de Bruijn's presentation

λ-term

$$\lambda x.x(\lambda y.xy(\lambda z.xyz))$$

de Bruijn term

$$\lambda.0(\lambda.10(\lambda.210))$$

de Bruijn terms are good for implementation, unreadable as a notation.

From λ-terms to de Bruijn terms

Definition

The translation $(\_)^*$: set of λ-terms $\longrightarrow$ set of de Bruijn terms is defined recursively as follows:

$$\begin{aligned}(x)^* &= x\\ (\lambda x.M)^* &= \lambda.\mathrm{abs}(0,x,M^*)\\ (MN)^* &= M^*N^*\end{aligned}$$

where

$$\begin{aligned}\mathrm{abs}(i,x,x) &= i\\ \mathrm{abs}(i,x,y) &= y\\ \mathrm{abs}(i,x,n) &= n\\ \mathrm{abs}(i,x,\lambda.M) &= \lambda.(\mathrm{abs}(i+1,x,M))\\ \mathrm{abs}(i,x,MN) &= \mathrm{abs}(i,x,M)\mathrm{abs}(i,x,N)\end{aligned}$$

Summary of implementation methods

1. ISWIM $\longrightarrow$ λ-calculus $\longrightarrow$ SECD machine;
2. λ-calculus $\longrightarrow$ de Bruijn notation;
3. λ-calculus $\longrightarrow$ combinators $\longrightarrow$ graph reduction.

de Bruijn terms

Definition

The **de Bruijn terms** are constructed recursively from a given set of variables V and the natural numbers. They may take one of the following forms:

x free variable;

n natural number;

$\lambda.t$ where t is a de Bruijn term;

t_1t_2 where t_1 and t_2 are de Bruijn terms.

Examples

1. $(\lambda x.x(\lambda y.uxy))^* = \lambda.0(\lambda.u10)$;

2. $(\lambda z.(\lambda xy.xz)(uz))^* = \lambda.(\lambda.\lambda.12)(u0)$;

3. $((\lambda x.x(\lambda y.uxy))v)^* = (\lambda.0(\lambda.u10))v$.