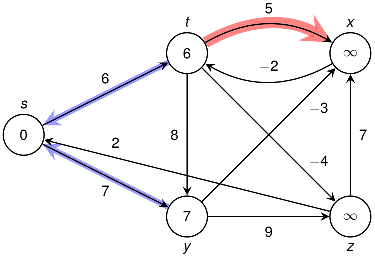


6.4: Single-Source Shortest Paths

Frank Stajano Thomas Sauerwald

Lent 2016

UNIVERSITY OF CAMBRIDGE

Outline

Dijkstra's Algorithm

Historical Remarks

Source: Wikipedia

- Dutch computer scientist
- developed Dijkstra's shortest path algorithm in 1956 (and published in 1959)
- many more fundamental contributions to computer science and engineering
- Turing Award (1972)

Edsger Wybe Dijkstra (1930-2002)

Some Quotes

"It is practically impossible to teach good programming to students that have had a prior exposure to BASIC: as potential programmers they are mentally mutilated beyond hope of regeneration."

"If you want more effective programmers, you will discover that they should not waste their time debugging, they should not introduce the bugs to start with."

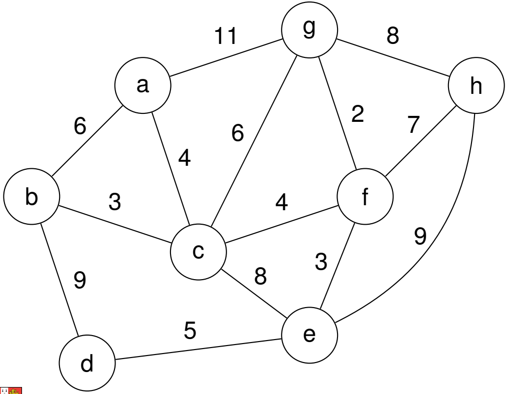
"FORTRAN's tragic fate has been its wide acceptance, mentally chaining thousands and thousands of programmers to our past mistakes."

"Programming is one of the most difficult branches of applied mathematics; the poorer mathematicians had better remain pure mathematicians."

Recap: Prim's Algorithm

Basic Strategy

- Start growing a tree from a designated root vertex



Recap: Prim's Algorithm

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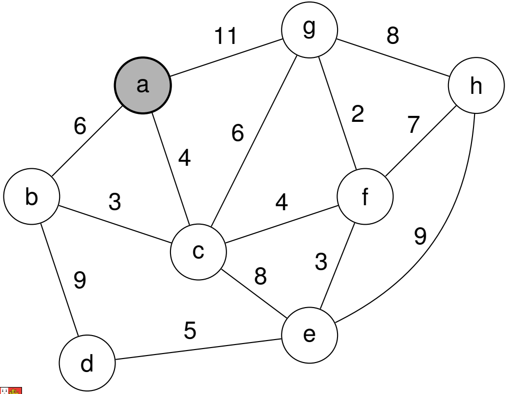
- At each step, add lightest edge linked to *A* that does not yield cycle



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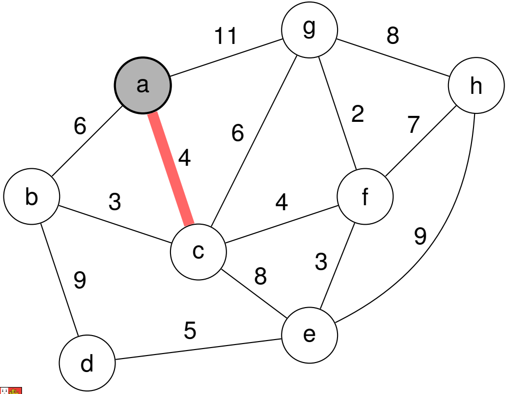

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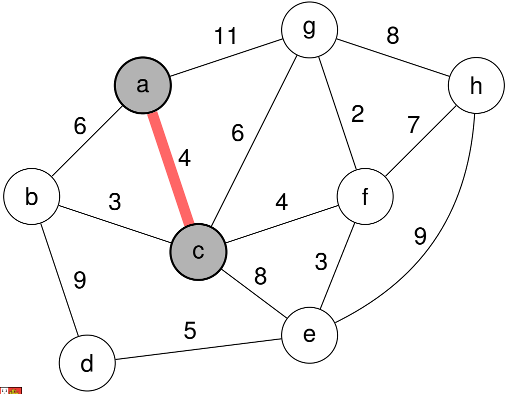

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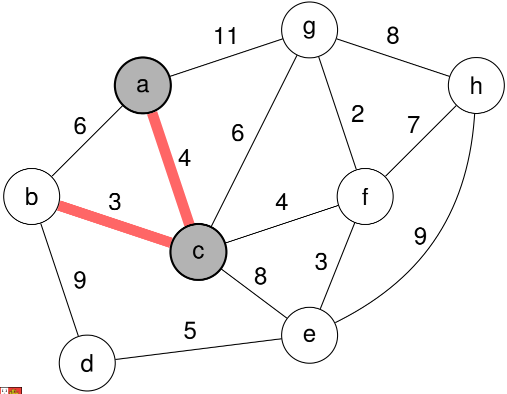

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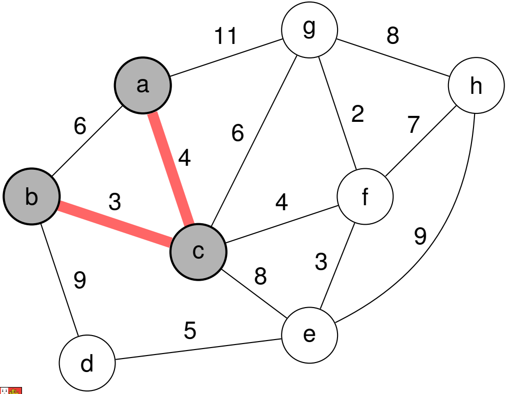

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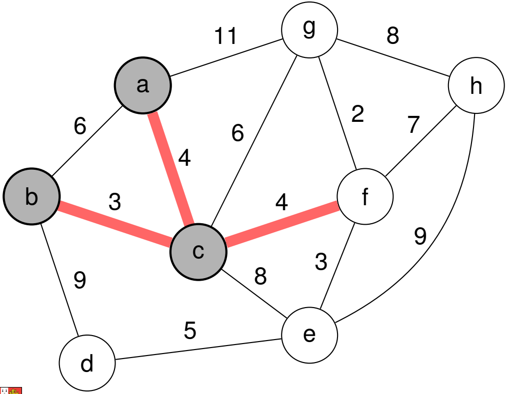

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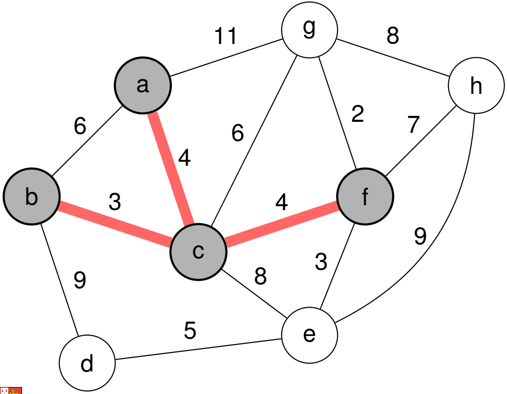

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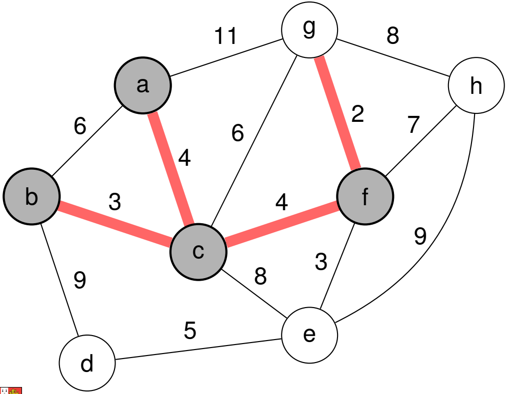

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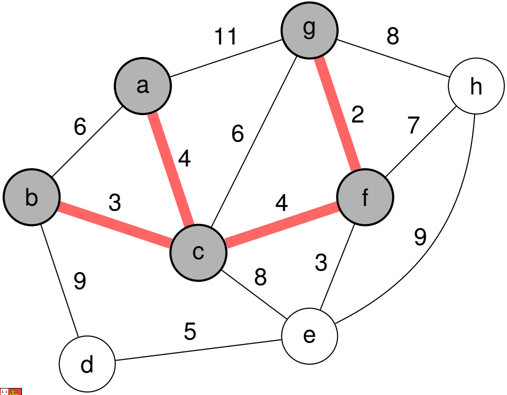

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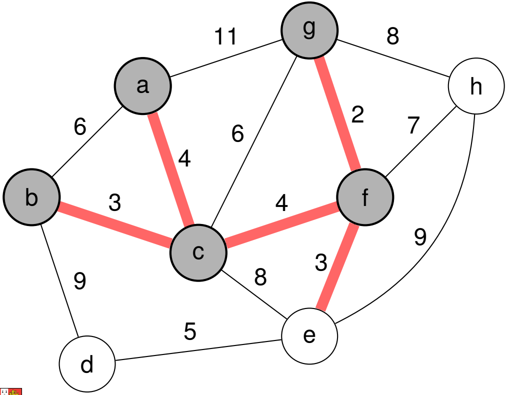

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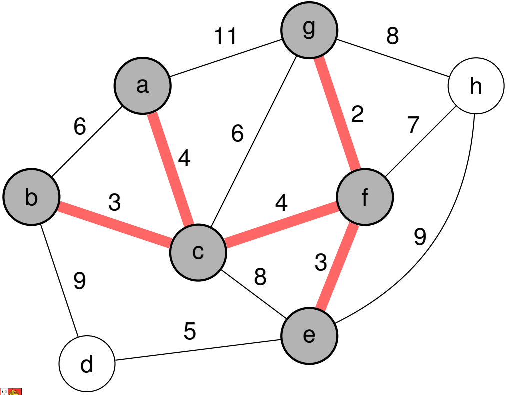

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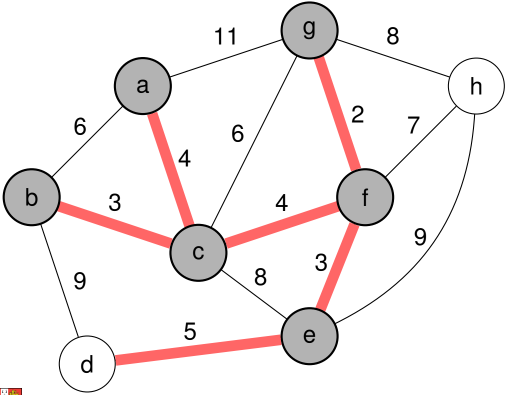

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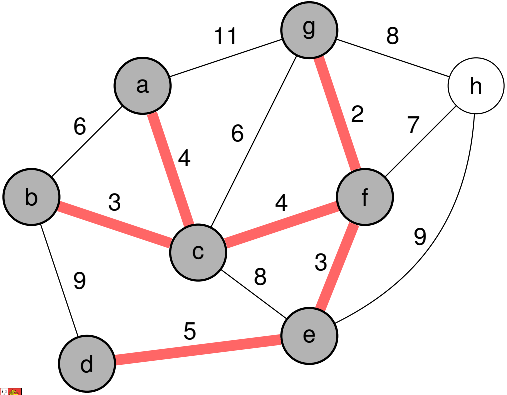

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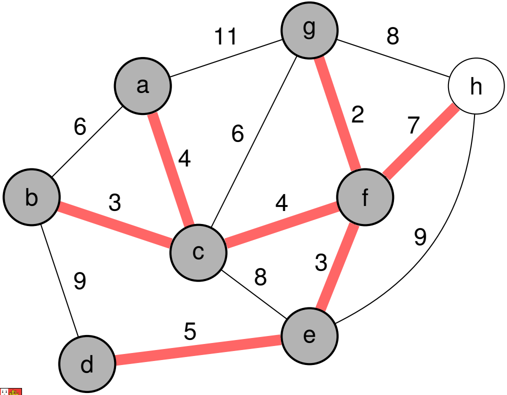

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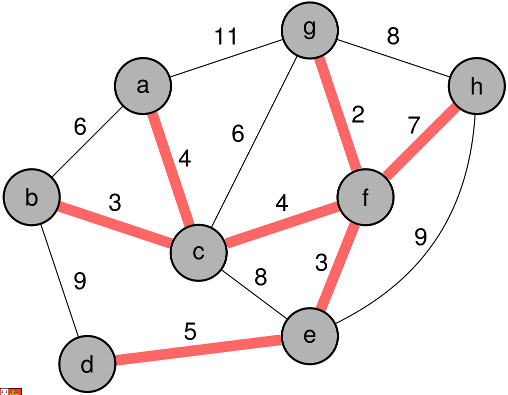

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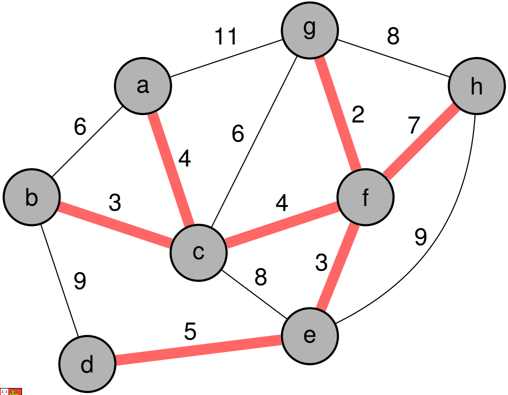
Implementation will be based on vertices!



Recap: Prim's Algorithm

Basic Strategy

- Start growing a tree from a designated root vertex
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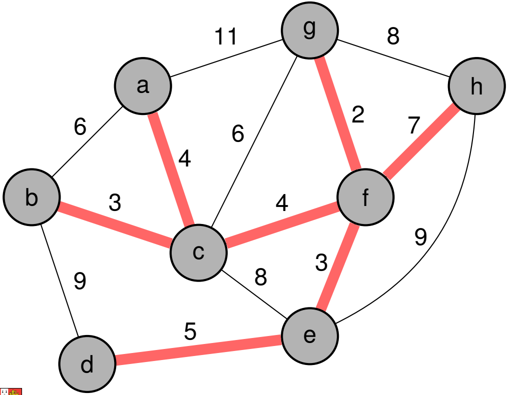
Assign every vertex not in A a key which is at all stages equal to the smallest weight of an edge connecting to A



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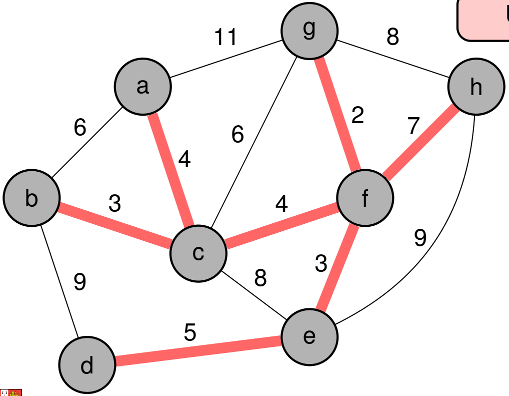
Use a Priority Queue!



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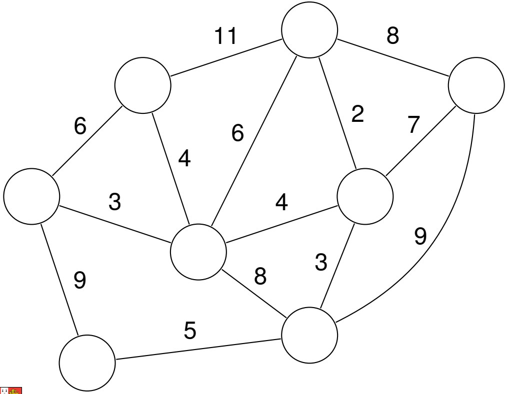

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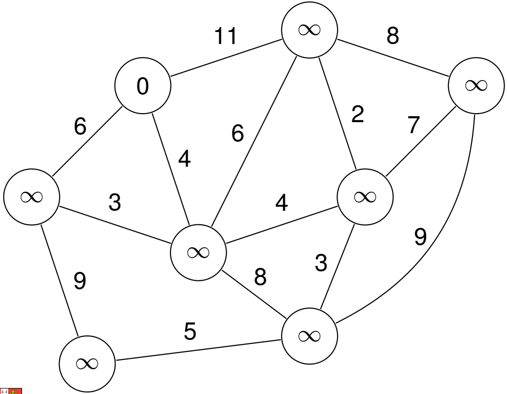

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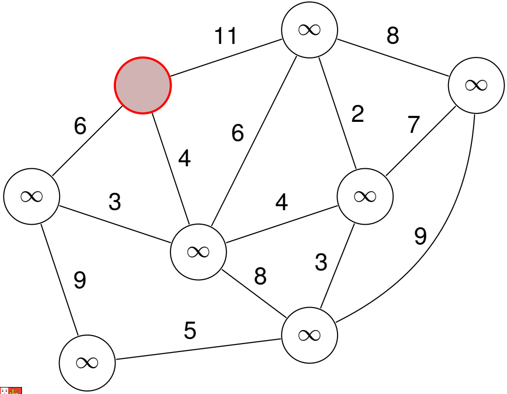

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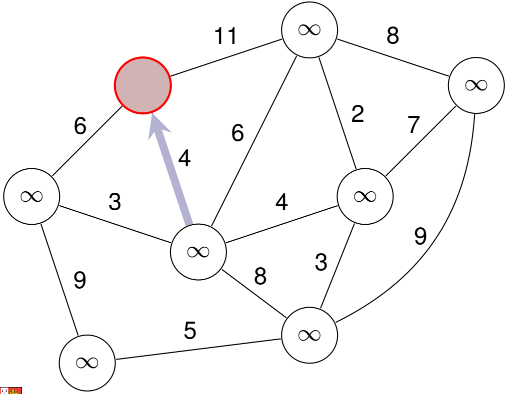

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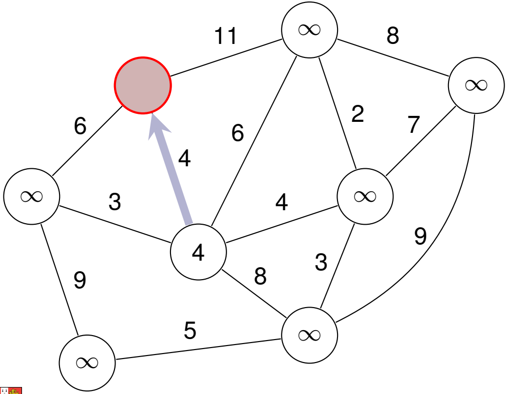

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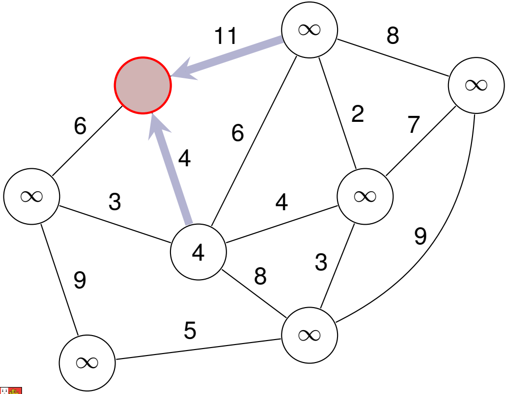

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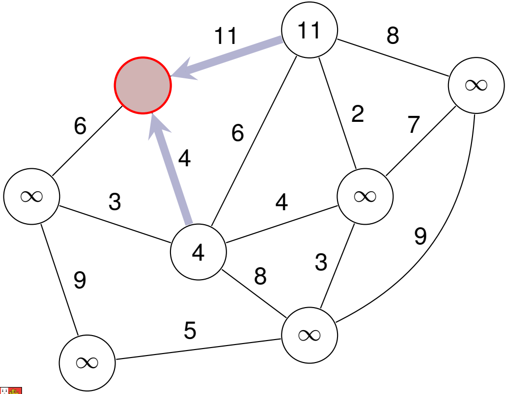

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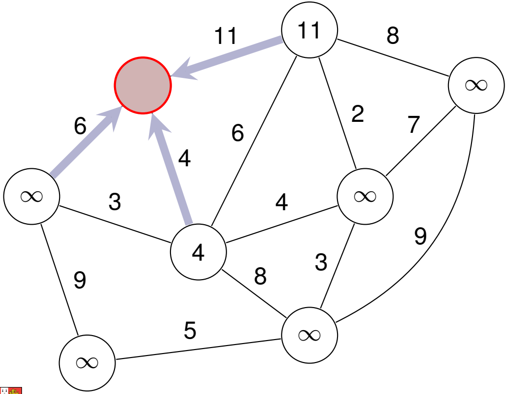

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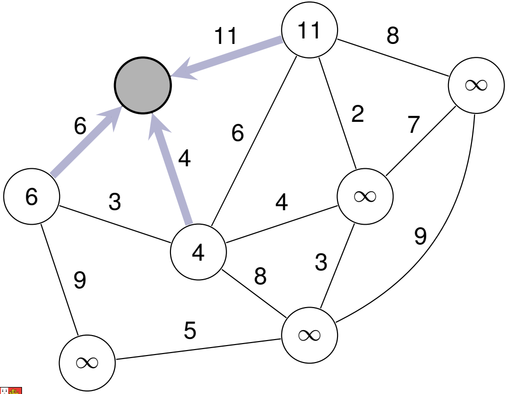

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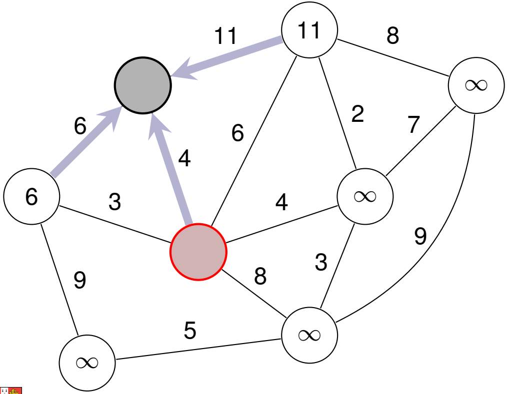

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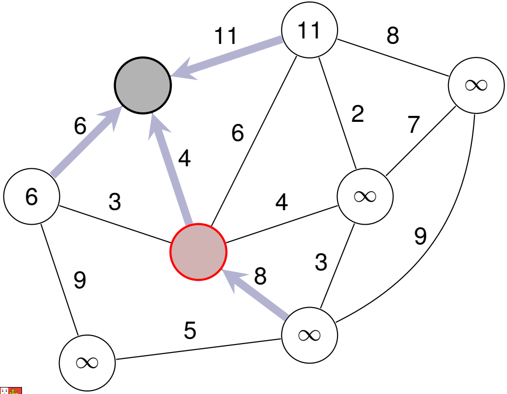

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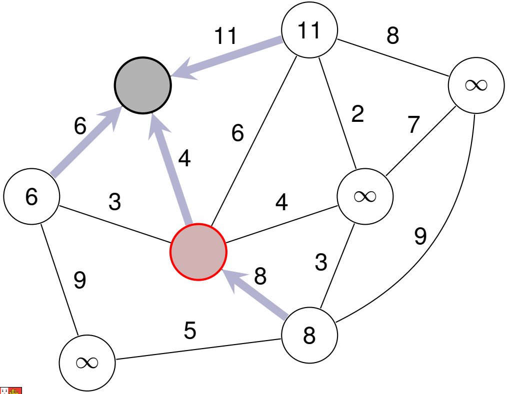

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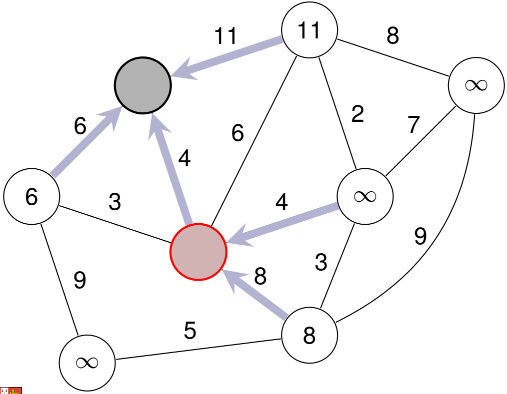

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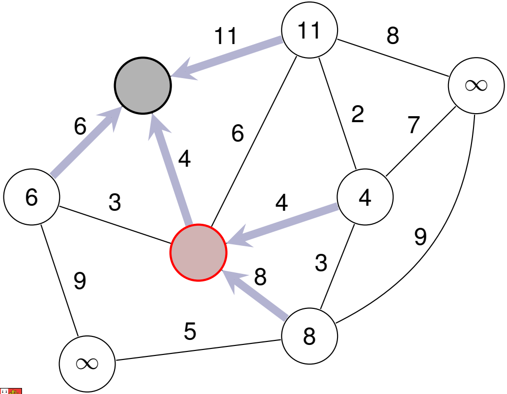

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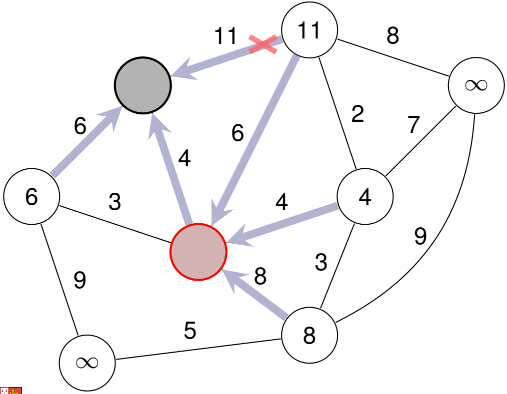

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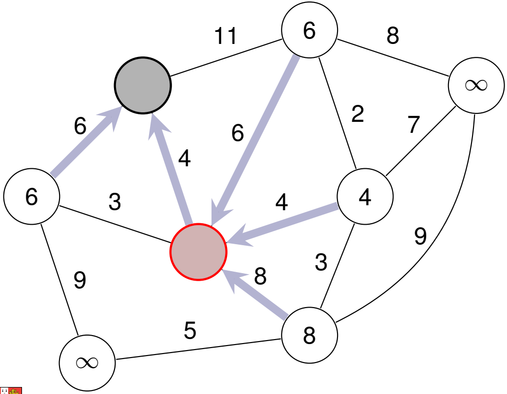

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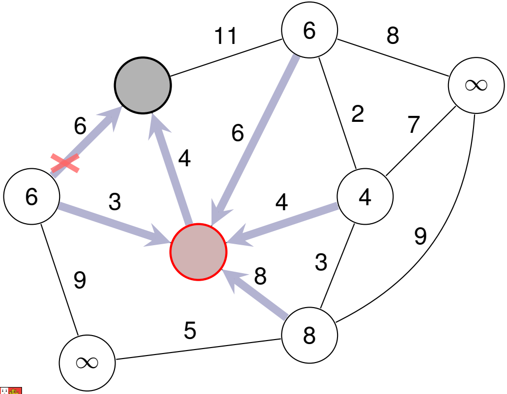

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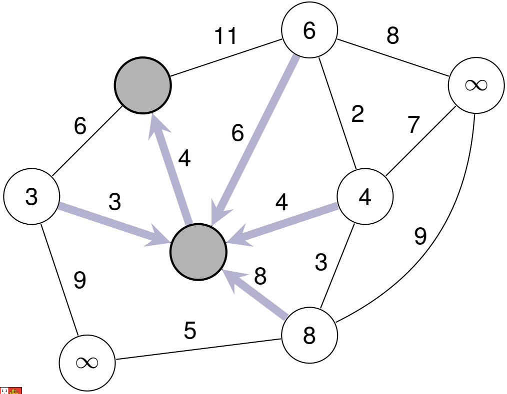

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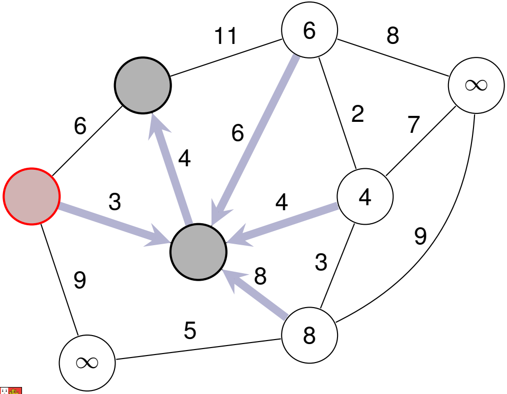

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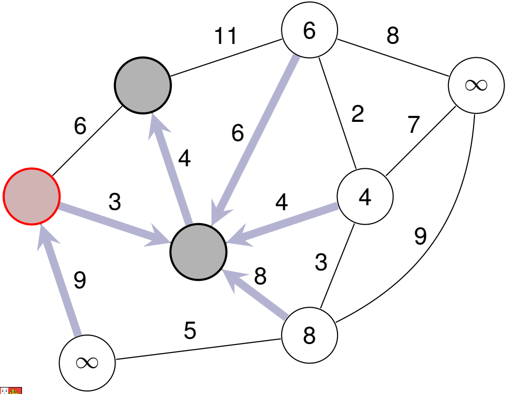

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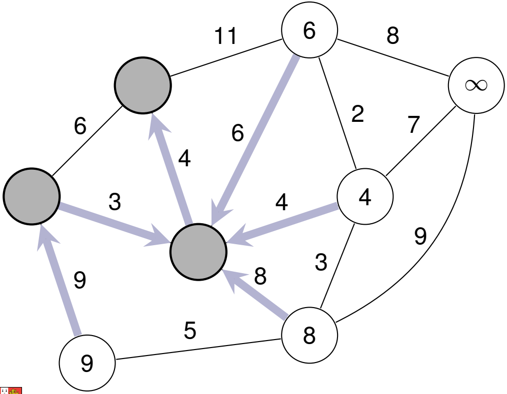

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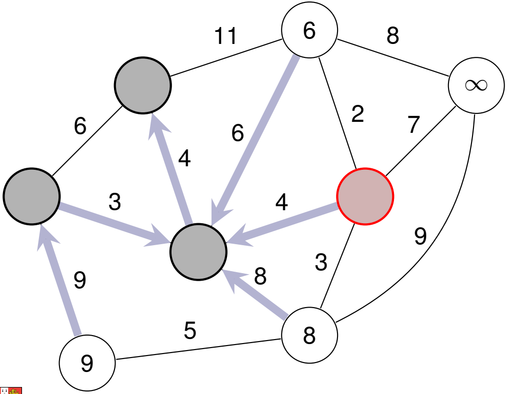

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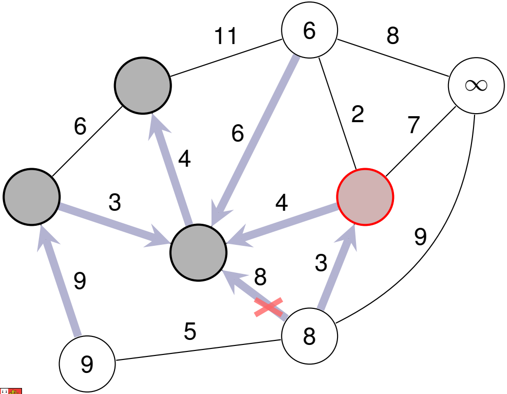

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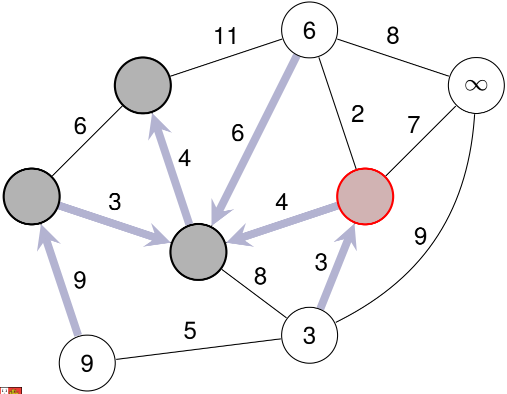

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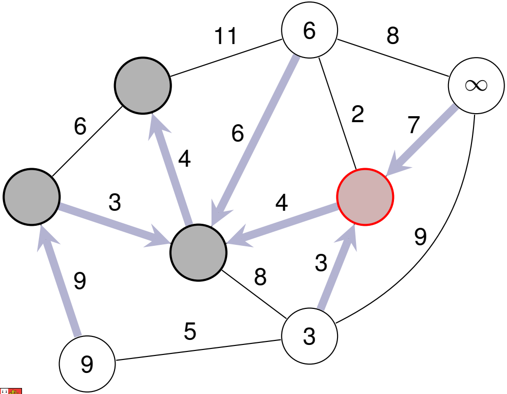

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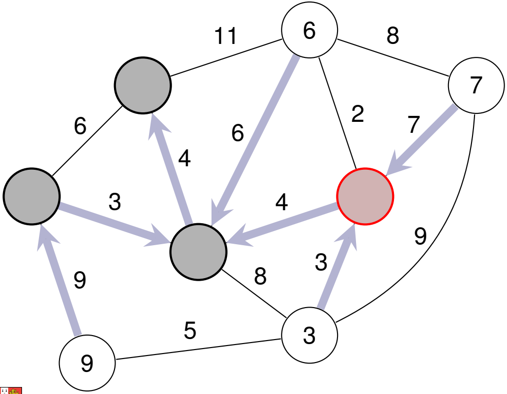

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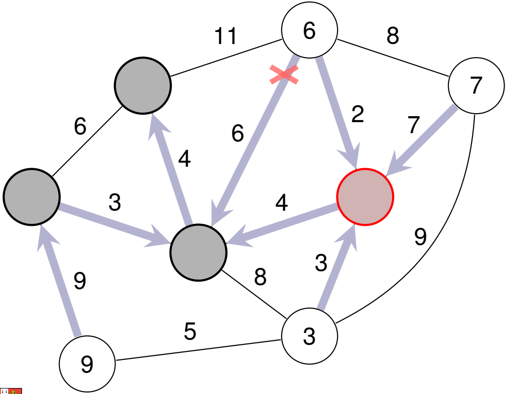

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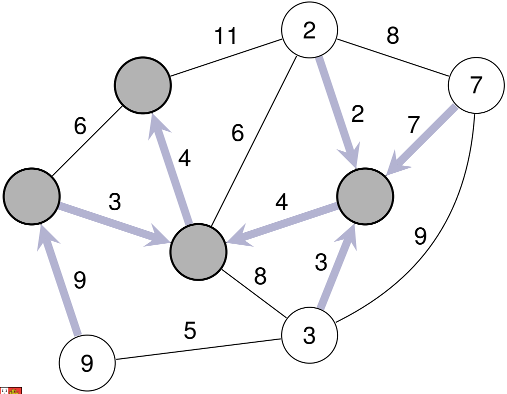

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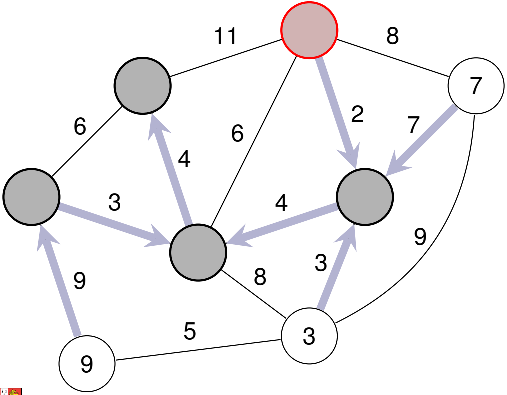

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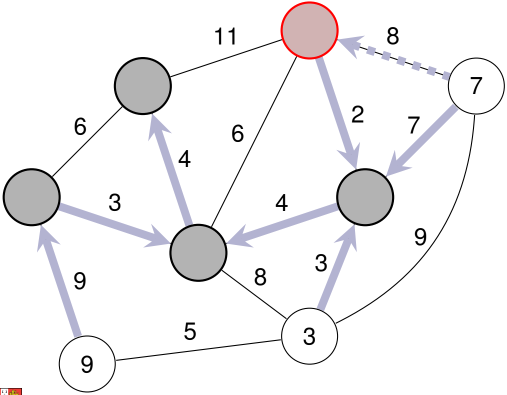

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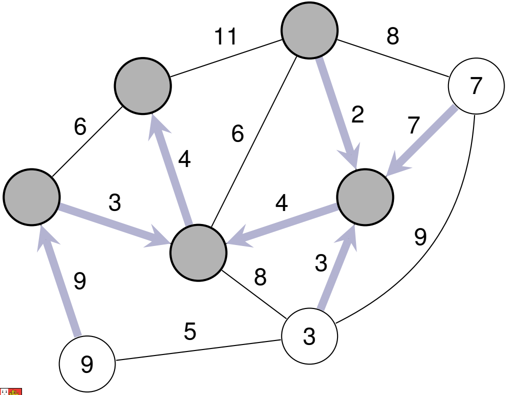

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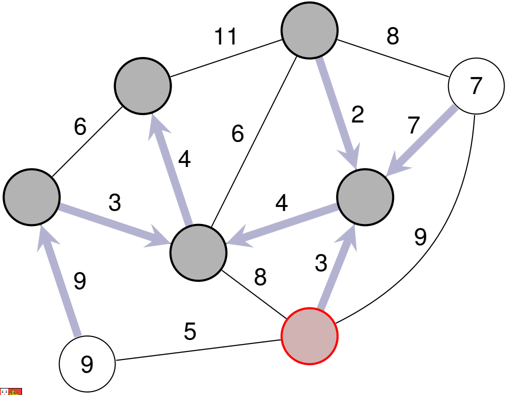

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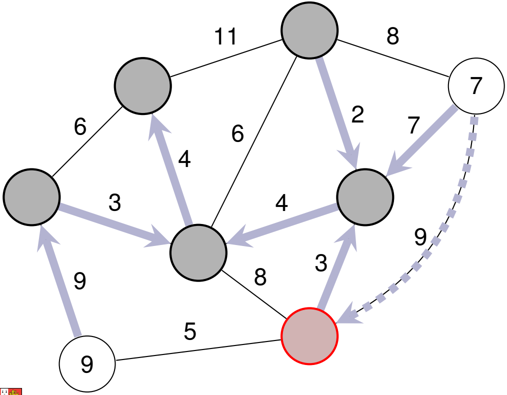

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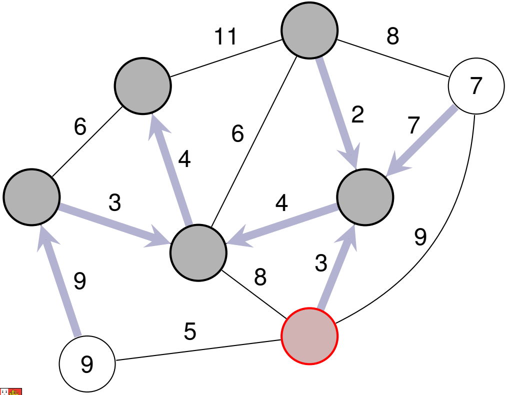

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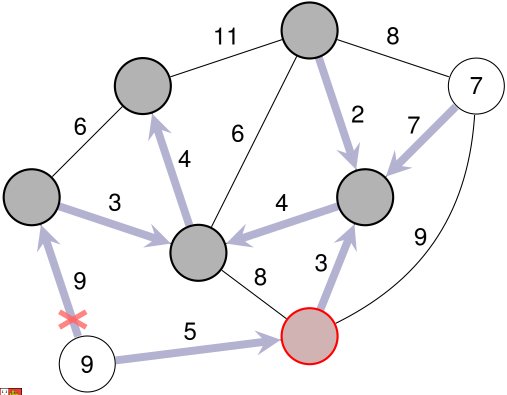

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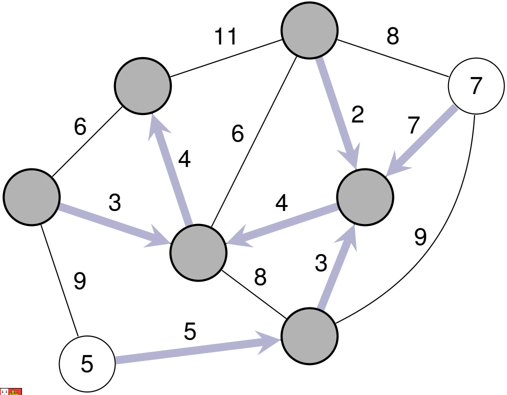

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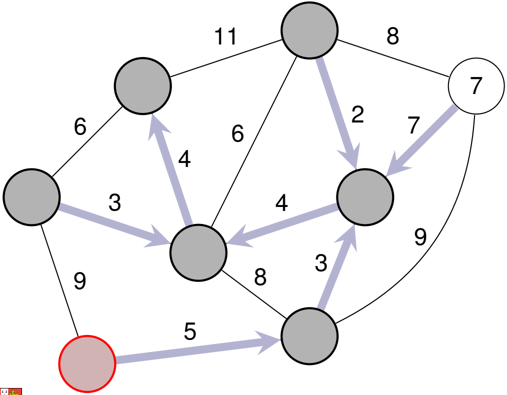

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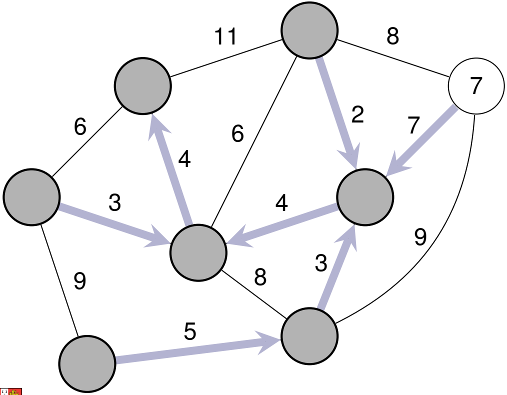

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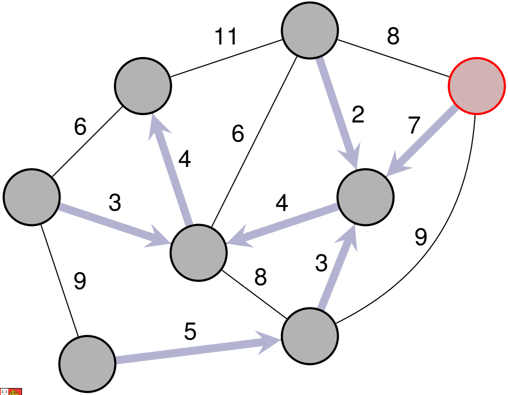

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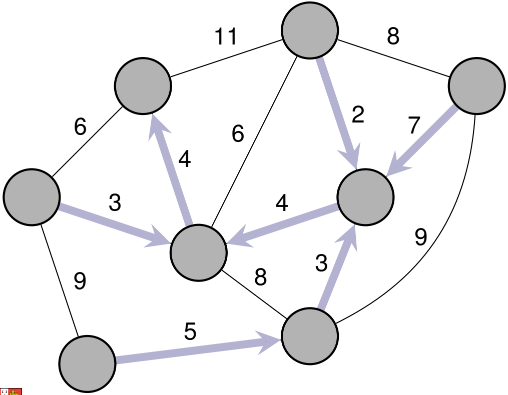

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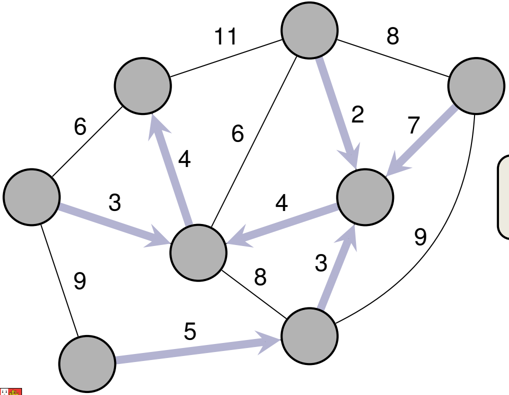

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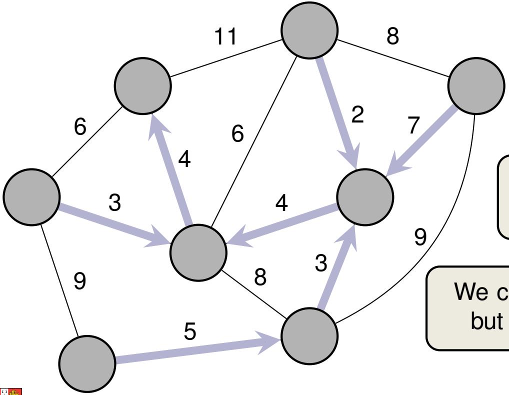

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We computed same MST as Kruskal, but in a completely different order!

Prim's Algorithms vs. Dijsktra's Algorithm

Prim's Algorithm

- Grows a tree that will eventually become a (minimum) spanning tree
- A is the set of vertices which have been connected so far
- Value of a vertex:
  - If $u \in A$, then it has no value.
  - If $u \notin A$, then it is equal to the smallest weight of an edge connecting to A (if such edge exists, otherwise ∞.)

Dijsktra's Algorithm

- Grows a tree that will eventually become a shortest-path tree
- S is the set of vertices in the (current) shortest-path tree
- Value of a vertex:
  - If $u \in S$, then it is the actual distance from the source s to u.
  - If $u \notin S$, then it may be any value (including ∞) that is at least the distance from the source s.

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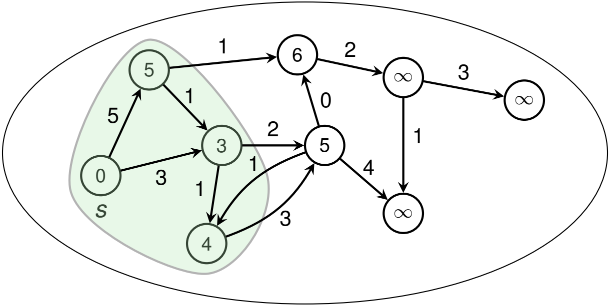

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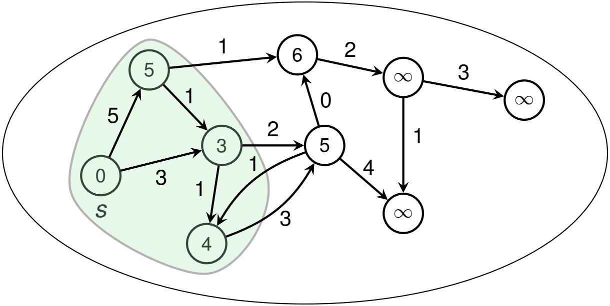

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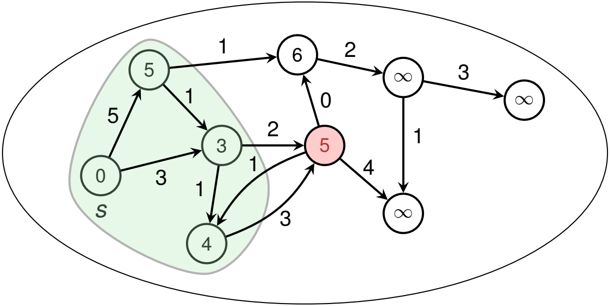

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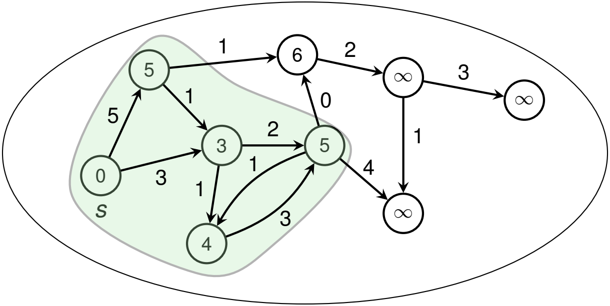

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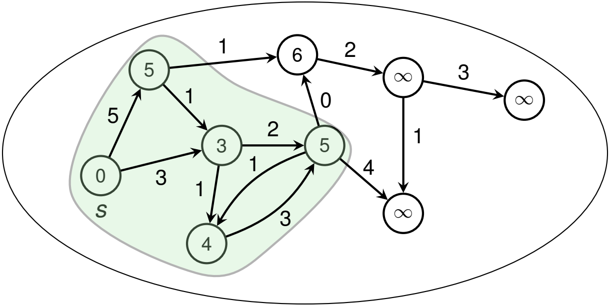

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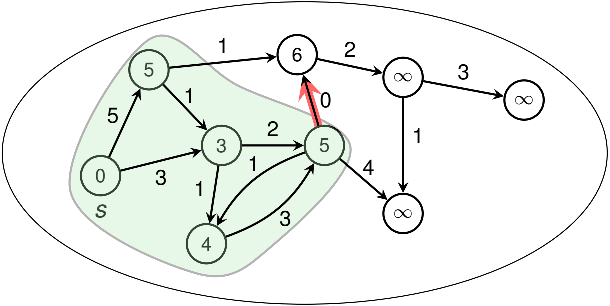

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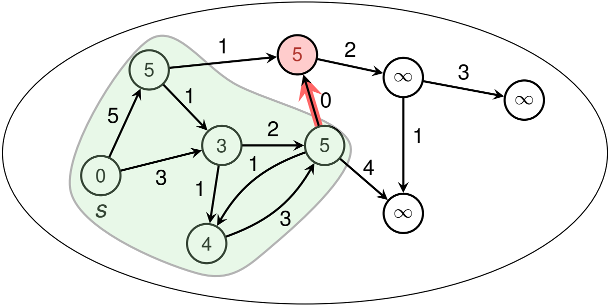

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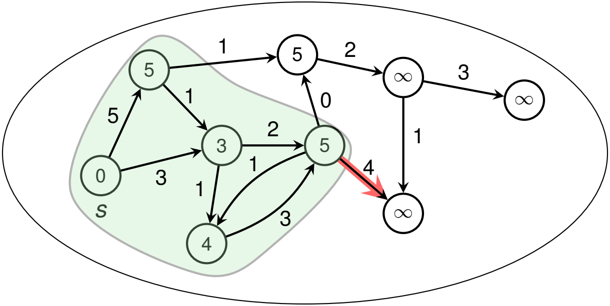

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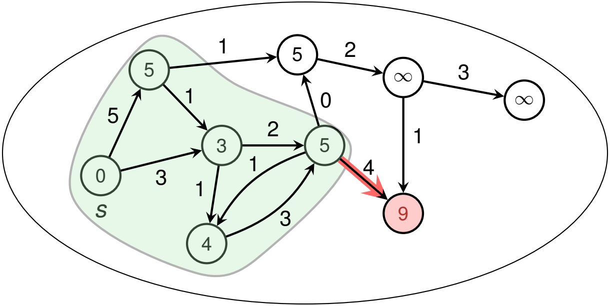

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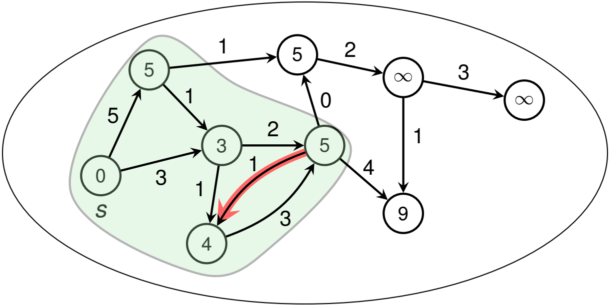

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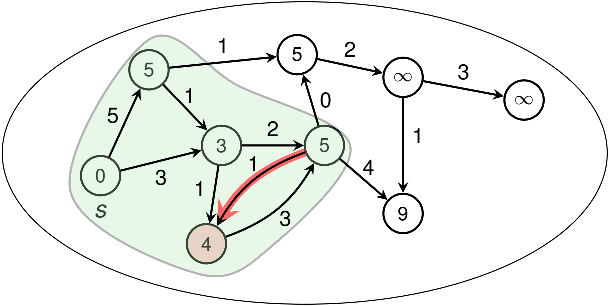

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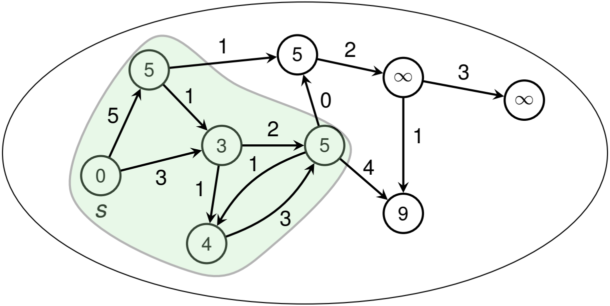

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0: INITIALIZE(G,s)
1: S = ∅
2: Q = V
3: while Q ≠ ∅ do
4:     u = Extract-Min(Q)
5:     S = S ∪ {u}
6:     for each v ∈ G.Adj[u] do
7:         RELAX(u, v, w)
8:     end for
9: end while
```

Runtime w. Fibonacci Heaps

- Initialization (l. 0-2): $\mathcal{O}(V)$
- ExtractMin (l. 4): $\mathcal{O}(V \cdot \log V)$
- DecreaseKey (l. 7): $\mathcal{O}(E \cdot 1)$

$\Rightarrow$ Overall: $\mathcal{O}(V \log V + E)$

Details of Dijkstra's Algorithm

As in Prim, use **priority queue** Q to keep track of the vertices' values.

```
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$\Rightarrow$ Overall: $\mathcal{O}(V \log V + E)$

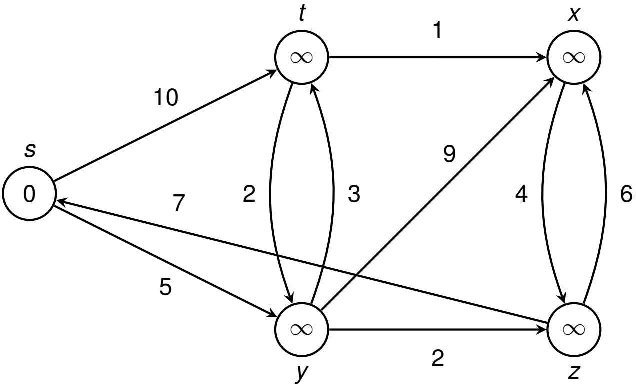
With a binary heap instead, the overall runtime would be $O(E \cdot \log V)$!

Prim's algorithm has the same runtime!

Execution of Dijkstra (Figure 24.6)

Priority Queue Q:

$(s, 0), (t, \infty), (x, \infty), (y, \infty), (z, \infty)$



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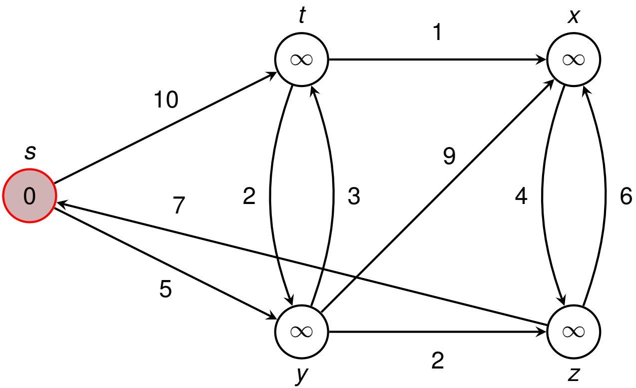
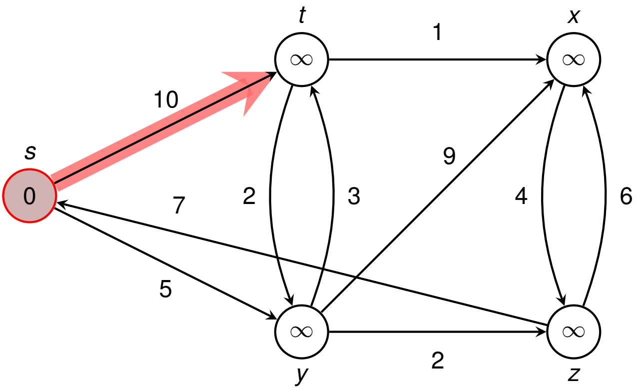


Execution of Dijkstra (Figure 24.6)

Priority Queue Q:

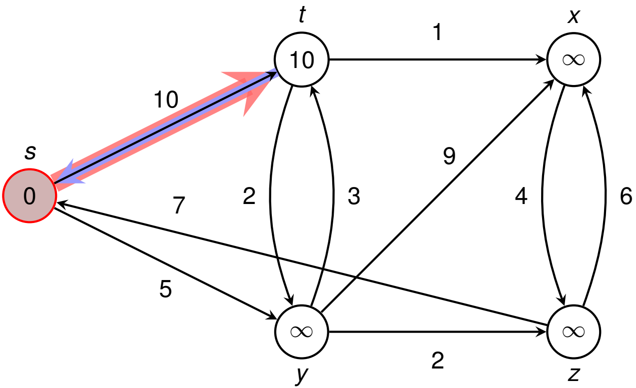
$\cancel{(s,0)}, (t,\infty), (x,\infty), (y,\infty), (z,\infty)$



Execution of Dijkstra (Figure 24.6)

Priority Queue Q:

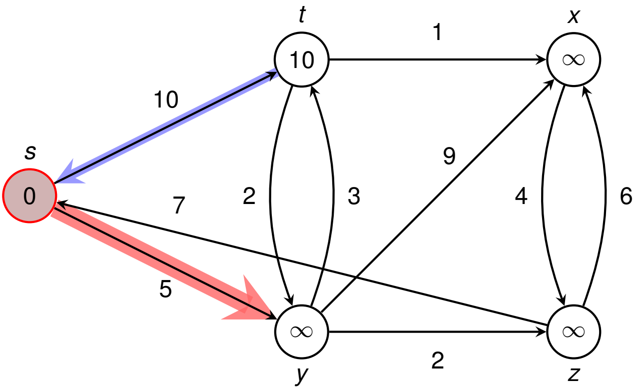
$\cancel{(s, 0)}, (t, 10), (x, \infty), (y, \infty), (z, \infty)$



Execution of Dijkstra (Figure 24.6)

Priority Queue Q:

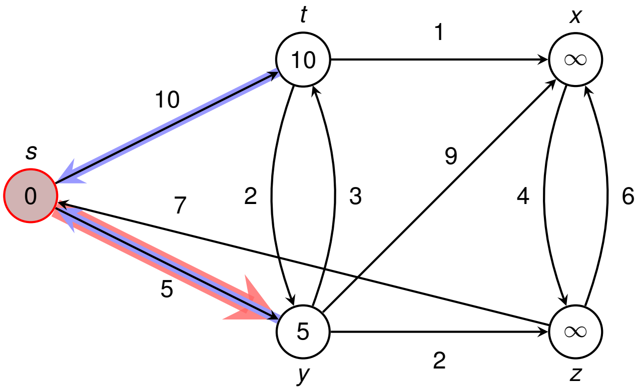
$\cancel{(s, 0)}, (t, 10), (x, \infty), (y, \infty), (z, \infty)$



Execution of Dijkstra (Figure 24.6)

Priority Queue Q:

$\cancel{(s,0)}, (t,10), (x,\infty), (y,5), (z,\infty)$



Execution of Dijkstra (Figure 24.6)

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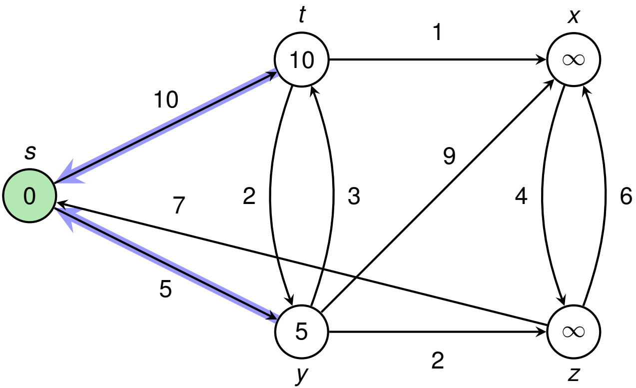
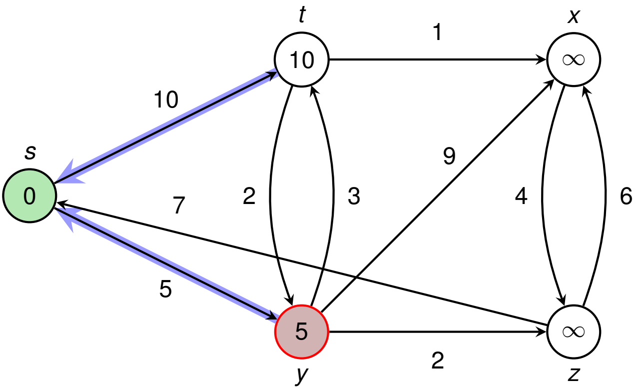


Execution of Dijkstra (Figure 24.6)

Priority Queue Q:

$(t, 10), (x, \infty), \cancel{(y, 5)}, (z, \infty)$



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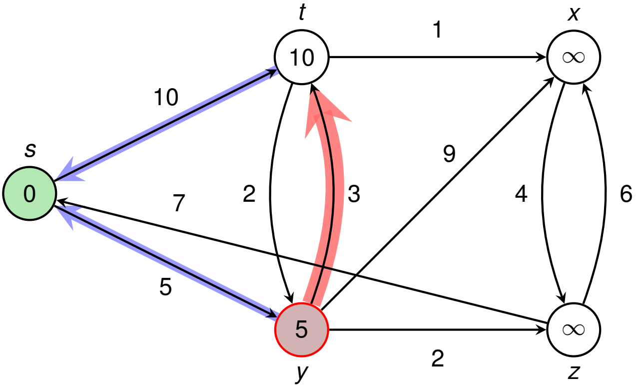
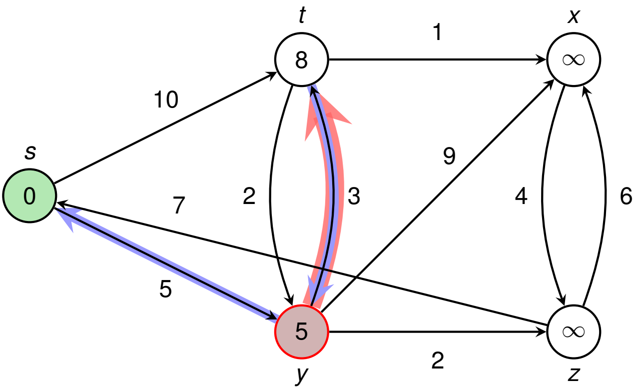


Execution of Dijkstra (Figure 24.6)

Priority Queue Q:

$(t, 8), (x, \infty), \cancel{(y, 5)}, (z, \infty)$



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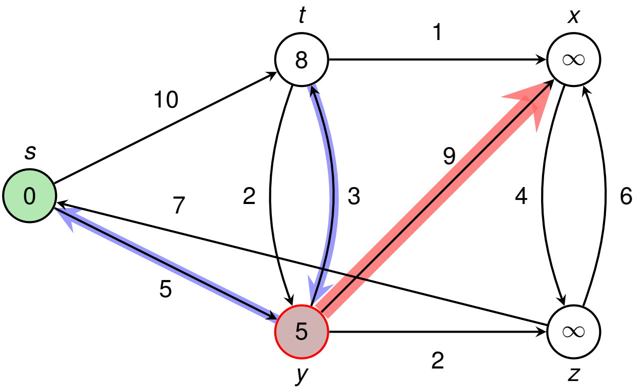
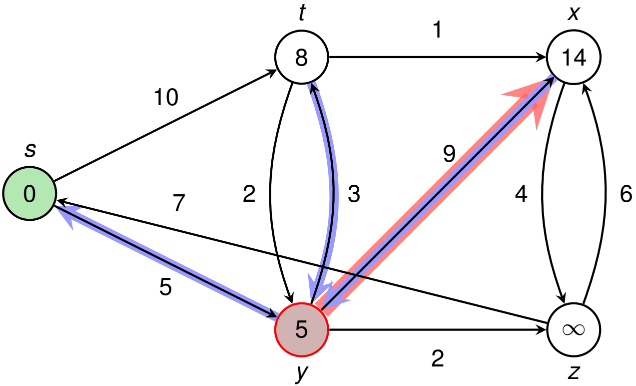


Execution of Dijkstra (Figure 24.6)

Priority Queue Q:

$(t, 8), (x, 14), \cancel{(y, 5)}, (z, \infty)$



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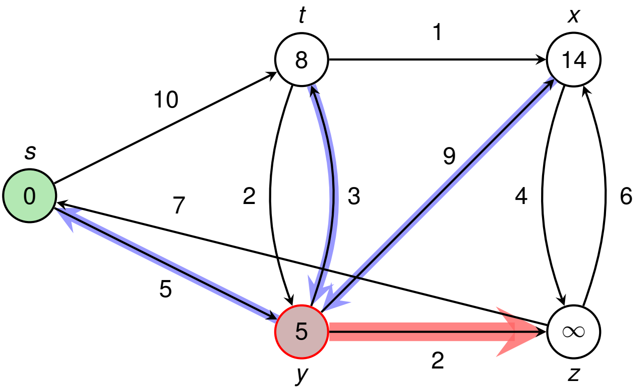
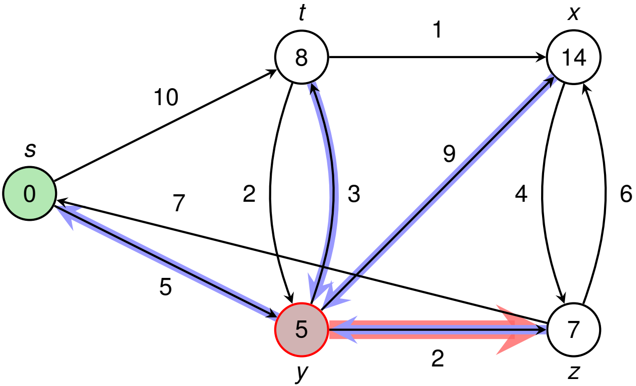


Execution of Dijkstra (Figure 24.6)

Priority Queue Q:

$(t, 8), (x, 14), \cancel{(y, 5)}, (z, 7)$



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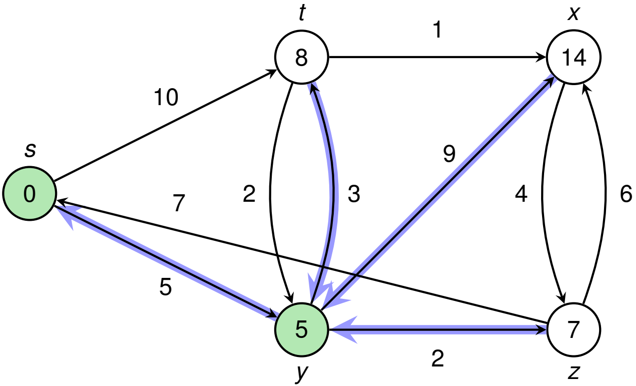

Execution of Dijkstra (Figure 24.6)

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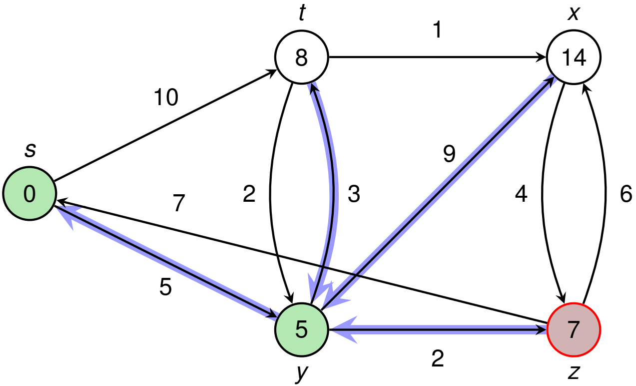
$(t, 8), (x, 14), \cancel{(z, 7)}$



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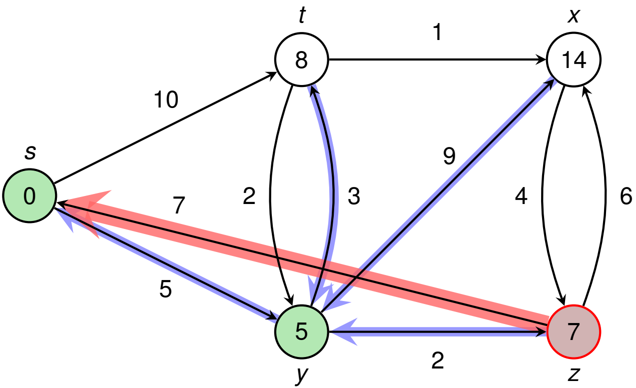

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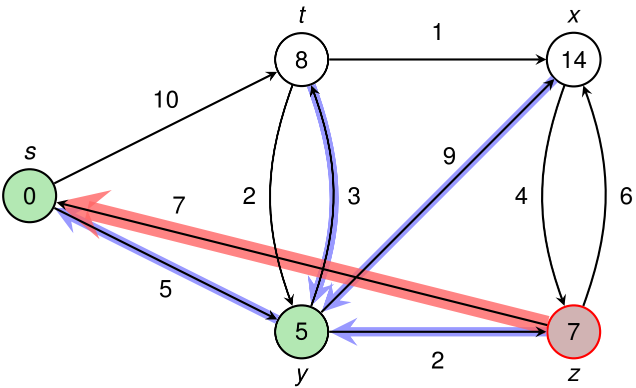

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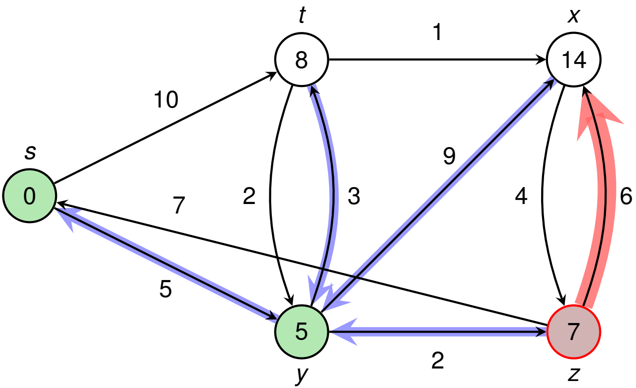
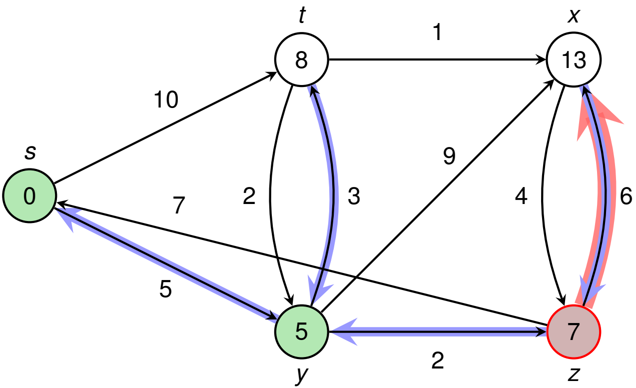


Execution of Dijkstra (Figure 24.6)

Priority Queue Q:

$(t, 8), (x, 13), \cancel{(z, 7)}$



Execution of Dijkstra (Figure 24.6)

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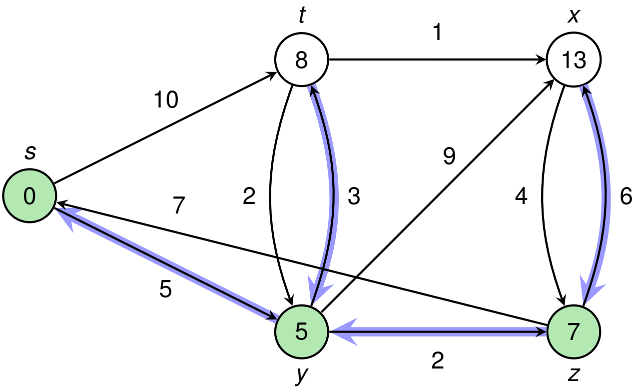
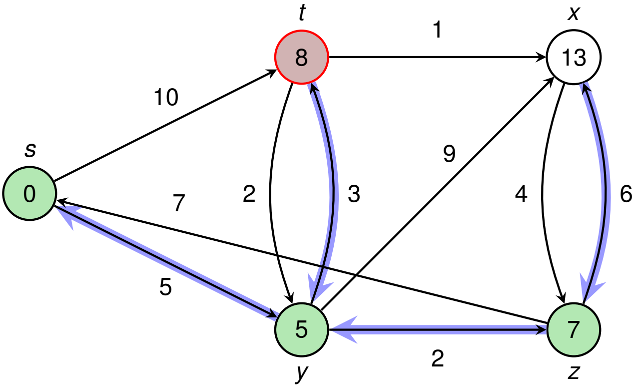


Execution of Dijkstra (Figure 24.6)

Priority Queue Q:

~~$(t, 8)$~~, $(x, 13)$



Execution of Dijkstra (Figure 24.6)

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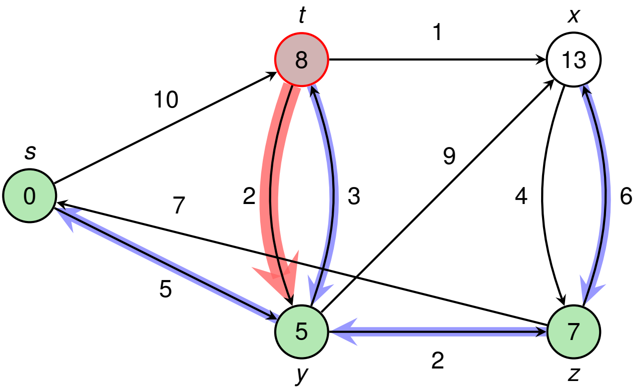

Execution of Dijkstra (Figure 24.6)

Priority Queue Q:

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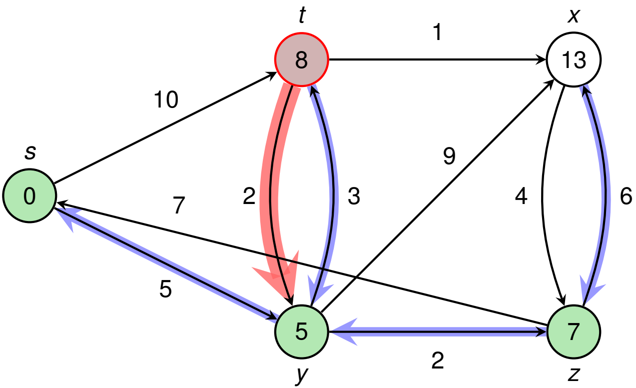
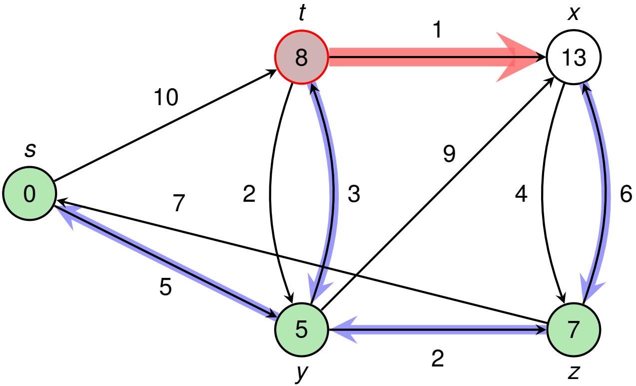


Execution of Dijkstra (Figure 24.6)

Priority Queue Q:

$\cancel{(t, 8)}, (x, 9)$



Execution of Dijkstra (Figure 24.6)

Priority Queue Q:

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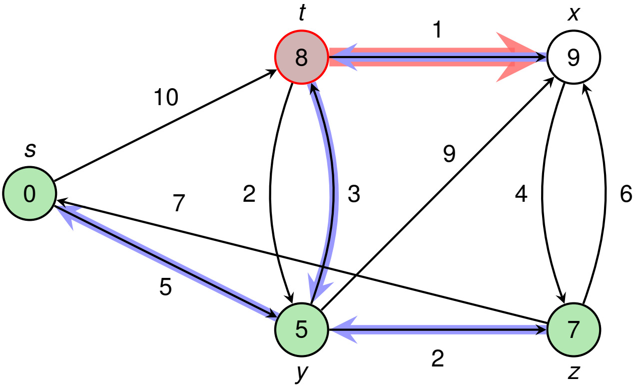

Execution of Dijkstra (Figure 24.6)

Priority Queue Q:

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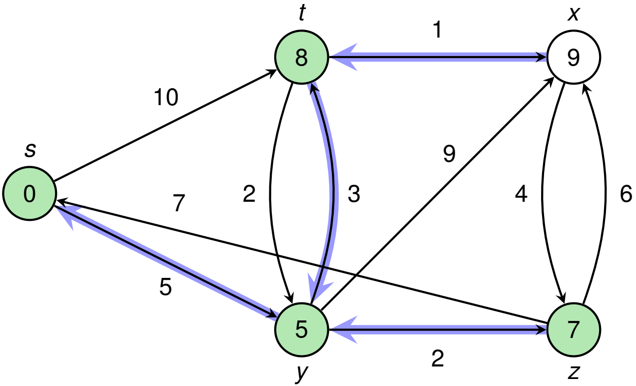
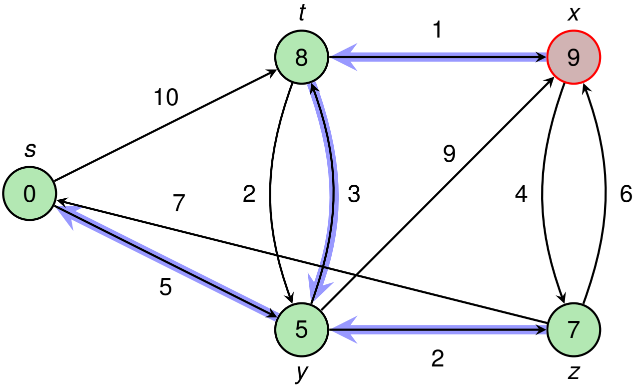


Execution of Dijkstra (Figure 24.6)

Priority Queue Q:

$\cancel{(x, 9)}$



Execution of Dijkstra (Figure 24.6)

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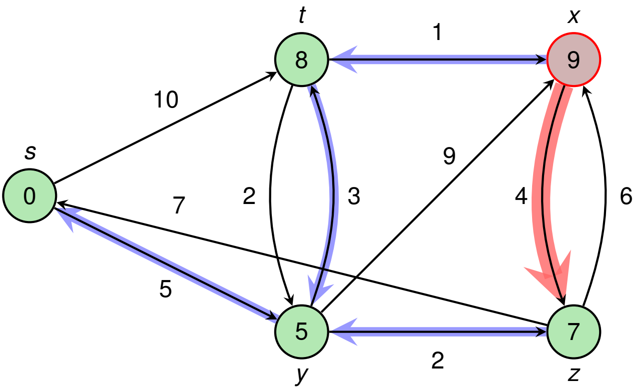

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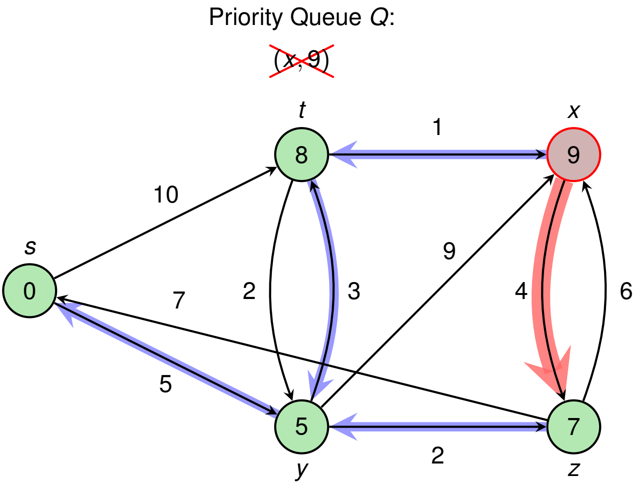

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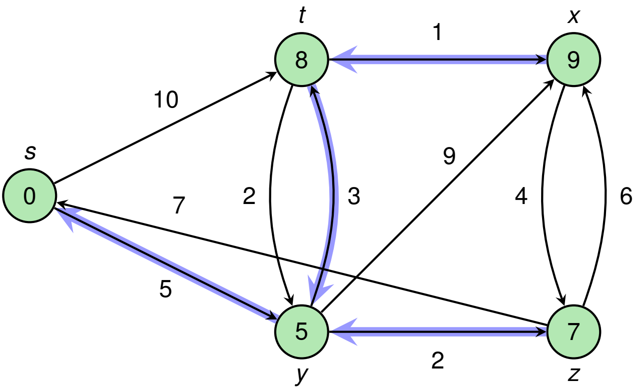

Dijkstra's Algorithm: Correctness

Correctness (Theorem 24.6)

For any directed graph $G = (V, E)$ with non-negative edge weights $w : E \to \mathbb{R}^+$ and source s, Dijkstra terminates with $u.d = u.\delta$ for all $u \in V$.

Proof: Invariant: If v is extracted, $v.d = v.\delta$

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- Suppose there is $u \in V$, when extracted,

$$u.d > u.\delta$$

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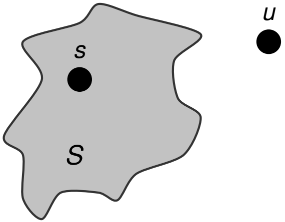
- Let u be the first vertex with this property



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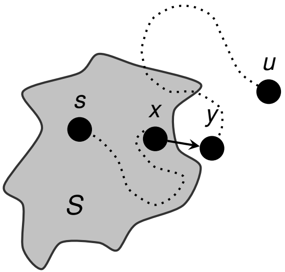

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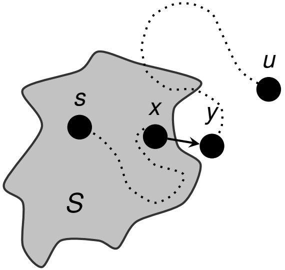
$\Rightarrow$

$$u.d$$



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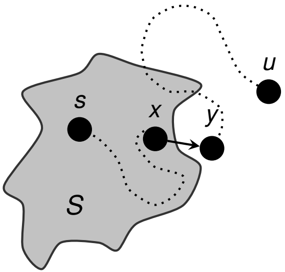
$\Rightarrow$

$$u.d \leq y.d$$



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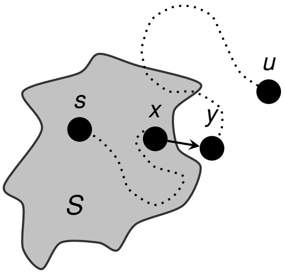

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$$u.d \leq y.d$$

u is extracted before y

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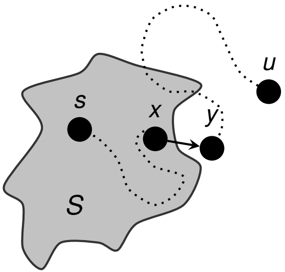

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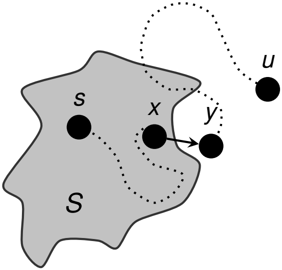
$\Rightarrow$

$$u.d \leq y.d = y.\delta$$

since $x.d = x.\delta$ when x is extracted, and then (x, y) is relaxed $\Rightarrow$ Convergence Property



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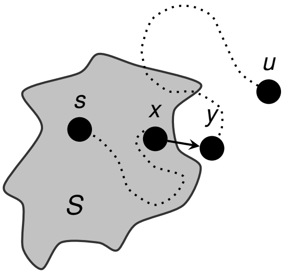
$\Rightarrow$

$$u.\delta < u.d \leq y.d = y.\delta$$



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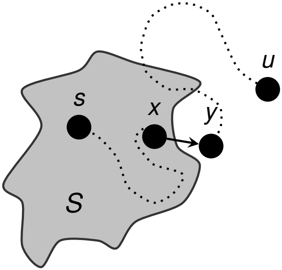

$\Rightarrow$

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This contradicts that y is on a shortest path from s to u. $\square$

Dijkstra's Algorithm: Correctness

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For any directed graph $G = (V, E)$ with non-negative edge weights $w : E \to \mathbb{R}^+$ and source s, Dijkstra terminates with $u.d = u.\delta$ for all $u \in V$.

There are edge cases like $s = x$ and/or $y = u$!

Proof: $\boxed{\text{Invariant: If } v \text{ is extracted, } v.d = v.\delta}$

- Suppose there is $u \in V$, when extracted,

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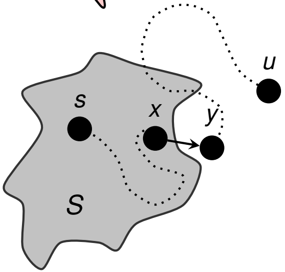
$\Rightarrow$

$$u.\delta < u.d \leq y.d = y.\delta$$

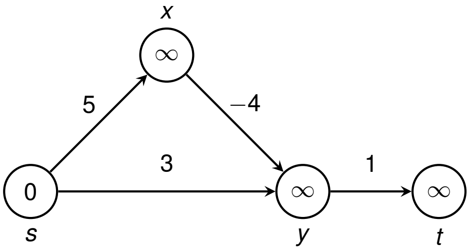


This contradicts that y is on a shortest path from s to u. $\square$

Why negative-weight edges don't work

Priority Queue Q:

$$(s, 0), (t, \infty), (x, \infty), (y, \infty)$$



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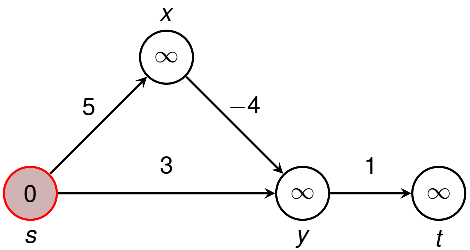

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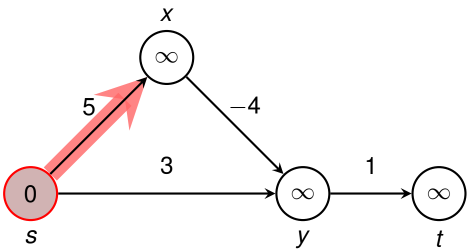

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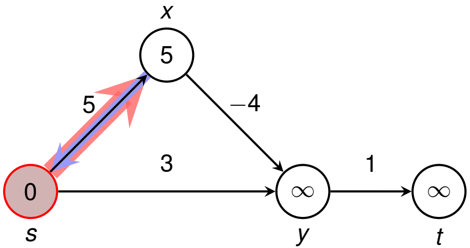

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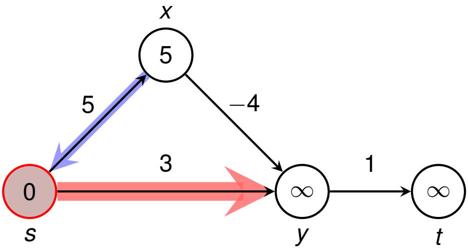

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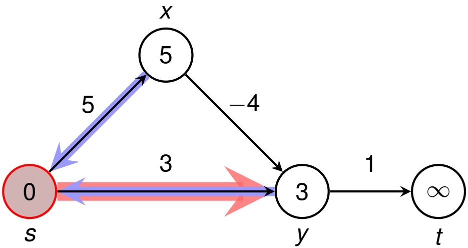

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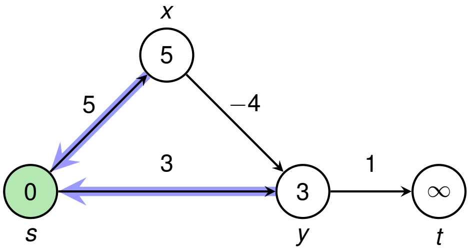

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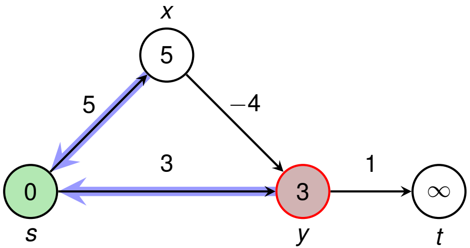

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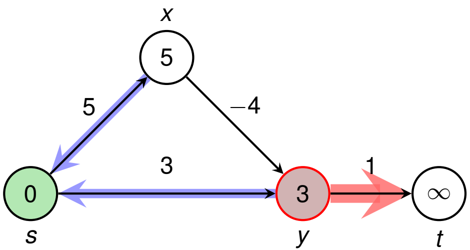

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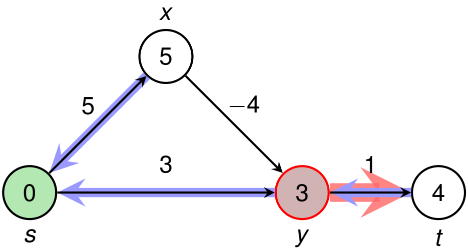

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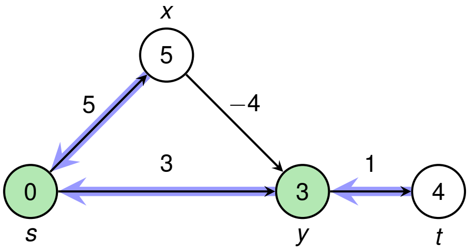

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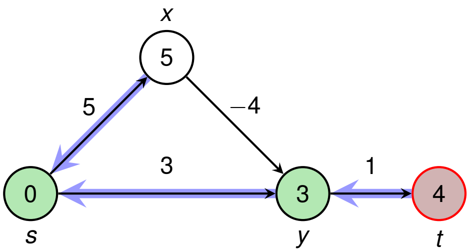

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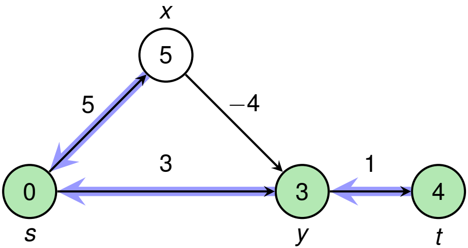

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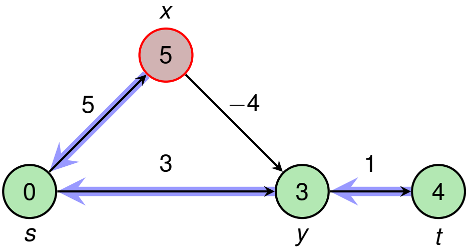
~~(x, 5)~~



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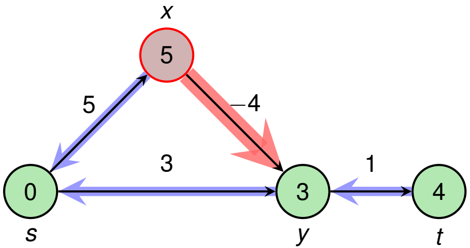

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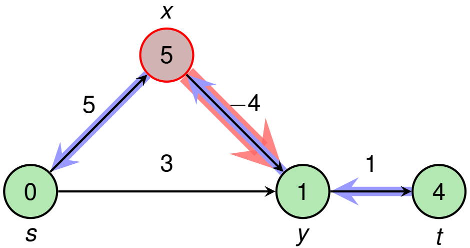

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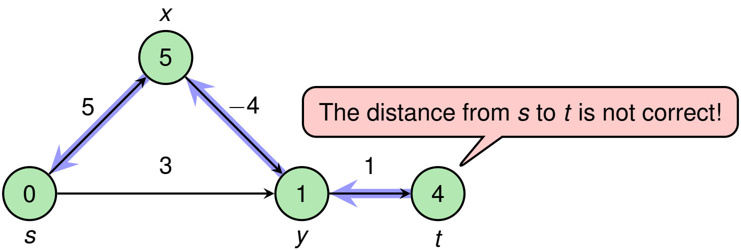

Summary of Single-Source Shortest Paths

Overview

- studied two algorithms for SSSP (single-source shortest path)
- basic operation: relaxing edges

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Dijkstra's Algorithm

- requires non-negative weights
- Greeedy strategy to choose which edge to relax (similar to Prim)
- Using Fibonacci Heaps $\Rightarrow$ Runtime $\mathcal{O}(V \log V + E)$