SUPERVISION GUIDE

PART IA PROBABILITY

The notes that accompany the twelve lectures of the course on Probability are in the form of an A4-sized book with yellow covers. A few copies are kept with the Student Administration Secretary. If there are none left, ask for the supply to be replenished.

Course Schedule

The schedule of the twelve lectures is provided on an accompanying sheet. This is a copy of the schedule given in the yellow-covered book.

The Course Notes and Exercises

The yellow-covered book incorporates twelve chapters, one for each lecture. At the end of most chapters there are exercises which relate to the material in the chapter. Supervisors should encourage students to work through these exercises.

Warning to Candidates

Throughout the course, almost all explicit probabilities are expressed as fractions and, in examples which make use of such fractions, working in fractions should be regarded as mandatory. Thus, in examinations, candidates who express the probability of a fair die landing with a 6 uppermost as 0.1667 (instead of $\frac{1}{6}$) will be at a severe disadvantage.

Solutions to the Exercises

The solutions to the end-of-chapter exercises are provided on accompanying sheets. These sheets are also handed to members of the class but only after they have been given a decent amount of time to attempt the exercises without the solutions to hand.

Recommended Books

The book which was most extensively consulted in the preparation of this course was *Probability: An Introduction* by G. Grimmet and D. Welsh. An important standard text is *An Introduction to Probability Theory, Volume I* by W. Feller.

F.H. King
7 January 1999

SCHEDULE OF LECTURES

1. **SINGLE RANDOM VARIABLE.** Probability in Computer Science. Sample space. Event space. Probability space. Relationship to set theory. Random variables: discrete *versus* continuous. The $\mathrm{P}(X{=}r)$ notation. Probability axioms. The inclusion-exclusion theorem. Conditional probability. Mapping.
2. **TWO OR MORE RANDOM VARIABLES.** The multiplication theorem. Independence and distinguishability. Array diagrams. Bayes's theorem. Event trees. Combinatorial numbers. Pascal's triangle. The binomial theorem.
3. **DISCRETE DISTRIBUTIONS.** Uniform distribution. Triangular distribution. Binomial distribution. Trinomial distribution. Multinomial distribution. Expectation or mean.
4. **MEANS AND VARIANCES.** Use of derived random variables and generalised expectation. Variance and standard deviation. Geometric distribution. Poisson distribution. Revision of summation (double-sigma sign). Mean and variance when there are two or more random variables. Covariance.
5. **CORRELATION.** Mean and variance of the Binomial distribution. Correlation coefficient. Complete positive and complete negative correlation. $\mathrm{P}(X{+}Y{=}t)$. A polynomial with probabilities as coefficients.
6. **PROBABILITY GENERATING FUNCTIONS.** Generating functions. Means and variances of distributions. Application of generating functions to $\mathrm{P}(X{+}Y{=}t)$.
7. **DIFFERENCE EQUATIONS.** Introduction to linear, second-order difference equations with constant coefficients. How these equations are found in Probability. Solving homogeneous and inhomogeneous difference equations.
8. **STOCHASTIC PROCESSES.** Random walks, recurrent *versus* transient. The gambler's ruin problem. Absorbing barriers. Probability of winning and losing. Expected length of a game.
9. **CONTINUOUS DISTRIBUTIONS.** The $\mathrm{P}(X{=}r)$ notation adapted to the continuous case. Probability density functions. Expectation and variance. Uniform distribution. Negative exponential distribution.
10. **BIVARIATE DISTRIBUTIONS.** Normal distribution. Standard form. The central limit theorem. Bivariate distributions. Illustrations.
11. **TRANSFORMING DENSITY FUNCTIONS.** Integration by substitution. Application to probability density functions. Transforming a uniform distribution. Transforming a Uniform distribution into a Normal distribution using Excel.
12. **TRANSFORMING BIVARIATE DENSITY FUNCTIONS.** Integration with two independent variables. Jacobians. Application to bivariate probability density functions. The Box–Muller Transformation.