Further Graphics 2022

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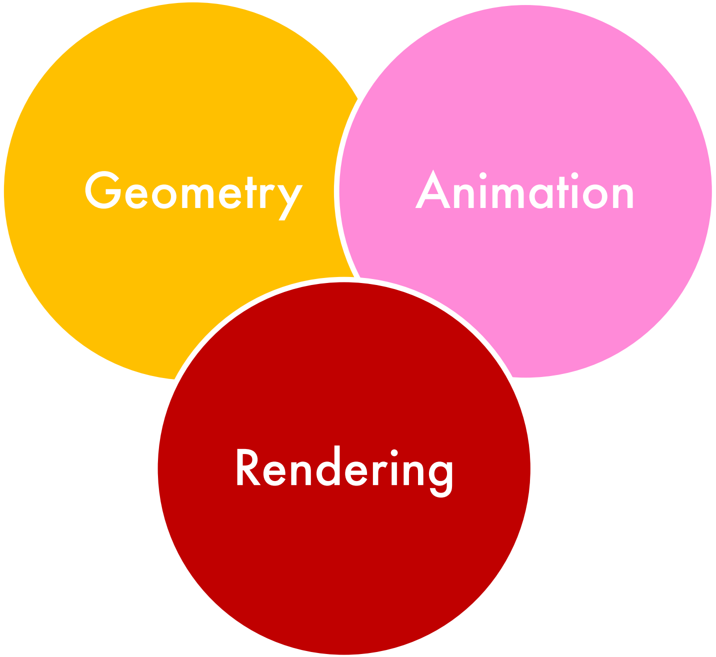
Cengiz Öztireli

Computer Graphics



Goal:

understand the fundamentals of **representing and rendering scenes** for computer aided image generation

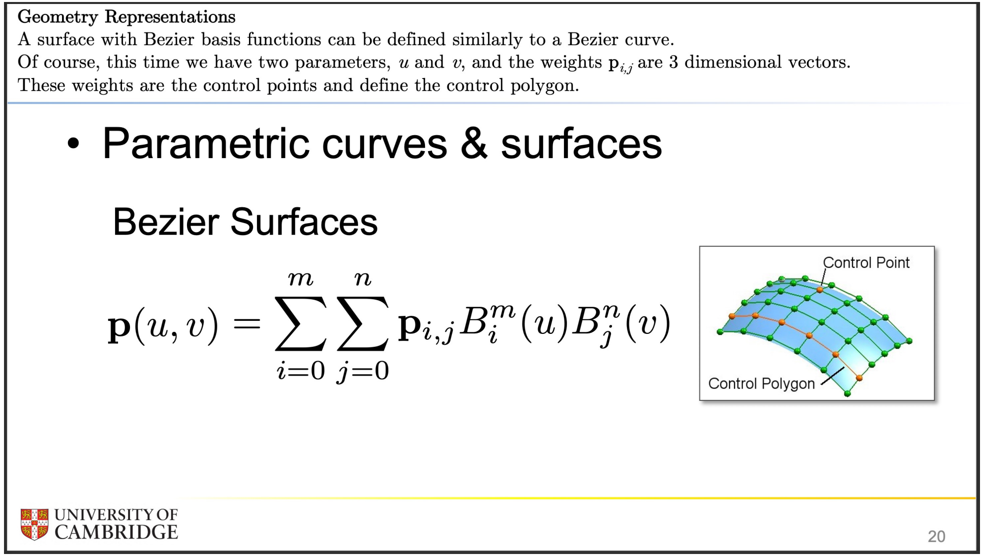
Lectures & Tick

Geometry Representations
Discrete Differential Geometry
Geometry Processing
Animation I
Animation II
The Rendering Equation
Distributed Ray Tracing
Inverse Rendering

Tick:
Approximately 2 weeks
Two coding exercises
Deadline:
Thursday, October 27, 12:00 PM

Lectures & Tick

Lecture notes with detailed notes for each slide shown in the lectures will be available via the **course webpage** and **Moodle**.



Geometry Representations

A surface with Bezier basis functions can be defined similarly to a Bezier curve.
Of course, this time we have two parameters, u and v, and the weights $\mathbf{p}_{i,j}$ are 3 dimensional vectors.
These weights are the control points and define the control polygon.

- Parametric curves & surfaces

Bezier Surfaces

$$\mathbf{p}(u,v) = \sum_{i=0}^{m}\sum_{j=0}^{n} \mathbf{p}_{i,j} B_i^m(u) B_j^n(v)$$

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Geometry Representations

Dr Cengiz Öztireli

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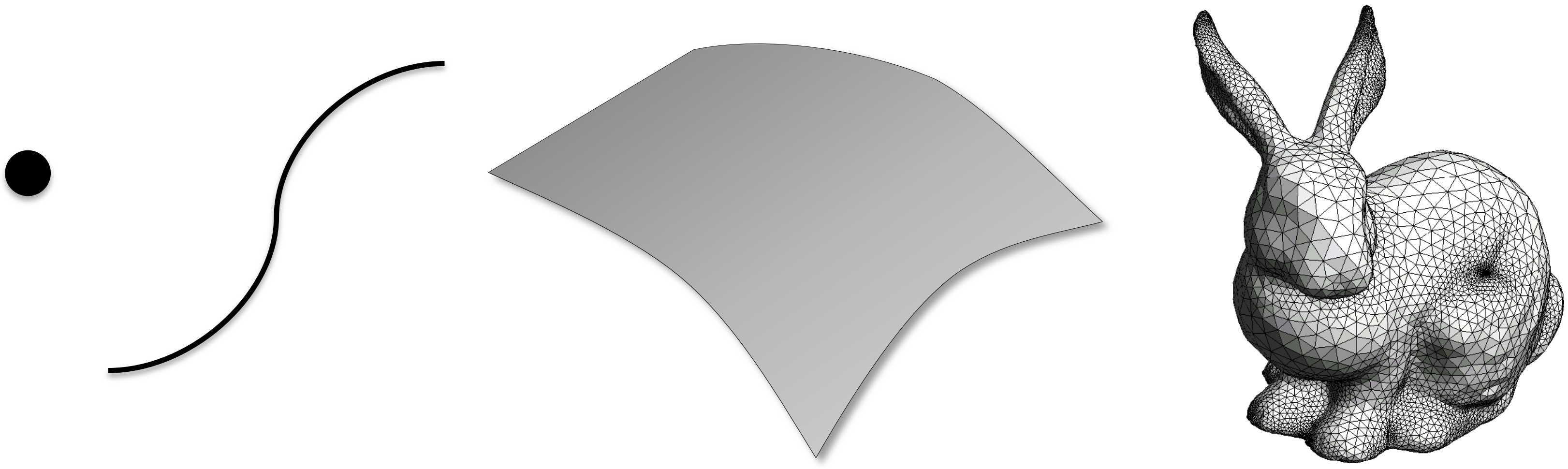
Geometry in Graphics

Geometry comes in many forms.

Examples, left to right: points, lines, continuous surfaces, surface meshes.



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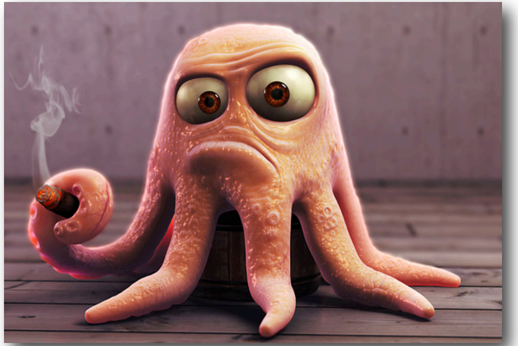
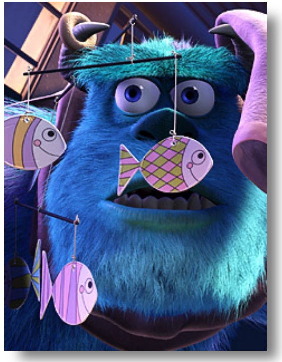
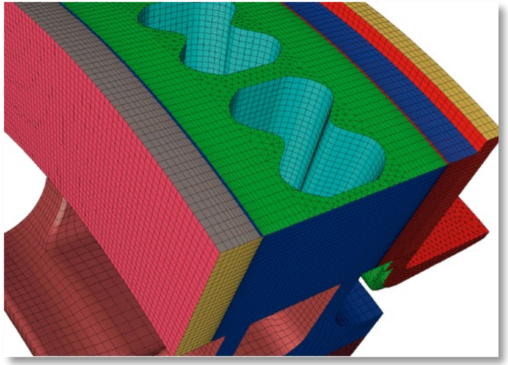
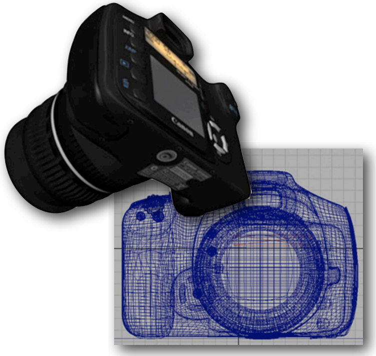
Applications





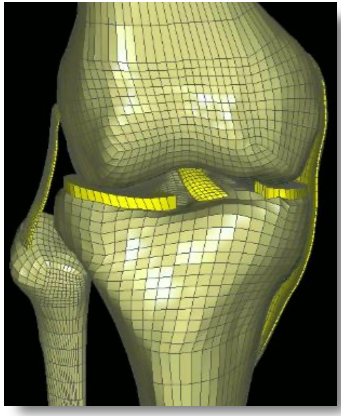
Games/Movies



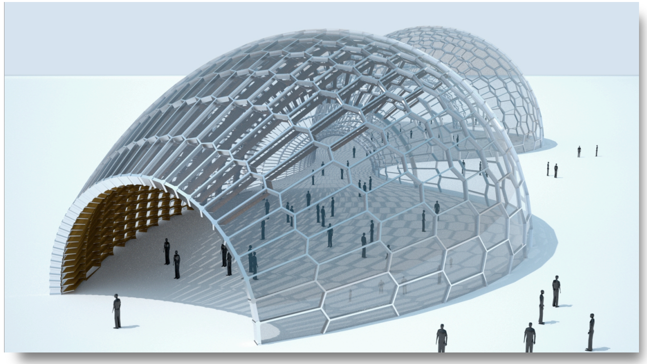
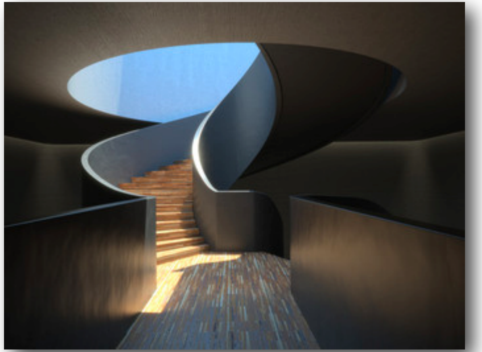


Engineering/Product design

Applications




Medicine/Biology

Architecture

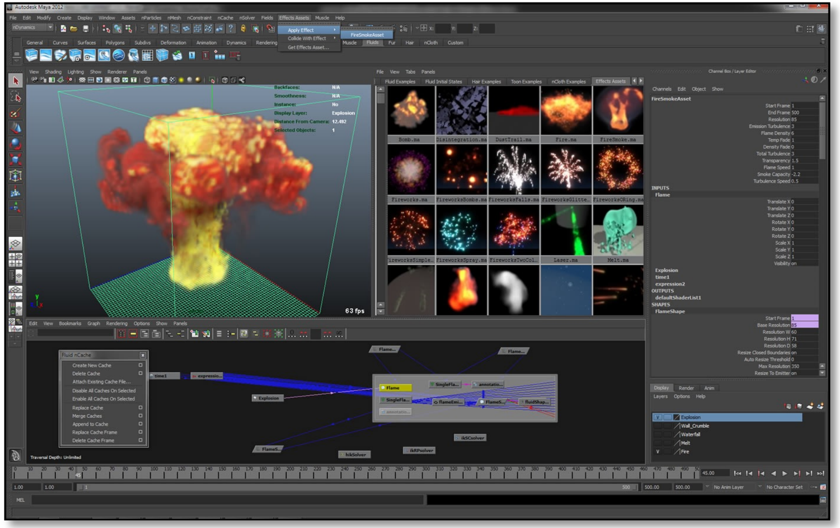
Sources of Geometry

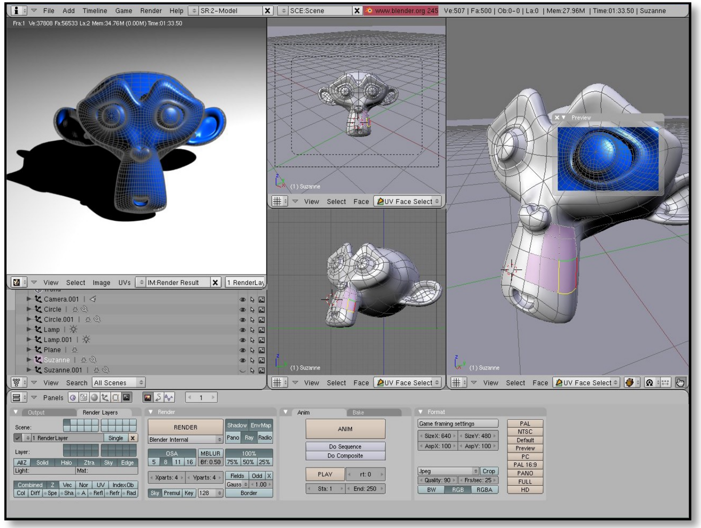
- Acquired real-world objects

3D Scanning

Sources of Geometry

- Digital 3D modeling



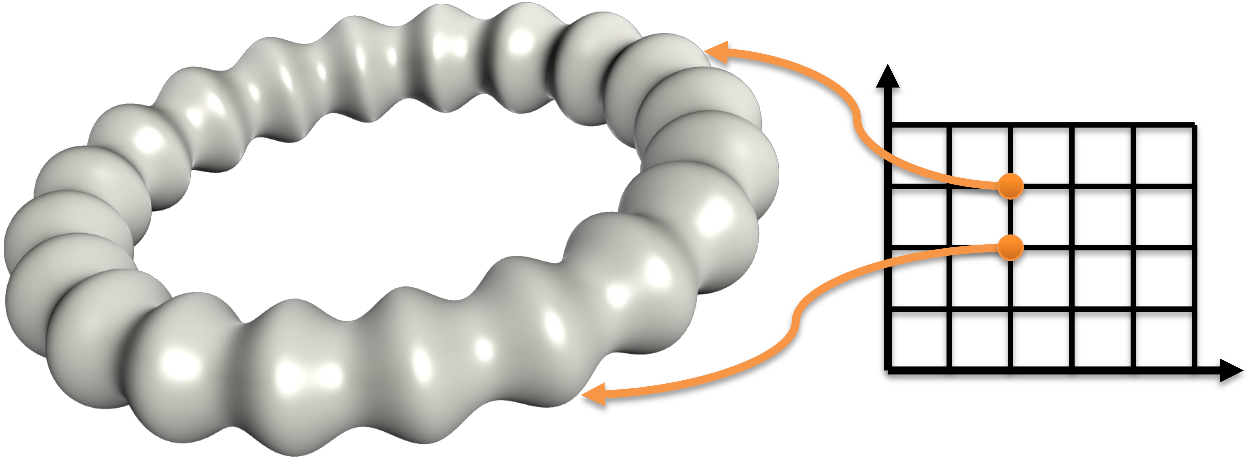


Geometry Representations

- Considerations
  - Storage
  - Acquisition of shapes
  - Creation of shapes
  - Editing shapes
  - Rendering shapes

The surface is represented as a map from (in this case) a 2D plane to a surface in 3D.
We thus have m degrees of freedom, although the object is embedded in an n - dimensional space.

- Parametric curves & surfaces



$$f : X \rightarrow Y, X \subseteq \mathbb{R}^m, Y \subseteq \mathbb{R}^n$$

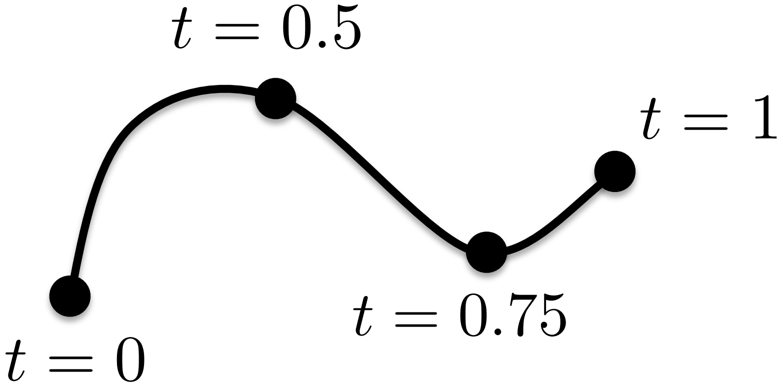
As another simple example, we have $m = 1$ and $n = 2$ for curves on a plane.
As we change t from 0 to 1, we trace a curve in 2D.

- Parametric curves & surfaces

Planar Curves

$$f : X \to Y, X \subseteq \mathbb{R}^m, Y \subseteq \mathbb{R}^n \quad m = 1, n = 2$$

$$\mathbf{p}(t) = (x(t), y(t))$$

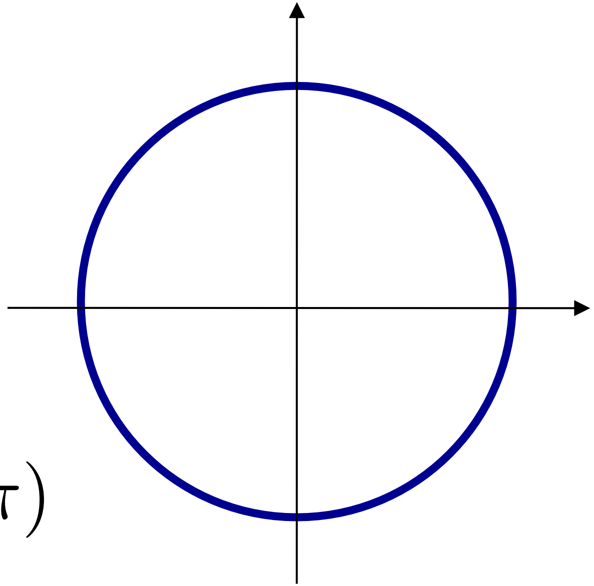


A special case of a planar curve is a circle.
As we change t from 0 to 2 pi, we trace a circle on a plane.

- Parametric curves & surfaces

Circle

$$\mathbf{p} : \mathbb{R} \to \mathbb{R}^2$$
$$t \mapsto \mathbf{p}(t) = (x(t), y(t))$$



$$\mathbf{p}(t) = r\,(\cos(t), \sin(t)) \quad t \in [0, 2\pi)$$

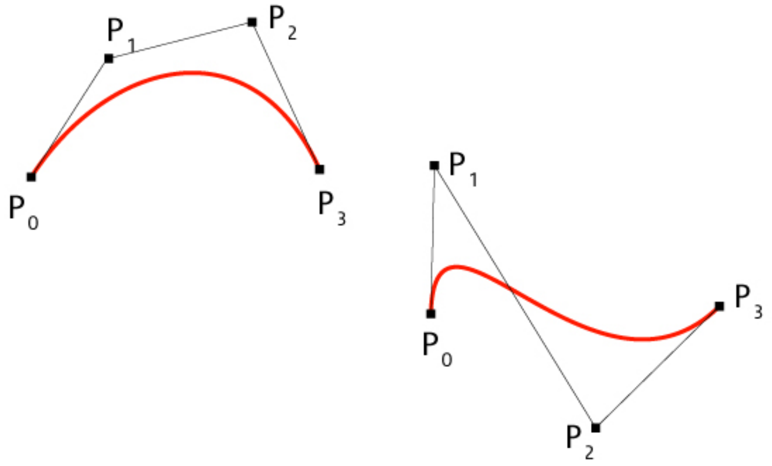
Geometry Representations
A more complex case is a Bezier curve, where we have a weighted combination of basis functions.
The weights $\mathbf{p}_i$, also called control points, are vectors in 2D.
Note: We always denote vectors with boldface lowercase letters.

- Parametric curves & surfaces

Bezier Curves

$$\mathbf{p}(t) = \sum_{i=0}^{n} \mathbf{p}_i B_i^n(t)$$

$$B_i^n(t) = \binom{n}{i} t^i (1-t)^{n-i}$$



We can similarly define a curve in 3D by having three functions, one for each coordinate.
For each t, we have a different point in 3D on the curve.

- Parametric curves & surfaces

Space Curves in 3D

$$f : X \to Y, X \subseteq \mathbb{R}^m, Y \subseteq \mathbb{R}^n$$

$$m = 1, n = 3$$

$$\mathbf{p}(t) = (x(t), y(t), z(t))$$

Geometry Representations

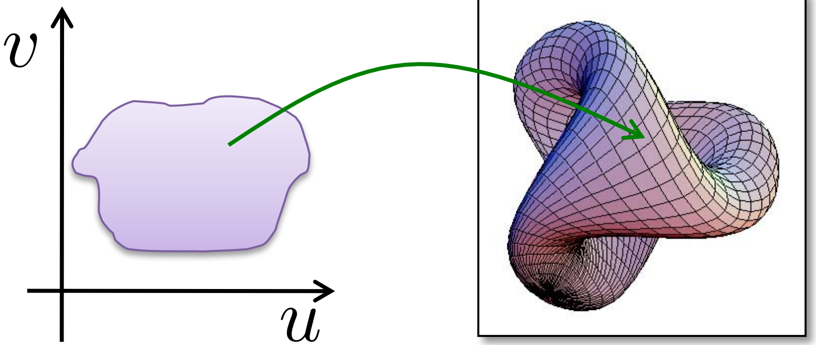
If we set $m = 2$, we get two degrees of freedom, which defines a surface.
Denoting the parameter space with u and v, each point on the surface in 3D is given by three functions x, y, z.

- Parametric curves & surfaces

Surfaces

$$f : X \to Y, X \subseteq \mathbb{R}^m, Y \subseteq \mathbb{R}^n \qquad m = 2, n = 3$$

$$\mathbf{p}(u, v) = (x(u, v), y(u, v), z(u, v))$$



A special case of a 2-surface embedded in 3D is a sphere.
Note the ranges of the $(u,\ v)$ pair.

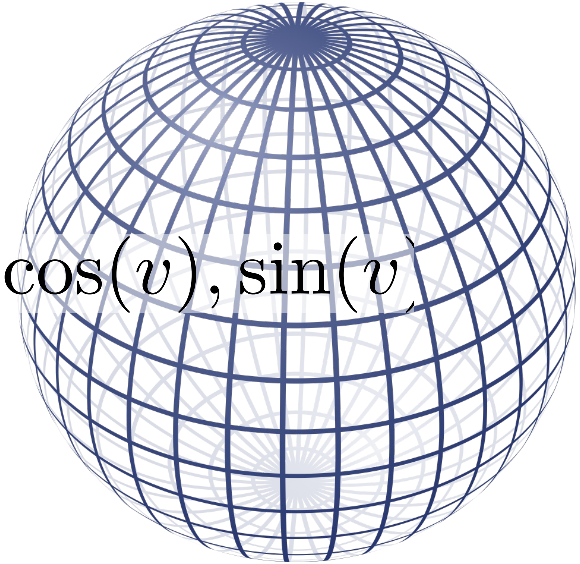
• Parametric curves & surfaces

Sphere

$$\mathbf{p} : \mathbb{R}^2 \rightarrow \mathbb{R}^3$$

$$\mathbf{p}(u,v) = r\left(\cos(u)\cos(v), \sin(u)\cos(v), \sin(v)\right)$$

$$(u,v) \in [0, 2\pi) \times [-\pi/2, \pi/2]$$

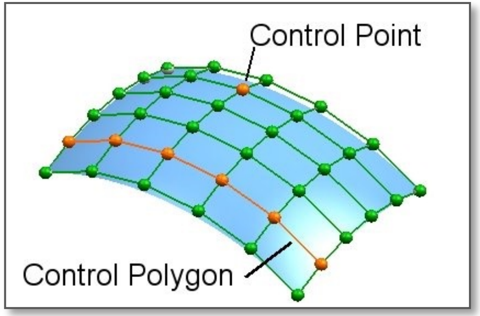


A surface with Bezier basis functions can be defined similarly to a Bezier curve.
Of course, this time we have two parameters, u and v, and the weights $\mathbf{p}_{i,j}$ are 3 dimensional vectors.
These weights are the control points and define the control polygon.

- Parametric curves & surfaces

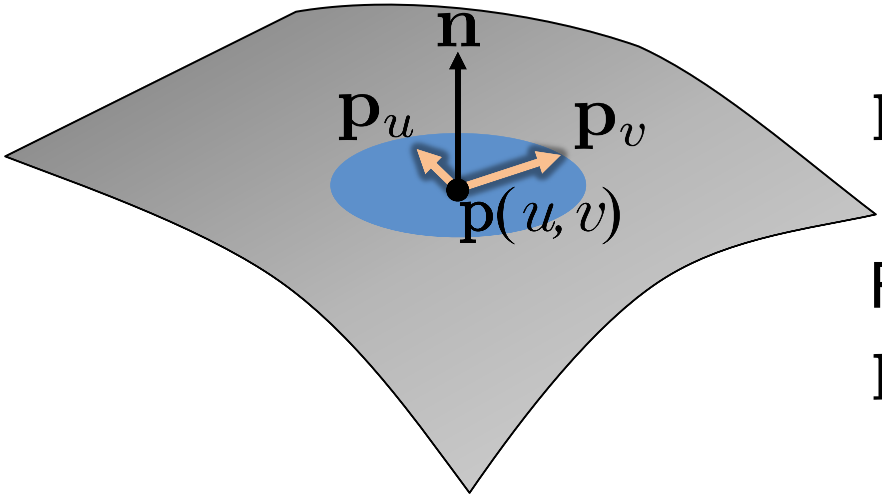
Bezier Surfaces

$$\mathbf{p}(u,v) = \sum_{i=0}^{m}\sum_{j=0}^{n}\mathbf{p}_{i,j}B_i^m(u)B_j^n(v)$$



Perhaps the most important local feature of a surface is a tangent plane.
It is defined as the plane spanned by the vectors computed by taking the derivatives w.r.t. u and v.
The surface normal $\mathbf{n}$ is defined as the vector orthogonal to the tangent plane.

- Parametric curves & surfaces



$$\mathbf{p}_u = \frac{\partial \mathbf{p}(u,v)}{\partial u}, \quad \mathbf{p}_v = \frac{\partial \mathbf{p}(u,v)}{\partial v}$$

Regular parametrization:

$$\mathbf{p}_u \times \mathbf{p}_v \neq 0$$

$$\mathbf{n}(u,v) = \frac{\mathbf{p}_u \times \mathbf{p}_v}{\|\mathbf{p}_u \times \mathbf{p}_v\|}$$

Volumetric representations of geometry can be thought of as fields in 3D, where at each point we have a value that determines e.g. the density of a tissue at that point. We can similarly define color fields, etc.

- Parametric curves & surfaces

Volumetric Representations

$$f : X \to Y, X \subseteq \mathbb{R}^m, Y \subseteq \mathbb{R}^n \quad m = 3, n = 1$$

Geometry Representations

- Parametric curves & surfaces
  - \+ Easy to generate points on a curve/surface
  - \+ Easy point-wise differential properties
  - \+ Easy to control by hand
  - − Hard to determine inside/outside
  - − Hard to determine if a point is on a curve/surface
  - − Hard to generate by reverse engineering

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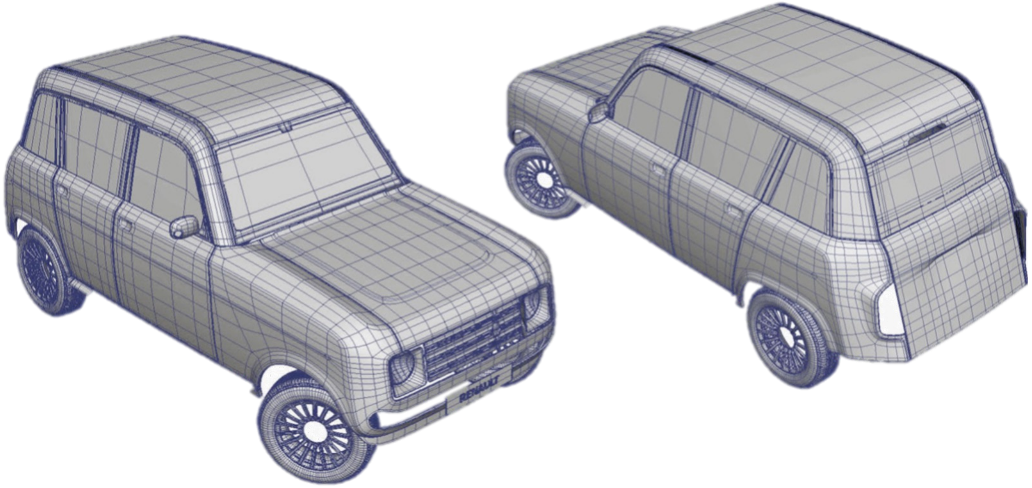
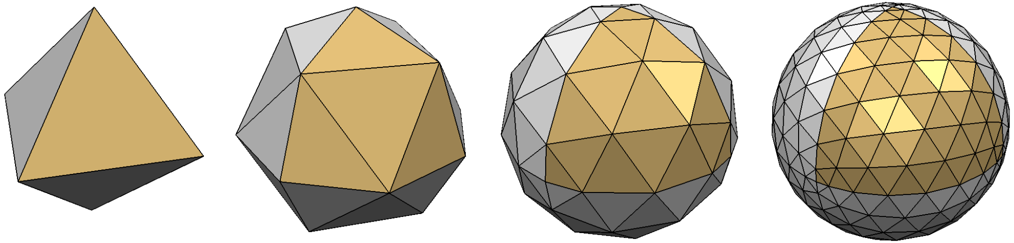
Meshes in computer graphics are points in 2D or 3D connected by edges forming polygons, e.g. triangles.
They represent the boundary of an object as a discrete surface.
They are the standard for modeling geometry e.g. for games.

- Polygonal Meshes



A polygonal mesh is a piecewise linear approximation of an underlying continuous surface.
As we increase the number of polygons, we get better approximations.

- Polygonal Meshes



Piecewise linear approximation

A surface mesh is a graph embedded in 3D. Each vertex in V is associated with a point $\mathbf{p}$ in 3D. The vertices are connected by edges in E. The edges form faces stored as a set F. Hence, each edge stores two vertex indices and each face stores three vertex indices.

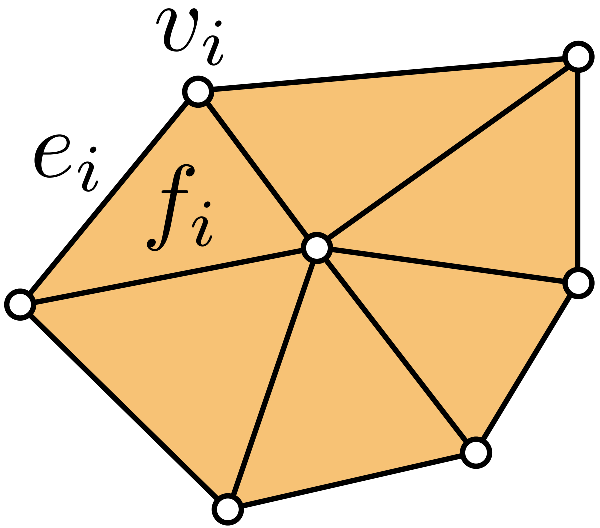
- Triangle Meshes

$$V = \{v_1, \ldots, v_n\}$$

$$E = \{e_1, \ldots, e_k\}, \quad e_i \in V \times V$$

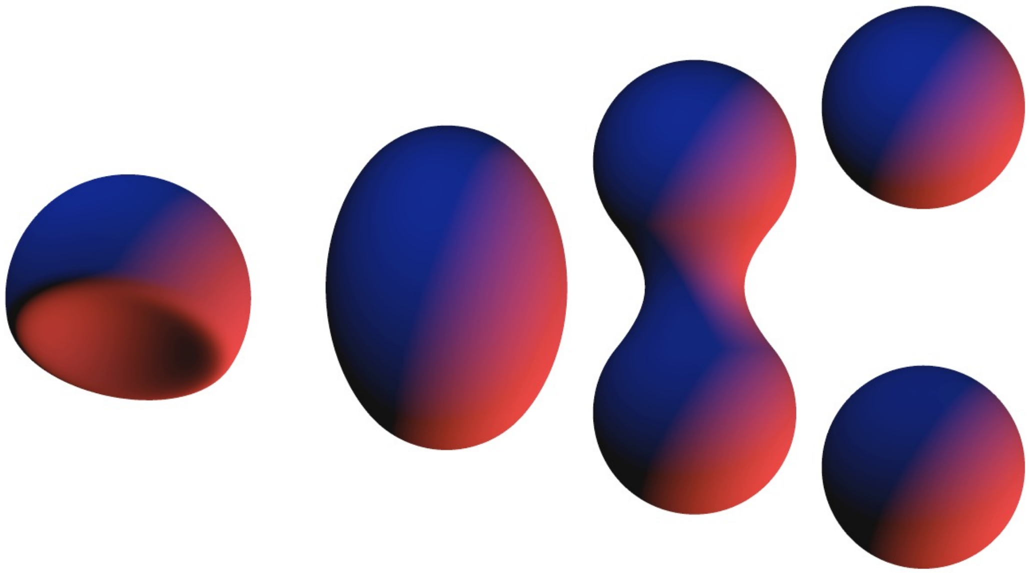
$$F = \{f_1, \ldots, f_m\}, \quad f_i \in V \times V \times V$$

$$P = \{\mathbf{p}_1, \ldots, \mathbf{p}_n\}, \quad \mathbf{p}_i \in \mathbb{R}^3$$



Geometry Representations

- Implicit surfaces

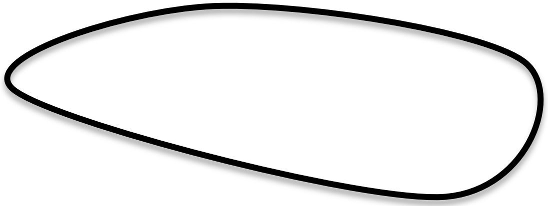


- Implicit curves & surfaces

$$f : \mathbb{R}^m \rightarrow \mathbb{R}$$

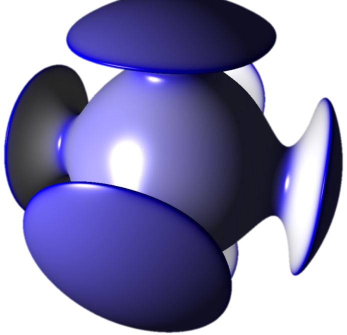
Planar Curves

$$S = \{x \in \mathbb{R}^2 | f(x) = 0\}$$



Surfaces in 3D

$$S = \{x \in \mathbb{R}^3 | f(x) = 0\}$$



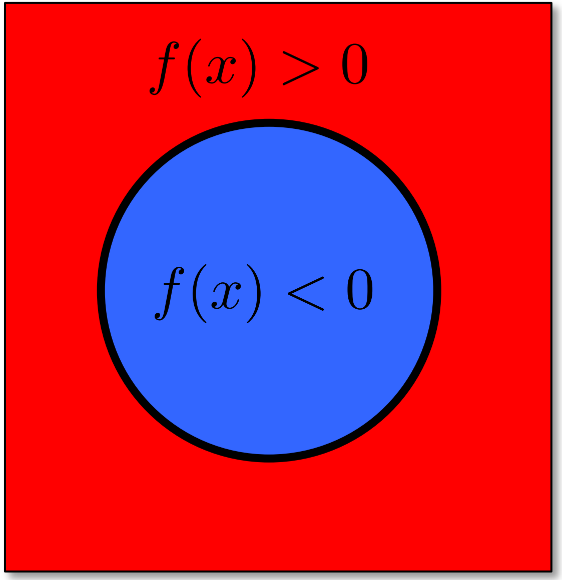
We can further define inside and outside of an object as the regions where the implicit function has different signs. Note that we can define the sign freely as both f and $-f$ represent the same implicit surface.

- Implicit curves & surfaces

$\{x \in \mathbb{R}^m | f(x) > 0\}$ Outside

$\{x \in \mathbb{R}^m | f(x) = 0\}$ Curve/Surface

$\{x \in \mathbb{R}^m | f(x) < 0\}$ Inside

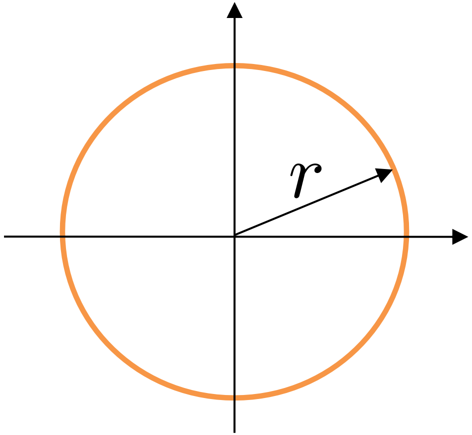


The forms of the implicit functions are quite simple for a circle and a sphere as compared to the parametric ones.

- Implicit curves & surfaces

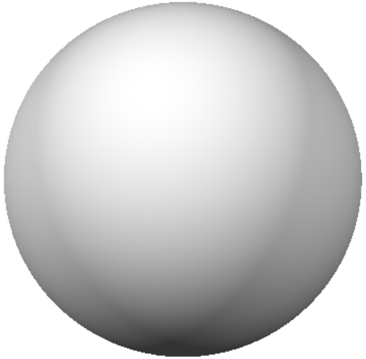
Circle

$$f(x, y) = x^2 + y^2 - r^2$$



Sphere

$$f(x, y, z) = x^2 + y^2 + z^2 - r^2$$



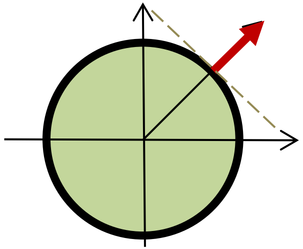
- Implicit curves & surfaces

Surface Normal

$$\nabla f(x,y,z) = \left(\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z}\right)^T$$

Sphere

$$f(x,y,z) = x^2 + y^2 + z^2 - r^2$$
$$\nabla f(x,y,z) = (2x, 2y, 2z)^T$$

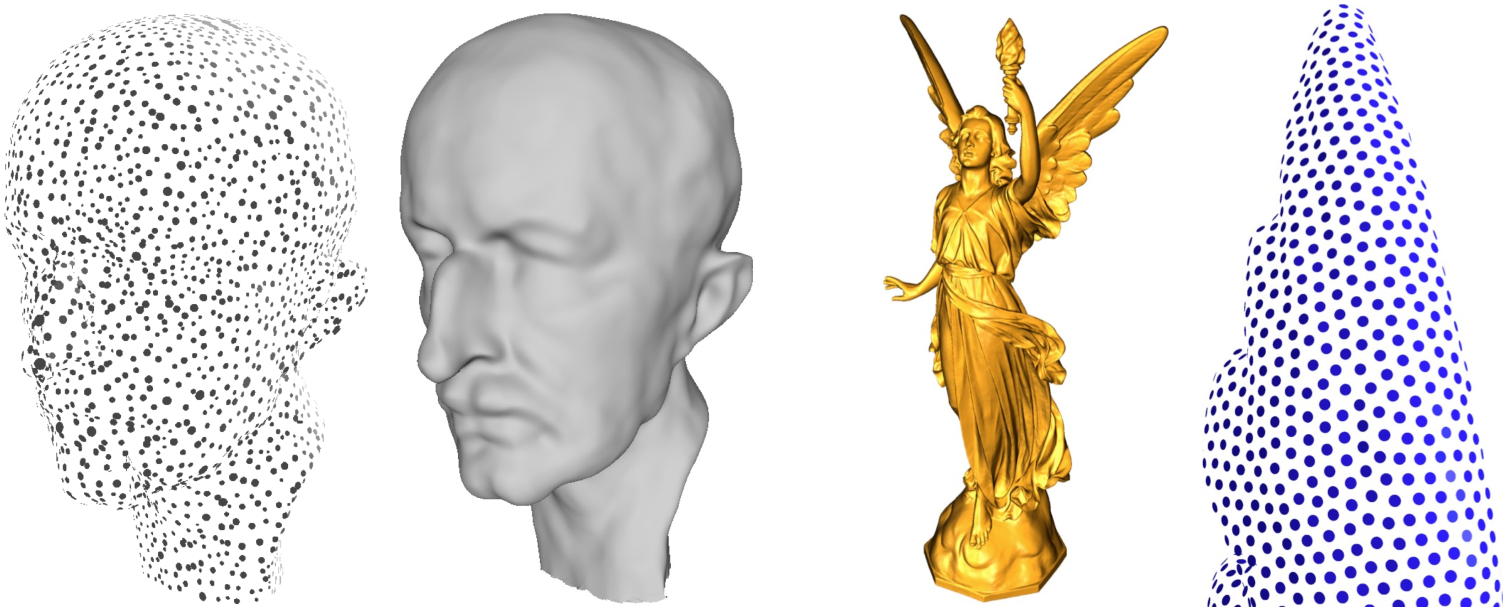


Geometry Representations

- Implicit curves & surfaces
  - + Easy to determine inside/outside
  - + Easy to determine if a point is on a curve/surface
  - + Easy to combine
  - − Hard to generate points on a curve/surface
  - − Limited set of surfaces
  - − Does not lend itself to (real-time) rendering

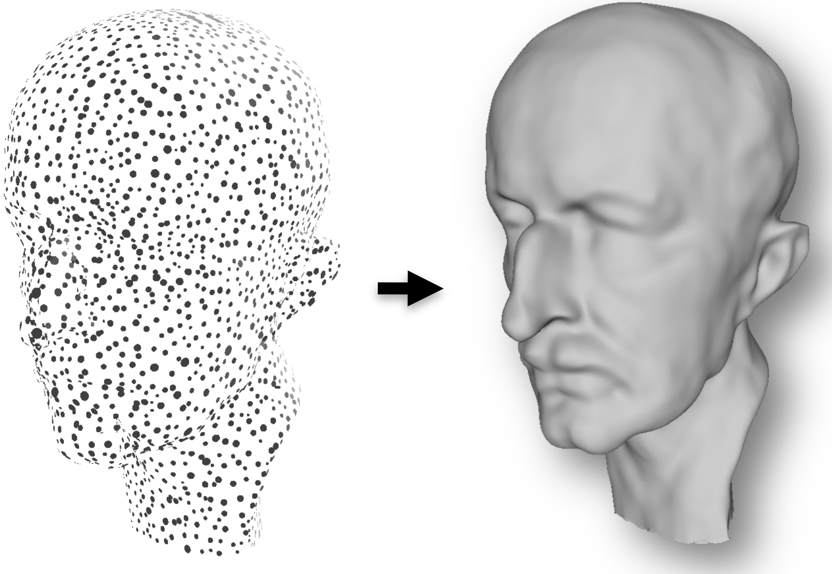
One practical representation that combines some advantages of parametric and implicit representations is point set surfaces.
A point set surface is defined in terms of a point cloud in 3D.

- Point Set Surfaces



A point cloud is a set of points possibly with point-wise attributes, e.g. position, surface normal, etc.
There are number of techniques from the approximation literature that define a point set surface.
These techniques work reliably with data acquired from the real world, so quite important in practice.

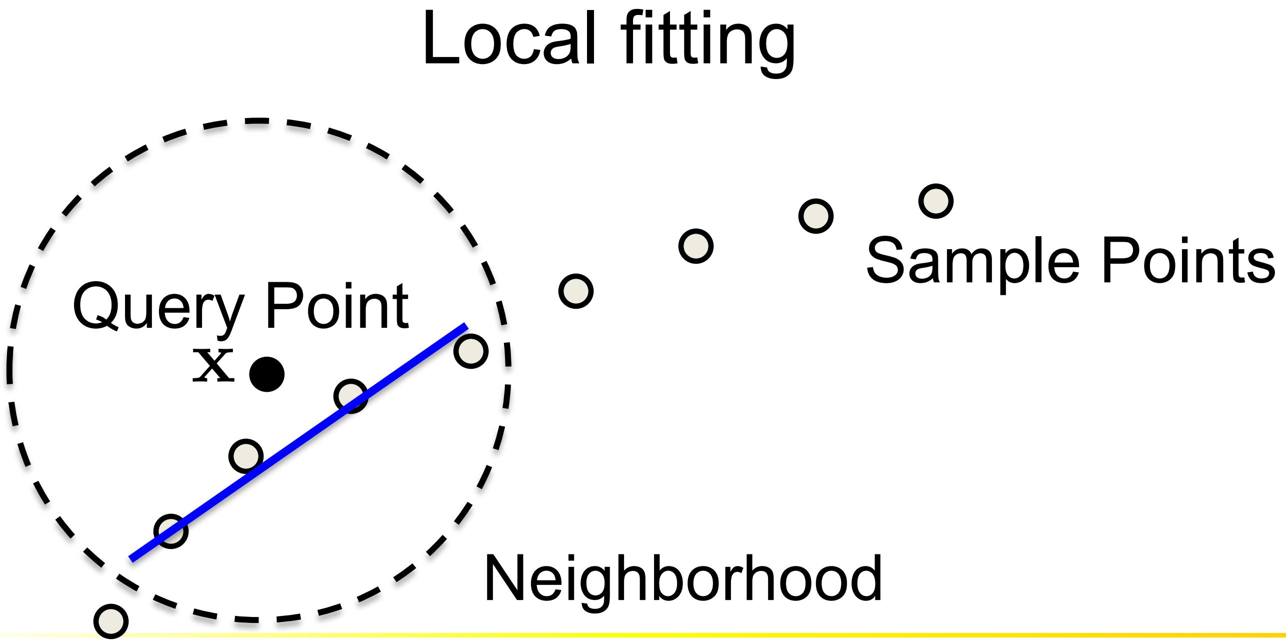
- Point Set Surfaces



Only point-wise attributes
Approximation methods
Smooth surfaces
Works on acquired data

One particular form a point set surface is defined via locally fitting a proxy surface.
We first choose a neighborhood around a query point and a simple proxy surface e.g. a line in this case.
The distance to the surface is then approximated with the distance to this local proxy surface.

- Point Set Surfaces

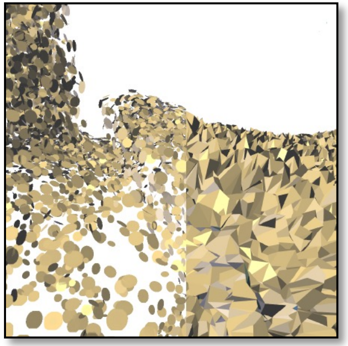


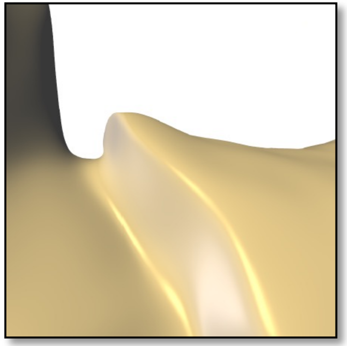
We thus get an implicit function that defines the surface.
It is also easy to generate points on the surface via projection, eliminating a limitation of implicit surfaces.

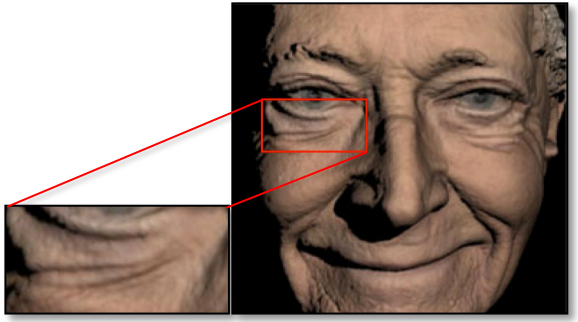
- Point Set Surfaces
  - Implicit representation & fast projection

A point set surface comes with the advantages of robustness to imperfect real-world point data, relatively fast rendering, and conversion to other surface representations such as meshes.

- Point Set Surfaces
  - Robust to noise
  - Direct rendering
  - Conversion to meshes







Geometry Representations

- Point Set Surfaces
  - \+ Easy to determine inside/outside
  - \+ Easy to determine if a point is on the curve/surface
  - \+ Easy to generate points on the curve/surface
  - \+ Suitable for reconstruction from general data
  - \+ Direct real-time rendering
  - − Not efficient to use in some modeling tasks