

## Complexity Theory

Lent 2004

### Suggested Exercises 2

1. Given a graph  $G = (V, E)$ , a set  $U \subseteq V$  of vertices is called a *vertex cover* of  $G$  if, for each edge  $(u, v) \in E$ , either  $u \in U$  or  $v \in U$ . That is, each edge has at least one end point in  $U$ . The decision problem **V-COVER** is defined as:

given a graph  $G = (V, E)$ , and an integer  $K$ , does  $G$  contain a vertex cover with  $K$  or *fewer* elements?

- (a) Show a polynomial time reduction from **IND** to **V-COVER**.
- (b) Use (a) to argue that **V-COVER** is **NP**-complete.

2. The problem of four dimensional matching, **4DM**, is defined analogously with **3DM**:

Given four sets,  $W, X, Y$  and  $Z$ , each with  $n$  elements, and a set of quadruples  $M \subseteq W \times X \times Y \times Z$ , is there a subset  $M' \subseteq M$ , such that each element of  $W, X, Y$  and  $Z$  appears in exactly one triple in  $M'$ .

Show that **4DM** is **NP**-complete.

3. Given a graph  $G = (V, E)$ , a *source vertex*  $s \in V$  and a *target vertex*  $t \in V$ , a *Hamiltonian Path* from  $s$  to  $t$  in  $G$  is a path that begins at  $s$ , ends at  $t$  and visits every vertex in  $V$  exactly once. We define the decision problem **HamPath** as:

given a graph  $G = (V, E)$  and vertices  $s, t \in V$ , does  $G$  contain a Hamiltonian path from  $s$  to  $t$ ?

- (a) Give a polynomial time reduction from the Hamiltonian cycle problem to **HamPath**.
- (b) Give a polynomial time reduction from **HamPath** to the problem of determining whether a graph has a Hamiltonian cycle.

*Hint:* consider adding a vertex to the graph.