2]
7	APP	1	[SUB2 6,SUB2 5,SUB2 4,SUB2 3,SUB2 2]
8	FIB	0	[SUB2 6,SUB2 5,SUB2 4,SUB2 3,PLUS 1]
9	APP	1	[SUB2 6,SUB2 5,SUB2 4,SUB2 3,PLUS 1]
10	APP	2	[SUB2 6,SUB2 5,SUB2 4,SUB2 3]
11	FIB	1	[SUB2 6,SUB2 5,SUB2 4,PLUS 2]
12	APP	1	[SUB2 6,SUB2 5,SUB2 4,PLUS 2]
13	APP	3	[SUB2 6,SUB2 5,SUB2 4]
14	FIB	2	[SUB2 6,SUB2 5,PLUS 3]
15	FIB	1	[SUB2 6,SUB2 5,PLUS 3,SUB2 2]
16	APP	1	[SUB2 6,SUB2 5,PLUS 3,SUB2 2]
17	FIB	0	[SUB2 6,SUB2 5,PLUS 3,PLUS 1]
18	APP	1	[SUB2 6,SUB2 5,PLUS 3,PLUS 1]
19	APP	2	[SUB2 6,SUB2 5,PLUS 3]
20	APP	5	[SUB2 6,SUB2 5]
21	FIB	3	[SUB2 6,PLUS 5]
22	FIB	2	[SUB2 6,PLUS 5,SUB2 3]
23	FIB	1	[SUB2 6,PLUS 5,SUB2 3,SUB2 2]
24	APP	1	[SUB2 6,PLUS 5,SUB2 3,SUB2 2]
25	FIB	0	[SUB2 6,PLUS 5,SUB2 3,PLUS 1]
26	APP	1	[SUB2 6,PLUS 5,SUB2 3,PLUS 1]
27	APP	2	[SUB2 6,PLUS 5,SUB2 3]
28	FIB	1	[SUB2 6,PLUS 5,PLUS 2]
29	APP	1	[SUB2 6,PLUS 5,PLUS 2]
30	APP	3	[SUB2 6,PLUS 5]

Tracing of fib_5 6


```
 1  FIB  6  []
 2  FIB  5  [SUB2 6]
 3  FIB  4  [SUB2 6,SUB2 5]
 4  FIB  3  [SUB2 6,SUB2 5,SUB2 4]
 5  FIB  2  [SUB2 6,SUB2 5,SUB2 4,SUB2 3]
 6  FIB  1  [SUB2 6,SUB2 5,SUB2 4,SUB2 3,SUB2 2]
 7  APP  1  [SUB2 6,SUB2 5,SUB2 4,SUB2 3,SUB2 2]
 8  FIB  0  [SUB2 6,SUB2 5,SUB2 4,SUB2 3,PLUS 1]
 9  APP  1  [SUB2 6,SUB2 5,SUB2 4,SUB2 3,PLUS 1]
10  APP  2  [SUB2 6,SUB2 5,SUB2 4,SUB2 3]
11  FIB  1  [SUB2 6,SUB2 5,SUB2 4,PLUS 2]
12  APP  1  [SUB2 6,SUB2 5,SUB2 4,PLUS 2]
13  APP  3  [SUB2 6,SUB2 5,SUB2 4]
14  FIB  2  [SUB2 6,SUB2 5,PLUS 3]
15  FIB  1  [SUB2 6,SUB2 5,PLUS 3,SUB2 2]
16  APP  1  [SUB2 6,SUB2 5,PLUS 3,SUB2 2]
17  FIB  0  [SUB2 6,SUB2 5,PLUS 3,PLUS 1]
18  APP  1  [SUB2 6,SUB2 5,PLUS 3,PLUS 1]
19  APP  2  [SUB2 6,SUB2 5,PLUS 3]
20  APP  5  [SUB2 6,SUB2 5]
21  FIB  3  [SUB2 6,PLUS 5]
22  FIB  2  [SUB2 6,PLUS 5,SUB2 3]
23  FIB  1  [SUB2 6,PLUS 5,SUB2 3,SUB2 2]
24  APP  1  [SUB2 6,PLUS 5,SUB2 3,SUB2 2]
25  FIB  0  [SUB2 6,PLUS 5,SUB2 3,PLUS 1]
26  APP  1  [SUB2 6,PLUS 5,SUB2 3,PLUS 1]
27  APP  2  [SUB2 6,PLUS 5,SUB2 3]
28  FIB  1  [SUB2 6,PLUS 5,PLUS 2]
29  APP  1  [SUB2 6,PLUS 5,PLUS 2]
30  APP  3  [SUB2 6,PLUS 5]
31  APP  8  [SUB2 6]
```

Tracing of fib_5 6


```
 1  FIB  6  []
 2  FIB  5  [SUB2 6]
 3  FIB  4  [SUB2 6,SUB2 5]
 4  FIB  3  [SUB2 6,SUB2 5,SUB2 4]
 5  FIB  2  [SUB2 6,SUB2 5,SUB2 4,SUB2 3]
 6  FIB  1  [SUB2 6,SUB2 5,SUB2 4,SUB2 3,SUB2 2]
 7  APP  1  [SUB2 6,SUB2 5,SUB2 4,SUB2 3,SUB2 2]
 8  FIB  0  [SUB2 6,SUB2 5,SUB2 4,SUB2 3,PLUS 1]
 9  APP  1  [SUB2 6,SUB2 5,SUB2 4,SUB2 3,PLUS 1]
10  APP  2  [SUB2 6,SUB2 5,SUB2 4,SUB2 3]
11  FIB  1  [SUB2 6,SUB2 5,SUB2 4,PLUS 2]
12  APP  1  [SUB2 6,SUB2 5,SUB2 4,PLUS 2]
13  APP  3  [SUB2 6,SUB2 5,SUB2 4]
14  FIB  2  [SUB2 6,SUB2 5,PLUS 3]
15  FIB  1  [SUB2 6,SUB2 5,PLUS 3,SUB2 2]
16  APP  1  [SUB2 6,SUB2 5,PLUS 3,SUB2 2]
17  FIB  0  [SUB2 6,SUB2 5,PLUS 3,PLUS 1]
18  APP  1  [SUB2 6,SUB2 5,PLUS 3,PLUS 1]
19  APP  2  [SUB2 6,SUB2 5,PLUS 3]
20  APP  5  [SUB2 6,SUB2 5]
21  FIB  3  [SUB2 6,PLUS 5]
22  FIB  2  [SUB2 6,PLUS 5,SUB2 3]
23  FIB  1  [SUB2 6,PLUS 5,SUB2 3,SUB2 2]
24  APP  1  [SUB2 6,PLUS 5,SUB2 3,SUB2 2]
25  FIB  0  [SUB2 6,PLUS 5,SUB2 3,PLUS 1]
26  APP  1  [SUB2 6,PLUS 5,SUB2 3,PLUS 1]
27  APP  2  [SUB2 6,PLUS 5,SUB2 3]
28  FIB  1  [SUB2 6,PLUS 5,PLUS 2]
29  APP  1  [SUB2 6,PLUS 5,PLUS 2]
30  APP  3  [SUB2 6,PLUS 5]
31  APP  8  [SUB2 6]
32  FIB  4  [PLUS 8]
```

Tracing of fib_5 6


```
 1  FIB  6  []
 2  FIB  5  [SUB2 6]
 3  FIB  4  [SUB2 6,SUB2 5]
 4  FIB  3  [SUB2 6,SUB2 5,SUB2 4]
 5  FIB  2  [SUB2 6,SUB2 5,SUB2 4,SUB2 3]
 6  FIB  1  [SUB2 6,SUB2 5,SUB2 4,SUB2 3,SUB2 2]
 7  APP  1  [SUB2 6,SUB2 5,SUB2 4,SUB2 3,SUB2 2]
 8  FIB  0  [SUB2 6,SUB2 5,SUB2 4,SUB2 3,PLUS 1]
 9  APP  1  [SUB2 6,SUB2 5,SUB2 4,SUB2 3,PLUS 1]
10  APP  2  [SUB2 6,SUB2 5,SUB2 4,SUB2 3]
11  FIB  1  [SUB2 6,SUB2 5,SUB2 4,PLUS 2]
12  APP  1  [SUB2 6,SUB2 5,SUB2 4,PLUS 2]
13  APP  3  [SUB2 6,SUB2 5,SUB2 4]
14  FIB  2  [SUB2 6,SUB2 5,PLUS 3]
15  FIB  1  [SUB2 6,SUB2 5,PLUS 3,SUB2 2]
16  APP  1  [SUB2 6,SUB2 5,PLUS 3,SUB2 2]
17  FIB  0  [SUB2 6,SUB2 5,PLUS 3,PLUS 1]
18  APP  1  [SUB2 6,SUB2 5,PLUS 3,PLUS 1]
19  APP  2  [SUB2 6,SUB2 5,PLUS 3]
20  APP  5  [SUB2 6,SUB2 5]
21  FIB  3  [SUB2 6,PLUS 5]
22  FIB  2  [SUB2 6,PLUS 5,SUB2 3]
23  FIB  1  [SUB2 6,PLUS 5,SUB2 3,SUB2 2]
24  APP  1  [SUB2 6,PLUS 5,SUB2 3,SUB2 2]
25  FIB  0  [SUB2 6,PLUS 5,SUB2 3,PLUS 1]
26  APP  1  [SUB2 6,PLUS 5,SUB2 3,PLUS 1]
27  APP  2  [SUB2 6,PLUS 5,SUB2 3]
28  FIB  1  [SUB2 6,PLUS 5,PLUS 2]
29  APP  1  [SUB2 6,PLUS 5,PLUS 2]
30  APP  3  [SUB2 6,PLUS 5]
31  APP  8  [SUB2 6]
32  FIB  4  [PLUS 8]
33  FIB  3  [PLUS 8,SUB2 4]
```

Tracing of fib_5 6


```
 1  FIB  6  []
 2  FIB  5  [SUB2 6]
 3  FIB  4  [SUB2 6,SUB2 5]
 4  FIB  3  [SUB2 6,SUB2 5,SUB2 4]
 5  FIB  2  [SUB2 6,SUB2 5,SUB2 4,SUB2 3]
 6  FIB  1  [SUB2 6,SUB2 5,SUB2 4,SUB2 3,SUB2 2]
 7  APP  1  [SUB2 6,SUB2 5,SUB2 4,SUB2 3,SUB2 2]
 8  FIB  0  [SUB2 6,SUB2 5,SUB2 4,SUB2 3,PLUS 1]
 9  APP  1  [SUB2 6,SUB2 5,SUB2 4,SUB2 3,PLUS 1]
10  APP  2  [SUB2 6,SUB2 5,SUB2 4,SUB2 3]
11  FIB  1  [SUB2 6,SUB2 5,SUB2 4,PLUS 2]
12  APP  1  [SUB2 6,SUB2 5,SUB2 4,PLUS 2]
13  APP  3  [SUB2 6,SUB2 5,SUB2 4]
14  FIB  2  [SUB2 6,SUB2 5,PLUS 3]
15  FIB  1  [SUB2 6,SUB2 5,PLUS 3,SUB2 2]
16  APP  1  [SUB2 6,SUB2 5,PLUS 3,SUB2 2]
17  FIB  0  [SUB2 6,SUB2 5,PLUS 3,PLUS 1]
18  APP  1  [SUB2 6,SUB2 5,PLUS 3,PLUS 1]
19  APP  2  [SUB2 6,SUB2 5,PLUS 3]
20  APP  5  [SUB2 6,SUB2 5]
21  FIB  3  [SUB2 6,PLUS 5]
22  FIB  2  [SUB2 6,PLUS 5,SUB2 3]
23  FIB  1  [SUB2 6,PLUS 5,SUB2 3,SUB2 2]
24  APP  1  [SUB2 6,PLUS 5,SUB2 3,SUB2 2]
25  FIB  0  [SUB2 6,PLUS 5,SUB2 3,PLUS 1]
26  APP  1  [SUB2 6,PLUS 5,SUB2 3,PLUS 1]
27  APP  2  [SUB2 6,PLUS 5,SUB2 3]
28  FIB  1  [SUB2 6,PLUS 5,PLUS 2]
29  APP  1  [SUB2 6,PLUS 5,PLUS 2]
30  APP  3  [SUB2 6,PLUS 5]
31  APP  8  [SUB2 6]
32  FIB  4  [PLUS 8]
33  FIB  3  [PLUS 8,SUB2 4]
34  FIB  2  [PLUS 8,SUB2 4,SUB2 3]
```

Tracing of fib_5 6


```
 1  FIB  6  []
 2  FIB  5  [SUB2 6]
 3  FIB  4  [SUB2 6,SUB2 5]
 4  FIB  3  [SUB2 6,SUB2 5,SUB2 4]
 5  FIB  2  [SUB2 6,SUB2 5,SUB2 4,SUB2 3]
 6  FIB  1  [SUB2 6,SUB2 5,SUB2 4,SUB2 3,SUB2 2]
 7  APP  1  [SUB2 6,SUB2 5,SUB2 4,SUB2 3,SUB2 2]
 8  FIB  0  [SUB2 6,SUB2 5,SUB2 4,SUB2 3,PLUS 1]
 9  APP  1  [SUB2 6,SUB2 5,SUB2 4,SUB2 3,PLUS 1]
10  APP  2  [SUB2 6,SUB2 5,SUB2 4,SUB2 3]
11  FIB  1  [SUB2 6,SUB2 5,SUB2 4,PLUS 2]
12  APP  1  [SUB2 6,SUB2 5,SUB2 4,PLUS 2]
13  APP  3  [SUB2 6,SUB2 5,SUB2 4]
14  FIB  2  [SUB2 6,SUB2 5,PLUS 3]
15  FIB  1  [SUB2 6,SUB2 5,PLUS 3,SUB2 2]
16  APP  1  [SUB2 6,SUB2 5,PLUS 3,SUB2 2]
17  FIB  0  [SUB2 6,SUB2 5,PLUS 3,PLUS 1]
18  APP  1  [SUB2 6,SUB2 5,PLUS 3,PLUS 1]
19  APP  2  [SUB2 6,SUB2 5,PLUS 3]
20  APP  5  [SUB2 6,SUB2 5]
21  FIB  3  [SUB2 6,PLUS 5]
22  FIB  2  [SUB2 6,PLUS 5,SUB2 3]
23  FIB  1  [SUB2 6,PLUS 5,SUB2 3,SUB2 2]
24  APP  1  [SUB2 6,PLUS 5,SUB2 3,SUB2 2]
25  FIB  0  [SUB2 6,PLUS 5,SUB2 3,PLUS 1]
26  APP  1  [SUB2 6,PLUS 5,SUB2 3,PLUS 1]
27  APP  2  [SUB2 6,PLUS 5,SUB2 3]
28  FIB  1  [SUB2 6,PLUS 5,PLUS 2]
29  APP  1  [SUB2 6,PLUS 5,PLUS 2]
30  APP  3  [SUB2 6,PLUS 5]
31  APP  8  [SUB2 6]
32  FIB  4  [PLUS 8]
33  FIB  3  [PLUS 8,SUB2 4]
34  FIB  2  [PLUS 8,SUB2 4,SUB2 3]
35  FIB  1  [PLUS 8,SUB2 4,SUB2 3,SUB2 2]
```

Tracing of fib_5 6


```
 1  FIB  6  []
 2  FIB  5  [SUB2 6]
 3  FIB  4  [SUB2 6,SUB2 5]
 4  FIB  3  [SUB2 6,SUB2 5,SUB2 4]
 5  FIB  2  [SUB2 6,SUB2 5,SUB2 4,SUB2 3]
 6  FIB  1  [SUB2 6,SUB2 5,SUB2 4,SUB2 3,SUB2 2]
 7  APP  1  [SUB2 6,SUB2 5,SUB2 4,SUB2 3,SUB2 2]
 8  FIB  0  [SUB2 6,SUB2 5,SUB2 4,SUB2 3,PLUS 1]
 9  APP  1  [SUB2 6,SUB2 5,SUB2 4,SUB2 3,PLUS 1]
10  APP  2  [SUB2 6,SUB2 5,SUB2 4,SUB2 3]
11  FIB  1  [SUB2 6,SUB2 5,SUB2 4,PLUS 2]
12  APP  1  [SUB2 6,SUB2 5,SUB2 4,PLUS 2]
13  APP  3  [SUB2 6,SUB2 5,SUB2 4]
14  FIB  2  [SUB2 6,SUB2 5,PLUS 3]
15  FIB  1  [SUB2 6,SUB2 5,PLUS 3,SUB2 2]
16  APP  1  [SUB2 6,SUB2 5,PLUS 3,SUB2 2]
17  FIB  0  [SUB2 6,SUB2 5,PLUS 3,PLUS 1]
18  APP  1  [SUB2 6,SUB2 5,PLUS 3,PLUS 1]
19  APP  2  [SUB2 6,SUB2 5,PLUS 3]
20  APP  5  [SUB2 6,SUB2 5]
21  FIB  3  [SUB2 6,PLUS 5]
22  FIB  2  [SUB2 6,PLUS 5,SUB2 3]
23  FIB  1  [SUB2 6,PLUS 5,SUB2 3,SUB2 2]
24  APP  1  [SUB2 6,PLUS 5,SUB2 3,SUB2 2]
25  FIB  0  [SUB2 6,PLUS 5,SUB2 3,PLUS 1]
26  APP  1  [SUB2 6,PLUS 5,SUB2 3,PLUS 1]
27  APP  2  [SUB2 6,PLUS 5,SUB2 3]
28  FIB  1  [SUB2 6,PLUS 5,PLUS 2]
29  APP  1  [SUB2 6,PLUS 5,PLUS 2]
30  APP  3  [SUB2 6,PLUS 5]
31  APP  8  [SUB2 6]
32  FIB  4  [PLUS 8]
33  FIB  3  [PLUS 8,SUB2 4]
34  FIB  2  [PLUS 8,SUB2 4,SUB2 3]
35  FIB  1  [PLUS 8,SUB2 4,SUB2 3,SUB2 2]
36  APP  1  [PLUS 8,SUB2 4,SUB2 3,SUB2 2]
```

Tracing of fib_5 6


```
 1  FIB  6  []
 2  FIB  5  [SUB2 6]
 3  FIB  4  [SUB2 6,SUB2 5]
 4  FIB  3  [SUB2 6,SUB2 5,SUB2 4]
 5  FIB  2  [SUB2 6,SUB2 5,SUB2 4,SUB2 3]
 6  FIB  1  [SUB2 6,SUB2 5,SUB2 4,SUB2 3,SUB2 2]
 7  APP  1  [SUB2 6,SUB2 5,SUB2 4,SUB2 3,SUB2 2]
 8  FIB  0  [SUB2 6,SUB2 5,SUB2 4,SUB2 3,PLUS 1]
 9  APP  1  [SUB2 6,SUB2 5,SUB2 4,SUB2 3,PLUS 1]
10  APP  2  [SUB2 6,SUB2 5,SUB2 4,SUB2 3]
11  FIB  1  [SUB2 6,SUB2 5,SUB2 4,PLUS 2]
12  APP  1  [SUB2 6,SUB2 5,SUB2 4,PLUS 2]
13  APP  3  [SUB2 6,SUB2 5,SUB2 4]
14  FIB  2  [SUB2 6,SUB2 5,PLUS 3]
15  FIB  1  [SUB2 6,SUB2 5,PLUS 3,SUB2 2]
16  APP  1  [SUB2 6,SUB2 5,PLUS 3,SUB2 2]
17  FIB  0  [SUB2 6,SUB2 5,PLUS 3,PLUS 1]
18  APP  1  [SUB2 6,SUB2 5,PLUS 3,PLUS 1]
19  APP  2  [SUB2 6,SUB2 5,PLUS 3]
20  APP  5  [SUB2 6,SUB2 5]
21  FIB  3  [SUB2 6,PLUS 5]
22  FIB  2  [SUB2 6,PLUS 5,SUB2 3]
23  FIB  1  [SUB2 6,PLUS 5,SUB2 3,SUB2 2]
24  APP  1  [SUB2 6,PLUS 5,SUB2 3,SUB2 2]
25  FIB  0  [SUB2 6,PLUS 5,SUB2 3,PLUS 1]
26  APP  1  [SUB2 6,PLUS 5,SUB2 3,PLUS 1]
27  APP  2  [SUB2 6,PLUS 5,SUB2 3]
28  FIB  1  [SUB2 6,PLUS 5,PLUS 2]
29  APP  1  [SUB2 6,PLUS 5,PLUS 2]
30  APP  3  [SUB2 6,PLUS 5]
31  APP  8  [SUB2 6]
32  FIB  4  [PLUS 8]
33  FIB  3  [PLUS 8,SUB2 4]
34  FIB  2  [PLUS 8,SUB2 4,SUB2 3]
35  FIB  1  [PLUS 8,SUB2 4,SUB2 3,SUB2 2]
36  APP  1  [PLUS 8,SUB2 4,SUB2 3,SUB2 2]
37  FIB  0  [PLUS 8,SUB2 4,SUB2 3,PLUS 1]
```

Tracing of fib_5 6


```
 1  FIB  6  []
 2  FIB  5  [SUB2 6]
 3  FIB  4  [SUB2 6,SUB2 5]
 4  FIB  3  [SUB2 6,SUB2 5,SUB2 4]
 5  FIB  2  [SUB2 6,SUB2 5,SUB2 4,SUB2 3]
 6  FIB  1  [SUB2 6,SUB2 5,SUB2 4,SUB2 3,SUB2 2]
 7  APP  1  [SUB2 6,SUB2 5,SUB2 4,SUB2 3,SUB2 2]
 8  FIB  0  [SUB2 6,SUB2 5,SUB2 4,SUB2 3,PLUS 1]
 9  APP  1  [SUB2 6,SUB2 5,SUB2 4,SUB2 3,PLUS 1]
10  APP  2  [SUB2 6,SUB2 5,SUB2 4,SUB2 3]
11  FIB  1  [SUB2 6,SUB2 5,SUB2 4,PLUS 2]
12  APP  1  [SUB2 6,SUB2 5,SUB2 4,PLUS 2]
13  APP  3  [SUB2 6,SUB2 5,SUB2 4]
14  FIB  2  [SUB2 6,SUB2 5,PLUS 3]
15  FIB  1  [SUB2 6,SUB2 5,PLUS 3,SUB2 2]
16  APP  1  [SUB2 6,SUB2 5,PLUS 3,SUB2 2]
17  FIB  0  [SUB2 6,SUB2 5,PLUS 3,PLUS 1]
18  APP  1  [SUB2 6,SUB2 5,PLUS 3,PLUS 1]
19  APP  2  [SUB2 6,SUB2 5,PLUS 3]
20  APP  5  [SUB2 6,SUB2 5]
21  FIB  3  [SUB2 6,PLUS 5]
22  FIB  2  [SUB2 6,PLUS 5,SUB2 3]
23  FIB  1  [SUB2 6,PLUS 5,SUB2 3,SUB2 2]
24  APP  1  [SUB2 6,PLUS 5,SUB2 3,SUB2 2]
25  FIB  0  [SUB2 6,PLUS 5,SUB2 3,PLUS 1]
26  APP  1  [SUB2 6,PLUS 5,SUB2 3,PLUS 1]
27  APP  2  [SUB2 6,PLUS 5,SUB2 3]
28  FIB  1  [SUB2 6,PLUS 5,PLUS 2]
29  APP  1  [SUB2 6,PLUS 5,PLUS 2]
30  APP  3  [SUB2 6,PLUS 5]
31  APP  8  [SUB2 6]
32  FIB  4  [PLUS 8]
33  FIB  3  [PLUS 8,SUB2 4]
34  FIB  2  [PLUS 8,SUB2 4,SUB2 3]
35  FIB  1  [PLUS 8,SUB2 4,SUB2 3,SUB2 2]
36  APP  1  [PLUS 8,SUB2 4,SUB2 3,SUB2 2]
37  FIB  0  [PLUS 8,SUB2 4,SUB2 3,PLUS 1]
38  APP  1  [PLUS 8,SUB2 4,SUB2 3,PLUS 1]
```

Tracing of fib_5 6


```
 1  FIB  6  []
 2  FIB  5  [SUB2 6]
 3  FIB  4  [SUB2 6,SUB2 5]
 4  FIB  3  [SUB2 6,SUB2 5,SUB2 4]
 5  FIB  2  [SUB2 6,SUB2 5,SUB2 4,SUB2 3]
 6  FIB  1  [SUB2 6,SUB2 5,SUB2 4,SUB2 3,SUB2 2]
 7  APP  1  [SUB2 6,SUB2 5,SUB2 4,SUB2 3,SUB2 2]
 8  FIB  0  [SUB2 6,SUB2 5,SUB2 4,SUB2 3,PLUS 1]
 9  APP  1  [SUB2 6,SUB2 5,SUB2 4,SUB2 3,PLUS 1]
10  APP  2  [SUB2 6,SUB2 5,SUB2 4,SUB2 3]
11  FIB  1  [SUB2 6,SUB2 5,SUB2 4,PLUS 2]
12  APP  1  [SUB2 6,SUB2 5,SUB2 4,PLUS 2]
13  APP  3  [SUB2 6,SUB2 5,SUB2 4]
14  FIB  2  [SUB2 6,SUB2 5,PLUS 3]
15  FIB  1  [SUB2 6,SUB2 5,PLUS 3,SUB2 2]
16  APP  1  [SUB2 6,SUB2 5,PLUS 3,SUB2 2]
17  FIB  0  [SUB2 6,SUB2 5,PLUS 3,PLUS 1]
18  APP  1  [SUB2 6,SUB2 5,PLUS 3,PLUS 1]
19  APP  2  [SUB2 6,SUB2 5,PLUS 3]
20  APP  5  [SUB2 6,SUB2 5]
21  FIB  3  [SUB2 6,PLUS 5]
22  FIB  2  [SUB2 6,PLUS 5,SUB2 3]
23  FIB  1  [SUB2 6,PLUS 5,SUB2 3,SUB2 2]
24  APP  1  [SUB2 6,PLUS 5,SUB2 3,SUB2 2]
25  FIB  0  [SUB2 6,PLUS 5,SUB2 3,PLUS 1]
26  APP  1  [SUB2 6,PLUS 5,SUB2 3,PLUS 1]
27  APP  2  [SUB2 6,PLUS 5,SUB2 3]
28  FIB  1  [SUB2 6,PLUS 5,PLUS 2]
29  APP  1  [SUB2 6,PLUS 5,PLUS 2]
30  APP  3  [SUB2 6,PLUS 5]
31  APP  8  [SUB2 6]
32  FIB  4  [PLUS 8]
33  FIB  3  [PLUS 8,SUB2 4]
34  FIB  2  [PLUS 8,SUB2 4,SUB2 3]
35  FIB  1  [PLUS 8,SUB2 4,SUB2 3,SUB2 2]
36  APP  1  [PLUS 8,SUB2 4,SUB2 3,SUB2 2]
37  FIB  0  [PLUS 8,SUB2 4,SUB2 3,PLUS 1]
38  APP  1  [PLUS 8,SUB2 4,SUB2 3,PLUS 1]
39  APP  2  [PLUS 8,SUB2 4,SUB2 3]
```

Tracing of fib_5 6


```
 1  FIB  6  []
 2  FIB  5  [SUB2 6]
 3  FIB  4  [SUB2 6,SUB2 5]
 4  FIB  3  [SUB2 6,SUB2 5,SUB2 4]
 5  FIB  2  [SUB2 6,SUB2 5,SUB2 4,SUB2 3]
 6  FIB  1  [SUB2 6,SUB2 5,SUB2 4,SUB2 3,SUB2 2]
 7  APP  1  [SUB2 6,SUB2 5,SUB2 4,SUB2 3,SUB2 2]
 8  FIB  0  [SUB2 6,SUB2 5,SUB2 4,SUB2 3,PLUS 1]
 9  APP  1  [SUB2 6,SUB2 5,SUB2 4,SUB2 3,PLUS 1]
10  APP  2  [SUB2 6,SUB2 5,SUB2 4,SUB2 3]
11  FIB  1  [SUB2 6,SUB2 5,SUB2 4,PLUS 2]
12  APP  1  [SUB2 6,SUB2 5,SUB2 4,PLUS 2]
13  APP  3  [SUB2 6,SUB2 5,SUB2 4]
14  FIB  2  [SUB2 6,SUB2 5,PLUS 3]
15  FIB  1  [SUB2 6,SUB2 5,PLUS 3,SUB2 2]
16  APP  1  [SUB2 6,SUB2 5,PLUS 3,SUB2 2]
17  FIB  0  [SUB2 6,SUB2 5,PLUS 3,PLUS 1]
18  APP  1  [SUB2 6,SUB2 5,PLUS 3,PLUS 1]
19  APP  2  [SUB2 6,SUB2 5,PLUS 3]
20  APP  5  [SUB2 6,SUB2 5]
21  FIB  3  [SUB2 6,PLUS 5]
22  FIB  2  [SUB2 6,PLUS 5,SUB2 3]
23  FIB  1  [SUB2 6,PLUS 5,SUB2 3,SUB2 2]
24  APP  1  [SUB2 6,PLUS 5,SUB2 3,SUB2 2]
25  FIB  0  [SUB2 6,PLUS 5,SUB2 3,PLUS 1]
26  APP  1  [SUB2 6,PLUS 5,SUB2 3,PLUS 1]
27  APP  2  [SUB2 6,PLUS 5,SUB2 3]
28  FIB  1  [SUB2 6,PLUS 5,PLUS 2]
29  APP  1  [SUB2 6,PLUS 5,PLUS 2]
30  APP  3  [SUB2 6,PLUS 5]
31  APP  8  [SUB2 6]
32  FIB  4  [PLUS 8]
33  FIB  3  [PLUS 8,SUB2 4]
34  FIB  2  [PLUS 8,SUB2 4,SUB2 3]
35  FIB  1  [PLUS 8,SUB2 4,SUB2 3,SUB2 2]
36  APP  1  [PLUS 8,SUB2 4,SUB2 3,SUB2 2]
37  FIB  0  [PLUS 8,SUB2 4,SUB2 3,PLUS 1]
38  APP  1  [PLUS 8,SUB2 4,SUB2 3,PLUS 1]
39  APP  2  [PLUS 8,SUB2 4,SUB2 3]
40  FIB  1  [PLUS 8,SUB2 4,PLUS 2]
```

Tracing of fib_5 6


```
 1  FIB  6  []
 2  FIB  5  [SUB2 6]
 3  FIB  4  [SUB2 6,SUB2 5]
 4  FIB  3  [SUB2 6,SUB2 5,SUB2 4]
 5  FIB  2  [SUB2 6,SUB2 5,SUB2 4,SUB2 3]
 6  FIB  1  [SUB2 6,SUB2 5,SUB2 4,SUB2 3,SUB2 2]
 7  APP  1  [SUB2 6,SUB2 5,SUB2 4,SUB2 3,SUB2 2]
 8  FIB  0  [SUB2 6,SUB2 5,SUB2 4,SUB2 3,PLUS 1]
 9  APP  1  [SUB2 6,SUB2 5,SUB2 4,SUB2 3,PLUS 1]
10  APP  2  [SUB2 6,SUB2 5,SUB2 4,SUB2 3]
11  FIB  1  [SUB2 6,SUB2 5,SUB2 4,PLUS 2]
12  APP  1  [SUB2 6,SUB2 5,SUB2 4,PLUS 2]
13  APP  3  [SUB2 6,SUB2 5,SUB2 4]
14  FIB  2  [SUB2 6,SUB2 5,PLUS 3]
15  FIB  1  [SUB2 6,SUB2 5,PLUS 3,SUB2 2]
16  APP  1  [SUB2 6,SUB2 5,PLUS 3,SUB2 2]
17  FIB  0  [SUB2 6,SUB2 5,PLUS 3,PLUS 1]
18  APP  1  [SUB2 6,SUB2 5,PLUS 3,PLUS 1]
19  APP  2  [SUB2 6,SUB2 5,PLUS 3]
20  APP  5  [SUB2 6,SUB2 5]
21  FIB  3  [SUB2 6,PLUS 5]
22  FIB  2  [SUB2 6,PLUS 5,SUB2 3]
23  FIB  1  [SUB2 6,PLUS 5,SUB2 3,SUB2 2]
24  APP  1  [SUB2 6,PLUS 5,SUB2 3,SUB2 2]
25  FIB  0  [SUB2 6,PLUS 5,SUB2 3,PLUS 1]
26  APP  1  [SUB2 6,PLUS 5,SUB2 3,PLUS 1]
27  APP  2  [SUB2 6,PLUS 5,SUB2 3]
28  FIB  1  [SUB2 6,PLUS 5,PLUS 2]
29  APP  1  [SUB2 6,PLUS 5,PLUS 2]
30  APP  3  [SUB2 6,PLUS 5]
31  APP  8  [SUB2 6]
32  FIB  4  [PLUS 8]
33  FIB  3  [PLUS 8,SUB2 4]
34  FIB  2  [PLUS 8,SUB2 4,SUB2 3]
35  FIB  1  [PLUS 8,SUB2 4,SUB2 3,SUB2 2]
36  APP  1  [PLUS 8,SUB2 4,SUB2 3,SUB2 2]
37  FIB  0  [PLUS 8,SUB2 4,SUB2 3,PLUS 1]
38  APP  1  [PLUS 8,SUB2 4,SUB2 3,PLUS 1]
39  APP  2  [PLUS 8,SUB2 4,SUB2 3]
40  FIB  1  [PLUS 8,SUB2 4,PLUS 2]
41  APP  1  [PLUS 8,SUB2 4,PLUS 2]
```

Tracing of fib_5 6


```
 1  FIB  6  []
 2  FIB  5  [SUB2 6]
 3  FIB  4  [SUB2 6,SUB2 5]
 4  FIB  3  [SUB2 6,SUB2 5,SUB2 4]
 5  FIB  2  [SUB2 6,SUB2 5,SUB2 4,SUB2 3]
 6  FIB  1  [SUB2 6,SUB2 5,SUB2 4,SUB2 3,SUB2 2]
 7  APP  1  [SUB2 6,SUB2 5,SUB2 4,SUB2 3,SUB2 2]
 8  FIB  0  [SUB2 6,SUB2 5,SUB2 4,SUB2 3,PLUS 1]
 9  APP  1  [SUB2 6,SUB2 5,SUB2 4,SUB2 3,PLUS 1]
10  APP  2  [SUB2 6,SUB2 5,SUB2 4,SUB2 3]
11  FIB  1  [SUB2 6,SUB2 5,SUB2 4,PLUS 2]
12  APP  1  [SUB2 6,SUB2 5,SUB2 4,PLUS 2]
13  APP  3  [SUB2 6,SUB2 5,SUB2 4]
14  FIB  2  [SUB2 6,SUB2 5,PLUS 3]
15  FIB  1  [SUB2 6,SUB2 5,PLUS 3,SUB2 2]
16  APP  1  [SUB2 6,SUB2 5,PLUS 3,SUB2 2]
17  FIB  0  [SUB2 6,SUB2 5,PLUS 3,PLUS 1]
18  APP  1  [SUB2 6,SUB2 5,PLUS 3,PLUS 1]
19  APP  2  [SUB2 6,SUB2 5,PLUS 3]
20  APP  5  [SUB2 6,SUB2 5]
21  FIB  3  [SUB2 6,PLUS 5]
22  FIB  2  [SUB2 6,PLUS 5,SUB2 3]
23  FIB  1  [SUB2 6,PLUS 5,SUB2 3,SUB2 2]
24  APP  1  [SUB2 6,PLUS 5,SUB2 3,SUB2 2]
25  FIB  0  [SUB2 6,PLUS 5,SUB2 3,PLUS 1]
26  APP  1  [SUB2 6,PLUS 5,SUB2 3,PLUS 1]
27  APP  2  [SUB2 6,PLUS 5,SUB2 3]
28  FIB  1  [SUB2 6,PLUS 5,PLUS 2]
29  APP  1  [SUB2 6,PLUS 5,PLUS 2]
30  APP  3  [SUB2 6,PLUS 5]
31  APP  8  [SUB2 6]
32  FIB  4  [PLUS 8]
33  FIB  3  [PLUS 8,SUB2 4]
34  FIB  2  [PLUS 8,SUB2 4,SUB2 3]
35  FIB  1  [PLUS 8,SUB2 4,SUB2 3,SUB2 2]
36  APP  1  [PLUS 8,SUB2 4,SUB2 3,SUB2 2]
37  FIB  0  [PLUS 8,SUB2 4,SUB2 3,PLUS 1]
38  APP  1  [PLUS 8,SUB2 4,SUB2 3,PLUS 1]
39  APP  2  [PLUS 8,SUB2 4,SUB2 3]
40  FIB  1  [PLUS 8,SUB2 4,PLUS 2]
41  APP  1  [PLUS 8,SUB2 4,PLUS 2]
42  APP  3  [PLUS 8,SUB2 4]
```

Tracing of fib_5 6


```
 1  FIB  6  []
 2  FIB  5  [SUB2 6]
 3  FIB  4  [SUB2 6,SUB2 5]
 4  FIB  3  [SUB2 6,SUB2 5,SUB2 4]
 5  FIB  2  [SUB2 6,SUB2 5,SUB2 4,SUB2 3]
 6  FIB  1  [SUB2 6,SUB2 5,SUB2 4,SUB2 3,SUB2 2]
 7  APP  1  [SUB2 6,SUB2 5,SUB2 4,SUB2 3,SUB2 2]
 8  FIB  0  [SUB2 6,SUB2 5,SUB2 4,SUB2 3,PLUS 1]
 9  APP  1  [SUB2 6,SUB2 5,SUB2 4,SUB2 3,PLUS 1]
10  APP  2  [SUB2 6,SUB2 5,SUB2 4,SUB2 3]
11  FIB  1  [SUB2 6,SUB2 5,SUB2 4,PLUS 2]
12  APP  1  [SUB2 6,SUB2 5,SUB2 4,PLUS 2]
13  APP  3  [SUB2 6,SUB2 5,SUB2 4]
14  FIB  2  [SUB2 6,SUB2 5,PLUS 3]
15  FIB  1  [SUB2 6,SUB2 5,PLUS 3,SUB2 2]
16  APP  1  [SUB2 6,SUB2 5,PLUS 3,SUB2 2]
17  FIB  0  [SUB2 6,SUB2 5,PLUS 3,PLUS 1]
18  APP  1  [SUB2 6,SUB2 5,PLUS 3,PLUS 1]
19  APP  2  [SUB2 6,SUB2 5,PLUS 3]
20  APP  5  [SUB2 6,SUB2 5]
21  FIB  3  [SUB2 6,PLUS 5]
22  FIB  2  [SUB2 6,PLUS 5,SUB2 3]
23  FIB  1  [SUB2 6,PLUS 5,SUB2 3,SUB2 2]
24  APP  1  [SUB2 6,PLUS 5,SUB2 3,SUB2 2]
25  FIB  0  [SUB2 6,PLUS 5,SUB2 3,PLUS 1]
26  APP  1  [SUB2 6,PLUS 5,SUB2 3,PLUS 1]
27  APP  2  [SUB2 6,PLUS 5,SUB2 3]
28  FIB  1  [SUB2 6,PLUS 5,PLUS 2]
29  APP  1  [SUB2 6,PLUS 5,PLUS 2]
30  APP  3  [SUB2 6,PLUS 5]
31  APP  8  [SUB2 6]
32  FIB  4  [PLUS 8]
33  FIB  3  [PLUS 8,SUB2 4]
34  FIB  2  [PLUS 8,SUB2 4,SUB2 3]
35  FIB  1  [PLUS 8,SUB2 4,SUB2 3,SUB2 2]
36  APP  1  [PLUS 8,SUB2 4,SUB2 3,SUB2 2]
37  FIB  0  [PLUS 8,SUB2 4,SUB2 3,PLUS 1]
38  APP  1  [PLUS 8,SUB2 4,SUB2 3,PLUS 1]
39  APP  2  [PLUS 8,SUB2 4,SUB2 3]
40  FIB  1  [PLUS 8,SUB2 4,PLUS 2]
41  APP  1  [PLUS 8,SUB2 4,PLUS 2]
42  APP  3  [PLUS 8,SUB2 4]
43  FIB  2  [PLUS 8,PLUS 3]
```

Tracing of fib_5 6


```
 1  FIB  6  []
 2  FIB  5  [SUB2 6]
 3  FIB  4  [SUB2 6,SUB2 5]
 4  FIB  3  [SUB2 6,SUB2 5,SUB2 4]
 5  FIB  2  [SUB2 6,SUB2 5,SUB2 4,SUB2 3]
 6  FIB  1  [SUB2 6,SUB2 5,SUB2 4,SUB2 3,SUB2 2]
 7  APP  1  [SUB2 6,SUB2 5,SUB2 4,SUB2 3,SUB2 2]
 8  FIB  0  [SUB2 6,SUB2 5,SUB2 4,SUB2 3,PLUS 1]
 9  APP  1  [SUB2 6,SUB2 5,SUB2 4,SUB2 3,PLUS 1]
10  APP  2  [SUB2 6,SUB2 5,SUB2 4,SUB2 3]
11  FIB  1  [SUB2 6,SUB2 5,SUB2 4,PLUS 2]
12  APP  1  [SUB2 6,SUB2 5,SUB2 4,PLUS 2]
13  APP  3  [SUB2 6,SUB2 5,SUB2 4]
14  FIB  2  [SUB2 6,SUB2 5,PLUS 3]
15  FIB  1  [SUB2 6,SUB2 5,PLUS 3,SUB2 2]
16  APP  1  [SUB2 6,SUB2 5,PLUS 3,SUB2 2]
17  FIB  0  [SUB2 6,SUB2 5,PLUS 3,PLUS 1]
18  APP  1  [SUB2 6,SUB2 5,PLUS 3,PLUS 1]
19  APP  2  [SUB2 6,SUB2 5,PLUS 3]
20  APP  5  [SUB2 6,SUB2 5]
21  FIB  3  [SUB2 6,PLUS 5]
22  FIB  2  [SUB2 6,PLUS 5,SUB2 3]
23  FIB  1  [SUB2 6,PLUS 5,SUB2 3,SUB2 2]
24  APP  1  [SUB2 6,PLUS 5,SUB2 3,SUB2 2]
25  FIB  0  [SUB2 6,PLUS 5,SUB2 3,PLUS 1]
26  APP  1  [SUB2 6,PLUS 5,SUB2 3,PLUS 1]
27  APP  2  [SUB2 6,PLUS 5,SUB2 3]
28  FIB  1  [SUB2 6,PLUS 5,PLUS 2]
29  APP  1  [SUB2 6,PLUS 5,PLUS 2]
30  APP  3  [SUB2 6,PLUS 5]
31  APP  8  [SUB2 6]
32  FIB  4  [PLUS 8]
33  FIB  3  [PLUS 8,SUB2 4]
34  FIB  2  [PLUS 8,SUB2 4,SUB2 3]
35  FIB  1  [PLUS 8,SUB2 4,SUB2 3,SUB2 2]
36  APP  1  [PLUS 8,SUB2 4,SUB2 3,SUB2 2]
37  FIB  0  [PLUS 8,SUB2 4,SUB2 3,PLUS 1]
38  APP  1  [PLUS 8,SUB2 4,SUB2 3,PLUS 1]
39  APP  2  [PLUS 8,SUB2 4,SUB2 3]
40  FIB  1  [PLUS 8,SUB2 4,PLUS 2]
41  APP  1  [PLUS 8,SUB2 4,PLUS 2]
42  APP  3  [PLUS 8,SUB2 4]
43  FIB  2  [PLUS 8,PLUS 3]
44  FIB  1  [PLUS 8,PLUS 3,SUB2 2]
```

Tracing of fib_5 6


```
 1  FIB  6  []
 2  FIB  5  [SUB2 6]
 3  FIB  4  [SUB2 6,SUB2 5]
 4  FIB  3  [SUB2 6,SUB2 5,SUB2 4]
 5  FIB  2  [SUB2 6,SUB2 5,SUB2 4,SUB2 3]
 6  FIB  1  [SUB2 6,SUB2 5,SUB2 4,SUB2 3,SUB2 2]
 7  APP  1  [SUB2 6,SUB2 5,SUB2 4,SUB2 3,SUB2 2]
 8  FIB  0  [SUB2 6,SUB2 5,SUB2 4,SUB2 3,PLUS 1]
 9  APP  1  [SUB2 6,SUB2 5,SUB2 4,SUB2 3,PLUS 1]
10  APP  2  [SUB2 6,SUB2 5,SUB2 4,SUB2 3]
11  FIB  1  [SUB2 6,SUB2 5,SUB2 4,PLUS 2]
12  APP  1  [SUB2 6,SUB2 5,SUB2 4,PLUS 2]
13  APP  3  [SUB2 6,SUB2 5,SUB2 4]
14  FIB  2  [SUB2 6,SUB2 5,PLUS 3]
15  FIB  1  [SUB2 6,SUB2 5,PLUS 3,SUB2 2]
16  APP  1  [SUB2 6,SUB2 5,PLUS 3,SUB2 2]
17  FIB  0  [SUB2 6,SUB2 5,PLUS 3,PLUS 1]
18  APP  1  [SUB2 6,SUB2 5,PLUS 3,PLUS 1]
19  APP  2  [SUB2 6,SUB2 5,PLUS 3]
20  APP  5  [SUB2 6,SUB2 5]
21  FIB  3  [SUB2 6,PLUS 5]
22  FIB  2  [SUB2 6,PLUS 5,SUB2 3]
23  FIB  1  [SUB2 6,PLUS 5,SUB2 3,SUB2 2]
24  APP  1  [SUB2 6,PLUS 5,SUB2 3,SUB2 2]
25  FIB  0  [SUB2 6,PLUS 5,SUB2 3,PLUS 1]
26  APP  1  [SUB2 6,PLUS 5,SUB2 3,PLUS 1]
27  APP  2  [SUB2 6,PLUS 5,SUB2 3]
28  FIB  1  [SUB2 6,PLUS 5,PLUS 2]
29  APP  1  [SUB2 6,PLUS 5,PLUS 2]
30  APP  3  [SUB2 6,PLUS 5]
31  APP  8  [SUB2 6]
32  FIB  4  [PLUS 8]
33  FIB  3  [PLUS 8,SUB2 4]
34  FIB  2  [PLUS 8,SUB2 4,SUB2 3]
35  FIB  1  [PLUS 8,SUB2 4,SUB2 3,SUB2 2]
36  APP  1  [PLUS 8,SUB2 4,SUB2 3,SUB2 2]
37  FIB  0  [PLUS 8,SUB2 4,SUB2 3,PLUS 1]
38  APP  1  [PLUS 8,SUB2 4,SUB2 3,PLUS 1]
39  APP  2  [PLUS 8,SUB2 4,SUB2 3]
40  FIB  1  [PLUS 8,SUB2 4,PLUS 2]
41  APP  1  [PLUS 8,SUB2 4,PLUS 2]
42  APP  3  [PLUS 8,SUB2 4]
43  FIB  2  [PLUS 8,PLUS 3]
44  FIB  1  [PLUS 8,PLUS 3,SUB2 2]
45  APP  1  [PLUS 8,PLUS 3,SUB2 2]
```

Tracing of fib_5 6


```
 1  FIB  6  []
 2  FIB  5  [SUB2 6]
 3  FIB  4  [SUB2 6,SUB2 5]
 4  FIB  3  [SUB2 6,SUB2 5,SUB2 4]
 5  FIB  2  [SUB2 6,SUB2 5,SUB2 4,SUB2 3]
 6  FIB  1  [SUB2 6,SUB2 5,SUB2 4,SUB2 3,SUB2 2]
 7  APP  1  [SUB2 6,SUB2 5,SUB2 4,SUB2 3,SUB2 2]
 8  FIB  0  [SUB2 6,SUB2 5,SUB2 4,SUB2 3,PLUS 1]
 9  APP  1  [SUB2 6,SUB2 5,SUB2 4,SUB2 3,PLUS 1]
10  APP  2  [SUB2 6,SUB2 5,SUB2 4,SUB2 3]
11  FIB  1  [SUB2 6,SUB2 5,SUB2 4,PLUS 2]
12  APP  1  [SUB2 6,SUB2 5,SUB2 4,PLUS 2]
13  APP  3  [SUB2 6,SUB2 5,SUB2 4]
14  FIB  2  [SUB2 6,SUB2 5,PLUS 3]
15  FIB  1  [SUB2 6,SUB2 5,PLUS 3,SUB2 2]
16  APP  1  [SUB2 6,SUB2 5,PLUS 3,SUB2 2]
17  FIB  0  [SUB2 6,SUB2 5,PLUS 3,PLUS 1]
18  APP  1  [SUB2 6,SUB2 5,PLUS 3,PLUS 1]
19  APP  2  [SUB2 6,SUB2 5,PLUS 3]
20  APP  5  [SUB2 6,SUB2 5]
21  FIB  3  [SUB2 6,PLUS 5]
22  FIB  2  [SUB2 6,PLUS 5,SUB2 3]
23  FIB  1  [SUB2 6,PLUS 5,SUB2 3,SUB2 2]
24  APP  1  [SUB2 6,PLUS 5,SUB2 3,SUB2 2]
25  FIB  0  [SUB2 6,PLUS 5,SUB2 3,PLUS 1]
26  APP  1  [SUB2 6,PLUS 5,SUB2 3,PLUS 1]
27  APP  2  [SUB2 6,PLUS 5,SUB2 3]
28  FIB  1  [SUB2 6,PLUS 5,PLUS 2]
29  APP  1  [SUB2 6,PLUS 5,PLUS 2]
30  APP  3  [SUB2 6,PLUS 5]
31  APP  8  [SUB2 6]
32  FIB  4  [PLUS 8]
33  FIB  3  [PLUS 8,SUB2 4]
34  FIB  2  [PLUS 8,SUB2 4,SUB2 3]
35  FIB  1  [PLUS 8,SUB2 4,SUB2 3,SUB2 2]
36  APP  1  [PLUS 8,SUB2 4,SUB2 3,SUB2 2]
37  FIB  0  [PLUS 8,SUB2 4,SUB2 3,PLUS 1]
38  APP  1  [PLUS 8,SUB2 4,SUB2 3,PLUS 1]
39  APP  2  [PLUS 8,SUB2 4,SUB2 3]
40  FIB  1  [PLUS 8,SUB2 4,PLUS 2]
41  APP  1  [PLUS 8,SUB2 4,PLUS 2]
42  APP  3  [PLUS 8,SUB2 4]
43  FIB  2  [PLUS 8,PLUS 3]
44  FIB  1  [PLUS 8,PLUS 3,SUB2 2]
45  APP  1  [PLUS 8,PLUS 3,SUB2 2]
46  FIB  0  [PLUS 8,PLUS 3,PLUS 1]
```

Tracing of fib_5 6


```
 1  FIB  6  []
 2  FIB  5  [SUB2 6]
 3  FIB  4  [SUB2 6,SUB2 5]
 4  FIB  3  [SUB2 6,SUB2 5,SUB2 4]
 5  FIB  2  [SUB2 6,SUB2 5,SUB2 4,SUB2 3]
 6  FIB  1  [SUB2 6,SUB2 5,SUB2 4,SUB2 3,SUB2 2]
 7  APP  1  [SUB2 6,SUB2 5,SUB2 4,SUB2 3,SUB2 2]
 8  FIB  0  [SUB2 6,SUB2 5,SUB2 4,SUB2 3,PLUS 1]
 9  APP  1  [SUB2 6,SUB2 5,SUB2 4,SUB2 3,PLUS 1]
10  APP  2  [SUB2 6,SUB2 5,SUB2 4,SUB2 3]
11  FIB  1  [SUB2 6,SUB2 5,SUB2 4,PLUS 2]
12  APP  1  [SUB2 6,SUB2 5,SUB2 4,PLUS 2]
13  APP  3  [SUB2 6,SUB2 5,SUB2 4]
14  FIB  2  [SUB2 6,SUB2 5,PLUS 3]
15  FIB  1  [SUB2 6,SUB2 5,PLUS 3,SUB2 2]
16  APP  1  [SUB2 6,SUB2 5,PLUS 3,SUB2 2]
17  FIB  0  [SUB2 6,SUB2 5,PLUS 3,PLUS 1]
18  APP  1  [SUB2 6,SUB2 5,PLUS 3,PLUS 1]
19  APP  2  [SUB2 6,SUB2 5,PLUS 3]
20  APP  5  [SUB2 6,SUB2 5]
21  FIB  3  [SUB2 6,PLUS 5]
22  FIB  2  [SUB2 6,PLUS 5,SUB2 3]
23  FIB  1  [SUB2 6,PLUS 5,SUB2 3,SUB2 2]
24  APP  1  [SUB2 6,PLUS 5,SUB2 3,SUB2 2]
25  FIB  0  [SUB2 6,PLUS 5,SUB2 3,PLUS 1]
26  APP  1  [SUB2 6,PLUS 5,SUB2 3,PLUS 1]
27  APP  2  [SUB2 6,PLUS 5,SUB2 3]
28  FIB  1  [SUB2 6,PLUS 5,PLUS 2]
29  APP  1  [SUB2 6,PLUS 5,PLUS 2]
30  APP  3  [SUB2 6,PLUS 5]
31  APP  8  [SUB2 6]
32  FIB  4  [PLUS 8]
33  FIB  3  [PLUS 8,SUB2 4]
34  FIB  2  [PLUS 8,SUB2 4,SUB2 3]
35  FIB  1  [PLUS 8,SUB2 4,SUB2 3,SUB2 2]
36  APP  1  [PLUS 8,SUB2 4,SUB2 3,SUB2 2]
37  FIB  0  [PLUS 8,SUB2 4,SUB2 3,PLUS 1]
38  APP  1  [PLUS 8,SUB2 4,SUB2 3,PLUS 1]
39  APP  2  [PLUS 8,SUB2 4,SUB2 3]
40  FIB  1  [PLUS 8,SUB2 4,PLUS 2]
41  APP  1  [PLUS 8,SUB2 4,PLUS 2]
42  APP  3  [PLUS 8,SUB2 4]
43  FIB  2  [PLUS 8,PLUS 3]
44  FIB  1  [PLUS 8,PLUS 3,SUB2 2]
45  APP  1  [PLUS 8,PLUS 3,SUB2 2]
46  FIB  0  [PLUS 8,PLUS 3,PLUS 1]
47  APP  1  [PLUS 8,PLUS 3,PLUS 1]
```

Tracing of fib_5 6


```
 1  FIB  6  []
 2  FIB  5  [SUB2 6]
 3  FIB  4  [SUB2 6,SUB2 5]
 4  FIB  3  [SUB2 6,SUB2 5,SUB2 4]
 5  FIB  2  [SUB2 6,SUB2 5,SUB2 4,SUB2 3]
 6  FIB  1  [SUB2 6,SUB2 5,SUB2 4,SUB2 3,SUB2 2]
 7  APP  1  [SUB2 6,SUB2 5,SUB2 4,SUB2 3,SUB2 2]
 8  FIB  0  [SUB2 6,SUB2 5,SUB2 4,SUB2 3,PLUS 1]
 9  APP  1  [SUB2 6,SUB2 5,SUB2 4,SUB2 3,PLUS 1]
10  APP  2  [SUB2 6,SUB2 5,SUB2 4,SUB2 3]
11  FIB  1  [SUB2 6,SUB2 5,SUB2 4,PLUS 2]
12  APP  1  [SUB2 6,SUB2 5,SUB2 4,PLUS 2]
13  APP  3  [SUB2 6,SUB2 5,SUB2 4]
14  FIB  2  [SUB2 6,SUB2 5,PLUS 3]
15  FIB  1  [SUB2 6,SUB2 5,PLUS 3,SUB2 2]
16  APP  1  [SUB2 6,SUB2 5,PLUS 3,SUB2 2]
17  FIB  0  [SUB2 6,SUB2 5,PLUS 3,PLUS 1]
18  APP  1  [SUB2 6,SUB2 5,PLUS 3,PLUS 1]
19  APP  2  [SUB2 6,SUB2 5,PLUS 3]
20  APP  5  [SUB2 6,SUB2 5]
21  FIB  3  [SUB2 6,PLUS 5]
22  FIB  2  [SUB2 6,PLUS 5,SUB2 3]
23  FIB  1  [SUB2 6,PLUS 5,SUB2 3,SUB2 2]
24  APP  1  [SUB2 6,PLUS 5,SUB2 3,SUB2 2]
25  FIB  0  [SUB2 6,PLUS 5,SUB2 3,PLUS 1]
26  APP  1  [SUB2 6,PLUS 5,SUB2 3,PLUS 1]
27  APP  2  [SUB2 6,PLUS 5,SUB2 3]
28  FIB  1  [SUB2 6,PLUS 5,PLUS 2]
29  APP  1  [SUB2 6,PLUS 5,PLUS 2]
30  APP  3  [SUB2 6,PLUS 5]
31  APP  8  [SUB2 6]
32  FIB  4  [PLUS 8]
33  FIB  3  [PLUS 8,SUB2 4]
34  FIB  2  [PLUS 8,SUB2 4,SUB2 3]
35  FIB  1  [PLUS 8,SUB2 4,SUB2 3,SUB2 2]
36  APP  1  [PLUS 8,SUB2 4,SUB2 3,SUB2 2]
37  FIB  0  [PLUS 8,SUB2 4,SUB2 3,PLUS 1]
38  APP  1  [PLUS 8,SUB2 4,SUB2 3,PLUS 1]
39  APP  2  [PLUS 8,SUB2 4,SUB2 3]
40  FIB  1  [PLUS 8,SUB2 4,PLUS 2]
41  APP  1  [PLUS 8,SUB2 4,PLUS 2]
42  APP  3  [PLUS 8,SUB2 4]
43  FIB  2  [PLUS 8,PLUS 3]
44  FIB  1  [PLUS 8,PLUS 3,SUB2 2]
45  APP  1  [PLUS 8,PLUS 3,SUB2 2]
46  FIB  0  [PLUS 8,PLUS 3,PLUS 1]
47  APP  1  [PLUS 8,PLUS 3,PLUS 1]
48  APP  2  [PLUS 8,PLUS 3]
```

Tracing of fib_5 6


```
 1  FIB  6  []
 2  FIB  5  [SUB2 6]
 3  FIB  4  [SUB2 6,SUB2 5]
 4  FIB  3  [SUB2 6,SUB2 5,SUB2 4]
 5  FIB  2  [SUB2 6,SUB2 5,SUB2 4,SUB2 3]
 6  FIB  1  [SUB2 6,SUB2 5,SUB2 4,SUB2 3,SUB2 2]
 7  APP  1  [SUB2 6,SUB2 5,SUB2 4,SUB2 3,SUB2 2]
 8  FIB  0  [SUB2 6,SUB2 5,SUB2 4,SUB2 3,PLUS 1]
 9  APP  1  [SUB2 6,SUB2 5,SUB2 4,SUB2 3,PLUS 1]
10  APP  2  [SUB2 6,SUB2 5,SUB2 4,SUB2 3]
11  FIB  1  [SUB2 6,SUB2 5,SUB2 4,PLUS 2]
12  APP  1  [SUB2 6,SUB2 5,SUB2 4,PLUS 2]
13  APP  3  [SUB2 6,SUB2 5,SUB2 4]
14  FIB  2  [SUB2 6,SUB2 5,PLUS 3]
15  FIB  1  [SUB2 6,SUB2 5,PLUS 3,SUB2 2]
16  APP  1  [SUB2 6,SUB2 5,PLUS 3,SUB2 2]
17  FIB  0  [SUB2 6,SUB2 5,PLUS 3,PLUS 1]
18  APP  1  [SUB2 6,SUB2 5,PLUS 3,PLUS 1]
19  APP  2  [SUB2 6,SUB2 5,PLUS 3]
20  APP  5  [SUB2 6,SUB2 5]
21  FIB  3  [SUB2 6,PLUS 5]
22  FIB  2  [SUB2 6,PLUS 5,SUB2 3]
23  FIB  1  [SUB2 6,PLUS 5,SUB2 3,SUB2 2]
24  APP  1  [SUB2 6,PLUS 5,SUB2 3,SUB2 2]
25  FIB  0  [SUB2 6,PLUS 5,SUB2 3,PLUS 1]
26  APP  1  [SUB2 6,PLUS 5,SUB2 3,PLUS 1]
27  APP  2  [SUB2 6,PLUS 5,SUB2 3]
28  FIB  1  [SUB2 6,PLUS 5,PLUS 2]
29  APP  1  [SUB2 6,PLUS 5,PLUS 2]
30  APP  3  [SUB2 6,PLUS 5]
31  APP  8  [SUB2 6]
32  FIB  4  [PLUS 8]
33  FIB  3  [PLUS 8,SUB2 4]
34  FIB  2  [PLUS 8,SUB2 4,SUB2 3]
35  FIB  1  [PLUS 8,SUB2 4,SUB2 3,SUB2 2]
36  APP  1  [PLUS 8,SUB2 4,SUB2 3,SUB2 2]
37  FIB  0  [PLUS 8,SUB2 4,SUB2 3,PLUS 1]
38  APP  1  [PLUS 8,SUB2 4,SUB2 3,PLUS 1]
39  APP  2  [PLUS 8,SUB2 4,SUB2 3]
40  FIB  1  [PLUS 8,SUB2 4,PLUS 2]
41  APP  1  [PLUS 8,SUB2 4,PLUS 2]
42  APP  3  [PLUS 8,SUB2 4]
43  FIB  2  [PLUS 8,PLUS 3]
44  FIB  1  [PLUS 8,PLUS 3,SUB2 2]
45  APP  1  [PLUS 8,PLUS 3,SUB2 2]
46  FIB  0  [PLUS 8,PLUS 3,PLUS 1]
47  APP  1  [PLUS 8,PLUS 3,PLUS 1]
48  APP  2  [PLUS 8,PLUS 3]
49  APP  5  [PLUS 8]
```

Tracing of fib_5 6


```
 1  FIB  6  []
 2  FIB  5  [SUB2 6]
 3  FIB  4  [SUB2 6,SUB2 5]
 4  FIB  3  [SUB2 6,SUB2 5,SUB2 4]
 5  FIB  2  [SUB2 6,SUB2 5,SUB2 4,SUB2 3]
 6  FIB  1  [SUB2 6,SUB2 5,SUB2 4,SUB2 3,SUB2 2]
 7  APP  1  [SUB2 6,SUB2 5,SUB2 4,SUB2 3,SUB2 2]
 8  FIB  0  [SUB2 6,SUB2 5,SUB2 4,SUB2 3,PLUS 1]
 9  APP  1  [SUB2 6,SUB2 5,SUB2 4,SUB2 3,PLUS 1]
10  APP  2  [SUB2 6,SUB2 5,SUB2 4,SUB2 3]
11  FIB  1  [SUB2 6,SUB2 5,SUB2 4,PLUS 2]
12  APP  1  [SUB2 6,SUB2 5,SUB2 4,PLUS 2]
13  APP  3  [SUB2 6,SUB2 5,SUB2 4]
14  FIB  2  [SUB2 6,SUB2 5,PLUS 3]
15  FIB  1  [SUB2 6,SUB2 5,PLUS 3,SUB2 2]
16  APP  1  [SUB2 6,SUB2 5,PLUS 3,SUB2 2]
17  FIB  0  [SUB2 6,SUB2 5,PLUS 3,PLUS 1]
18  APP  1  [SUB2 6,SUB2 5,PLUS 3,PLUS 1]
19  APP  2  [SUB2 6,SUB2 5,PLUS 3]
20  APP  5  [SUB2 6,SUB2 5]
21  FIB  3  [SUB2 6,PLUS 5]
22  FIB  2  [SUB2 6,PLUS 5,SUB2 3]
23  FIB  1  [SUB2 6,PLUS 5,SUB2 3,SUB2 2]
24  APP  1  [SUB2 6,PLUS 5,SUB2 3,SUB2 2]
25  FIB  0  [SUB2 6,PLUS 5,SUB2 3,PLUS 1]
26  APP  1  [SUB2 6,PLUS 5,SUB2 3,PLUS 1]
27  APP  2  [SUB2 6,PLUS 5,SUB2 3]
28  FIB  1  [SUB2 6,PLUS 5,PLUS 2]
29  APP  1  [SUB2 6,PLUS 5,PLUS 2]
30  APP  3  [SUB2 6,PLUS 5]
31  APP  8  [SUB2 6]
32  FIB  4  [PLUS 8]
33  FIB  3  [PLUS 8,SUB2 4]
34  FIB  2  [PLUS 8,SUB2 4,SUB2 3]
35  FIB  1  [PLUS 8,SUB2 4,SUB2 3,SUB2 2]
36  APP  1  [PLUS 8,SUB2 4,SUB2 3,SUB2 2]
37  FIB  0  [PLUS 8,SUB2 4,SUB2 3,PLUS 1]
38  APP  1  [PLUS 8,SUB2 4,SUB2 3,PLUS 1]
39  APP  2  [PLUS 8,SUB2 4,SUB2 3]
40  FIB  1  [PLUS 8,SUB2 4,PLUS 2]
41  APP  1  [PLUS 8,SUB2 4,PLUS 2]
42  APP  3  [PLUS 8,SUB2 4]
43  FIB  2  [PLUS 8,PLUS 3]
44  FIB  1  [PLUS 8,PLUS 3,SUB2 2]
45  APP  1  [PLUS 8,PLUS 3,SUB2 2]
46  FIB  0  [PLUS 8,PLUS 3,PLUS 1]
47  APP  1  [PLUS 8,PLUS 3,PLUS 1]
48  APP  2  [PLUS 8,PLUS 3]
49  APP  5  [PLUS 8]
50  APP  13 []
```

We turned the recursive cps into a function that uses no OCaml stack space

The transformed cps function carries its own stack as an extra argument

We transformed cps incrementally, with each step easily proved correct

Next time: application to **interpreter 0**