

Mathematical Tripos Part IA

1988 Paper 6 Question 10

Discrete Mathematics

Let R be a relation on a set X . Define the reflexive, symmetric and transitive closures $r(R)$, $s(R)$ and $t(R)$ of R . Let Δ be the relation $\{(x, x) \mid x \in X\}$.

Prove that:

$$(a) \quad R \circ \Delta = R,$$

$$(b) \quad (R \cup \Delta)^n = \Delta \cup \bigsqcup \{R^i \mid 1 \leq i \leq n\}, \text{ for all } n \geq 1,$$

$$(c) \quad tr(R) = rt(R).$$

Show also that $st(R) \subseteq ts(R)$. If $X = \mathbb{N}$ and

$$R = \Delta \cup \{(x, y) \mid x, y \in \mathbb{N} \text{ s.t. } y = p.x \text{ for some prime } p \in \mathbb{N}\},$$

describe $st(R)$ and $ts(R)$.

[Notation. In this question $rt(R)$ stands for $r(t(R))$, and so on.]