Complexity Theory
Easter 2001
Suggested Exercises 1

1. In the lecture, a proof was sketched showing a $\Omega(n \log n)$ lower bound on the complexity of the sorting problem. It was also stated that a similar analysis could be used to establish the same bound for the Travelling Salesman Problem. Give a detailed sketch of such an argument.

2. On slide 24 of the notes, a number of functions are listed as being constructible. Show that this is the case by giving, for each one, a description of an appropriate Turing machine.

Prove that if f and g are constructible functions and $f(n) \geq n$, then so are $f(g)$, $f + g$, $f \cdot g$ and 2^f.

3. For any constructible function f, and any language $L \in \mathsf{TIME(f(n))}$, there is a machine M that accepts L and halts in time $O(f(n))$ for all inputs of length n. Give a detailed argument for this statement, describing how M might be obtained from a machine accepting L in time $f(n)$.

4. Consider the language L in the alphabet $\{a, b\}$ given by $L = \{a^n b^n \mid n \in \mathbb{N}\}$. $L \notin \mathsf{SPACE(c)}$ for any constant c. Why?