

Complexity Theory

Easter 2001

Suggested Exercises 2

1. In the lecture, a proof of the Time Hierarchy Theorem was sketched. Give a similar argument for the following Space Hierarchy Theorem:

Space Hierarchy. For every constructible function f , there is a language in $\text{SPACE}(f(n) \cdot \log f(n))$ that is not in $\text{SPACE}(f(n))$.

Could you replace the factor of $\log f(n)$ in this statement with something even smaller?

2. Consider the algorithm presented in the lecture which establishes that Reachability is in $\text{SPACE}((\log n)^2)$. What is the time complexity of this algorithm?

Can you generalise the time bound to the entire complexity class? That is, give a class of functions F , such that

$$\text{SPACE}((\log n)^2) \subseteq \bigcup_{f \in F} \text{TIME}(f)$$

3. Show that, if $\text{SPACE}((\log n)^2) \subseteq \text{P}$, then $\text{L} \neq \text{P}$. (Hint: use the Space Hierarchy Theorem from Exercise sheet 1.)

4. Show that, for every nondeterministic machine M which uses $O(\log n)$ work space, there is a machine R with three tapes (input, work and output) which works as follows. On input x , R produces on its output tape a description of the configuration graph for M, x , and R uses $O(\log |x|)$ space on its work tape.

Explain why this means that if Reachability is in L , then $\text{L} = \text{NL}$.