Complexity Theory
Easter 2000
Suggested Exercises 1

1. On slide 24 of the notes, a number of functions are listed as being constructible. Show that this is the case by giving, for each one, a description of an appropriate Turing machine.

2. Prove that if f and g are constructible functions, then so are $f(g)$, $f+g$, $f \cdot g$ and 2^f.

3. In the lecture, a proof of the Time Hierarchy Theorem was sketched. Give a similar argument for the following Space Hierarchy Theorem:

> **Space Hierarchy.** For every constructible function f, there is a language in $\mathsf{SPACE}(f(n) \cdot \log f(n))$ that is not in $\mathsf{SPACE}(f(n))$.

Could you replace the factor of $\log f(n)$ in this statement with something even smaller?