Slide 1

An introduction to the surface representations used in Computer Aided Design

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with considerable help
from Sabin, Barthe,
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Slide 2

Applications

✦ **Computer-aided design (CAD) of artistically attractive surfaces**

✦ **Principally the design of car bodies**

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Slide 3

Applications

✦ **The design of industrial artefacts and animated characters**

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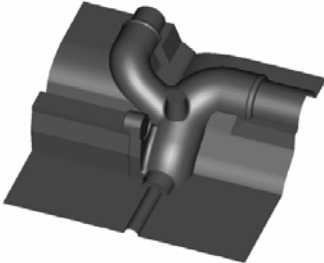
Slide 4

Applications

✦ **Computer-aided design of industrial artefacts which do not need to be beautiful**

◆ Such as internal engine parts

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Desirable features in CAD

+ **Need to handle *any* surface**
+ **Need guaranteed continuity**
  - Continuity of slope (C1)
    - Smooth surfaces
  - Continuity of curvature (C2)
    - Smoothly reflecting surfaces
    - Required for some aerodynamics
+ **Need to allow for discontinuities**
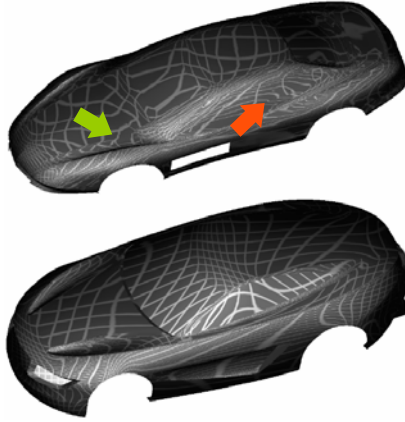 - Edges, creases and holes
+ **Needs to be easy to use**

Reflection patterns on car models

+ **The top car has problems**
  - Some discontinuities in curvature ●
  - Some badly distorted areas owing to rapidly changing curvature ●
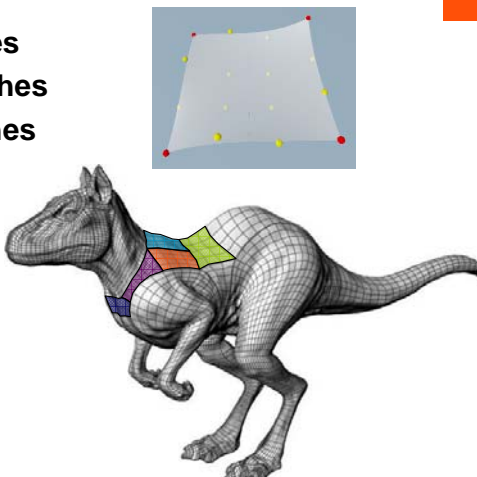+ **Only the bottom car is a "Class A" surface**
  - The reflection lines look nice

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Traditional tools

+ **Bezier patches**
+ **B-spline patches**
+ **NURBS patches**
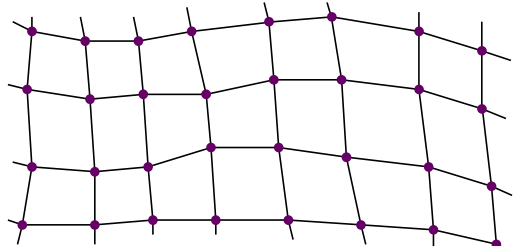
Bezier & B-spline patches

+ **A rectangular array of control points**
+ **A mathematical function determines where the surface goes based on those points**
+ **Move a control point to change the surface**

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The first problem

✦ **Very few objects are made up of a single rectangular patch, so we need to join patches together**

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The mathematics of joins

✦ **We want to preserve certain types of mathematical continuity across joins**
  ◆ C0: continuity of position
    ▪ Prevents holes at the join
  ◆ C1: continuity of slope
    ▪ Prevents a sharp edge at the join
  ◆ C2: continuity of curvature
    ▪ Strongly related to aesthetics
    ▪ Most often visible in reflections
    ▪ Prevents sharp edges in reflected lines
  ◆ These are continuity of the zeroth, first and second derivatives

C0 discontinuity

C1 discontinuity

C2 discontinuity

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Joining two Bezier patches

✦ **C0 but not C1**
  ◆ Four edge points are the same

✦ **C0 and C1**
  ◆ Four edge points are the same
  ◆ Next four points out in either direction are constrained

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Joining four Bezier patches

Constraints

C0 ●

C1 ● ●

C2 ● ● ●
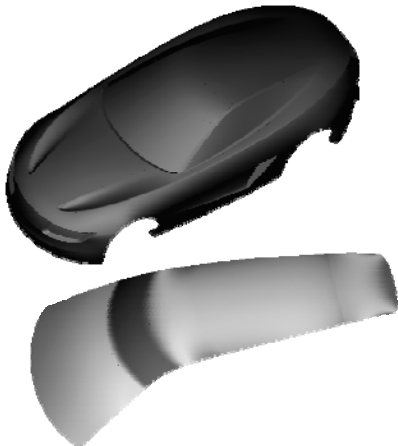
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An example: the car's roof
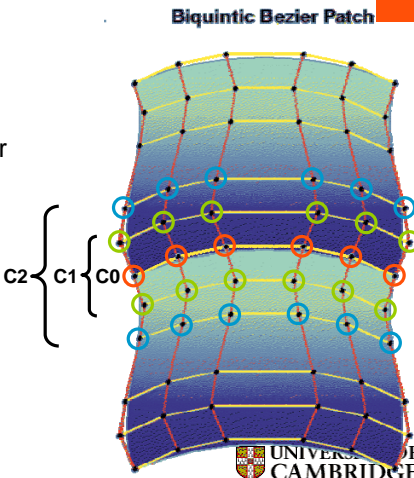
✦ **The car**

✦ **Curvature plot of its roof**

Definition of the car's roof

✦ **5×2 grid of biquintic Bezier patches**

  ◆ 36 control points per patch

  ◆ 286 control points overall

  ◆ Moving one point also moves several others to maintain C2 continuity
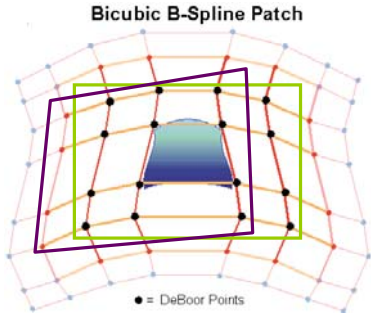
Biquintic Bezier Patch

C2 { C1 { C0

B-spline patches

✦ **A rectangular array of points define a rectangular array of *automatically joined* patches**

✦ **Example**

  ◆ The black points control the blue patch

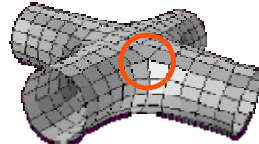 ◆ All points together define a surface of many joined patches

Bicubic B-Spline Patch

● = DeBoor Points

The second problem

✦ **What do we do at special points where other than four patches meet?**

  ◆ Either we cannot get C2
    ▪ Which means that curvature is not continuous
  ◆ Or we get C2 be forcing curvature to be zero
    ▪ Which produces a flat spot
  ◆ Or we get C2 using very high degree patches
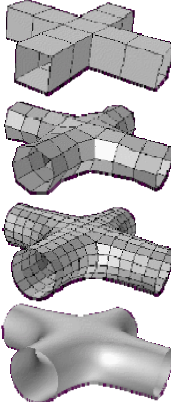   ▪ Which are very hard for a designer to control

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Subdivision surfaces

- Developed in the 1970s, adopted in computer animation in 1990s
- Replace the patch-based representation of B-splines and Beziers
- Base a curve or surface solely on its control points and their connectivity
- A simple mechanism produces a larger, more refined set of control points from the current set
- Iterate refinement until the appropriate level of detail is achieved
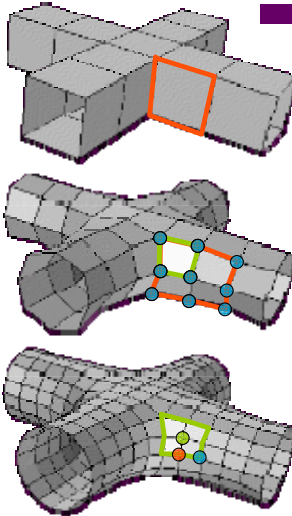

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Subdivision

- ✦ **Introduce new points**
  - At face-centres
  - At mid-edges
- ✦ **Adjust positions of original points**
- ✦ **Repeat until sufficiently detailed**

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Subdivision

- ✦ **Advantages**
  - Reproduces everything which can be done by B-splines
  - Handles extraordinary points much more easily
- ✦ **Disadvantages**
  - Cannot get C2 unless you produce a flat spot
  - Generates other visual artefacts, not seen in B-spline surfaces
- ✦ **Commercial position**
  - Subdivision is replacing B-splines in computer animation
  - Subdivision is **not** replacing B-splines in CAD

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