COMPUTER SCIENCE TRIPOS, Part II (General)
DIPLOMA IN COMPUTER SCIENCE

Mathematics for Computation Theory (KM 1999)

Past Tripos and Diploma questions

Questions on Discrete Mathematics have in the past been set in Part IA (for both Mathematics and Computer Science) and in the Diploma and Part II (General).

Questions on Regular Languages were set on a course given both to Part IB and to the Diploma and Part II (General) until the year 1994–95. From the year 1995–96 that course was moved to CST IA (the 50% option): the present course covers similar material from a more algebraic viewpoint, the intention being to illustrate discrete mathematical structures and the associated proof techniques.

Many past examples are suitable for the present Diploma and Part II (General) course.

Part A

Relations on sets (including equivalence relations and transitive closure):

Maths Part IA 1988	Paper 6	Question 10.
CST Part IA 1990	Paper 1	Question 11.
CST Part IA 1993	Paper 2	Question 3.
Diploma 1989	Paper 2	Question 7.
Diploma 1990	Paper 3	Question 4.
Diploma 1993	Paper 1	Question 11.
Diploma 1998	Paper 1	Question 10.
Diploma 1999	Paper 1	Question 10.

Partially Ordered sets:

CST Part IA 1989	Paper 1	Question 11.
CST Part IA 1992	Paper 2	Question 3.
CST Part IA 1996	Paper 1	Question 7.
CST Part IA 1997	Paper 1	Question 7.
Diploma 1991	Paper 1	Question 8.

Induction:

Maths Part IA 1988	Paper 6	Question 9.
Diploma 1989	Paper 3	Question 8.
Diploma 1992	Paper 2	Question 8.
Diploma 1997	Paper 1	Question 9.

All of these papers should be available with the exception of Maths Part IA 1988 Paper 6. The relevant questions from that paper are reproduced on the other side of this sheet.

9 State the principle of mathematical induction. Prove your statement, assuming that every non-empty subset of the natural numbers has a least element.

The Master of Regent's College and his wife invite n Fellows and their spouses to a party. After the party the Master asks everyone (including his own wife) how many people they shook hands with, and receives $2n{+}1$ different answers. Of course, no woman shook hands with her own husband. Show that the person who shook the most hands was **not** the Master's wife. How many hands did the Master shake?

10 Let R be a relation on a set X. Define the reflexive, symmetric and transitive closures $r(R)$, $s(R)$ and $t(R)$ of R. Let Δ be the relation $\{(x, x) \mid x \in X\}$. Prove that:

(a) $R \circ \Delta = R$,

(b) $(R \cup \Delta)^n = \Delta \cup \bigsqcup \{R^i \mid 1 \leq i \leq n\}$, for all $n \geq 1$,

(c) $tr(R) = rt(R)$.

Show also that $st(R) \subseteq ts(R)$. If $X = \mathbb{N}$ and

$R = \Delta \cup \{(x, y) \mid x, y \in \mathbb{N}$ s.t. $y = p.x$ for some prime $p \in \mathbb{N}\}$, describe $st(R)$ and $ts(R)$.

[Notation. In this question $rt(R)$ stands for $r(t(R))$, and so on.]

Part B

Regular Languages and Finite Automata:

CST Part IA 1995	Paper 2	Question 27.
Diploma 1989	Paper 4	Question 11.
Diploma 1991	Paper 2	Question 6.
Diploma 1992	Paper 2	Question 9.
Diploma 1993	Paper 4	Question 12.
Diploma 1996	Paper 1	Question 9.
Diploma 1996	Paper 2	Question 8.
Diploma 1997	Paper 2	Question 8.
Diploma 1998	Paper 1	Question 10.
Diploma 1998	Paper 2	Question 8.
Diploma 1999	Paper 2	Question 8.