Exercises: sheet 1

1. Insert the full amount of parentheses in the following abbreviated terms:
   (a) $ux(yz)(\lambda v.vy)$.
   (b) $(\lambda xyz.xz(yz))uvw$.
   (c) $w(\lambda xyz.xz(yz))uv$.

2. (a) Evaluate (up to α-conversion) the substitutions
      i. $(\lambda y.x(\lambda x.x))[(\lambda y.xy)/x]$;
      ii. $(y(\lambda v.xv))[(\lambda y.vy)/x]$.
   (b) Under what conditions does the equality $M[N/x][L/y] \equiv_\alpha M[L/y][N/x]$ hold?

3. Reduce the following λ-terms to normal form, or indicate that a normal form does not exist:
   (a) $P \equiv (\lambda x.x(xy))I$ where $I \equiv \lambda u.u$.
   (b) $Y \equiv \lambda f.QQ$ where $Q \equiv (\lambda x.f(xx))$.
   (c) $L \equiv (\lambda x.xxy)(\lambda x.xxy)$.
   (d) $(\lambda x.xL)M$ where $M \equiv \lambda x.y$ and L is defined in 3c.

4. Find all possible reduction paths to normal form for the following λ-terms, where $I \equiv \lambda u.u$:
   (a) $(\lambda x.xx)((\lambda x.xy)I)$.
   (b) $I((\lambda y.Ix)z)$.
   (c) $(\lambda u.v)L$ where $L \equiv (\lambda x.xxy)(\lambda x.xxy)$.

   State and comment on which reduction paths are shortest.

5. Consider the λ-terms given in part 3. Show that the following pairs of λ-terms are equal or prove otherwise:
   (a) P and $(\lambda x.xL)M$.
   (b) L and I;
   (c) Yf and $f(Yf)$.

6. For the λ-terms in part 4, identify which reduction paths satisfy the call-by-name and call-by-value reduction strategies.