# Databases 2015 

Timothy G. Griffin

Computer Laboratory
University of Cambridge, UK

Databases, Lent 2015

## Lecture 01 : What is a DBMS?

- DB vs. IR
- Relational Databases
- ACID properties
- Two fundamental trade-offs
- OLTP vs. OLAP
- Behond ACID/Relational model ...


## Example Database Management Systems (DBMSs)

## A few database examples

- Banking : supporting customer accounts, deposits and withdrawals
- University : students, past and present, marks, academic status
- Business : products, sales, suppliers
- Real Estate : properties, leases, owners, renters
- Aviation : flights, seat reservations, passenger info, prices, payments
- Aviation : Aircraft, maintenance history, parts suppliers, parts orders


## Some observations about these DBMSs ...

- They contains highly structured data that has been engineered to model some restricted aspect of the real world
- They support the activity of an organization in an essential way
- They support concurrent access, both read and write
- They often outlive their designers
- Users need to know very little about the DBMS technology used
- Well designed database systems are nearly transparent, just part of our infrastructure


## Databases vs Information Retrieval

Always ask What problem am I solving?

| DBMS |  |
| :--- | :--- |
| exact query results |  |
| optimized for concurrent updates |  | data models a narrow domain generates documents (reports) increase control over information

IR system
fuzzy query results
optimized for concurrent reads domain often open-ended search existing documents reduce information overload

And of course there are many systems that combine elements of DB and IR.

## Still the dominant approach : Relational DBMSs

## your relational application

relational interface

Database Management System (DBMS)

- The problem : in 1970 you could not write a database application without knowing a great deal about the low-level physical implementation of the data.
- Codd's radical idea [C1970]: give users a model of data and a language for manipulating that data which is completely independent of the details of its physical representation/implementation.
- This decouples development of Database Management Systems (DBMSs) from the development of database applications (at least in an idealized world).


## What "services" do applications expect from a DBMS?

## Transactions - ACID properties

Atomicity Either all actions are carried out, or none are

- logs needed to undo operations, if needed

Consistency If each transaction is consistent, and the database is initially consistent, then it is left consistent

- Applications designers must exploit the DBMS's capabilities.
Isolation Transactions are isolated, or protected, from the effects of other scheduled transactions
- Serializability, 2-phase commit protocol

Durability If a transactions completes successfully, then its effects persist

- Logging and crash recovery

These concepts should be familiar from Concurrent Systems and Applications.

## Relational Database Design

## Our tools

Entity-Relationship (ER) modeling high-level, diagram-based design
Relational modeling
SQL implementation formal model normal forms based on Functional Dependencies (FDs) Where the rubber meets the road

The ER and FD approaches are complementary

- ER facilitates design by allowing communication with domain experts who may know little about database technology.
- FD allows us formally explore general design trade-offs. Such as - A Fundamental Trade-off in Database Design: the more we reduce data redundancy, the harder it is to enforce some types of data integrity. (An example of this is made precise when we look at 3 NF vs. BCNF.)


## ER Demo Diagram (Notation follows SKS book) ${ }^{1}$


${ }^{1}$ By PÃ $j$ vel Calado,
http://www.texample.net/tikz/examples/entity-relationship-diagram

## A Fundamental Trade-off in Database Implementation - Query response vs. update throughput

## Redundancy is a Bad Thing.

- One of the main goals of ER and FD modeling is to reduce data redundancy. We seek normalized designs.
- A normalized database can support high update throughput and greatly facilitates the task of ensuring semantic consistency and data integrity.
- Update throughput is increased because in a normalized database a typical transaction need only lock a few data items perhaps just one field of one row in a very large table.


## Redundancy is a Good Thing.

- A de-normalized database can greatly improve the response time of read-only queries.
a Colnativen and nontrallad do normolizatinn in nftan ranioirnd in


## OLAP vs. OLTP

OLTP Online Transaction Processing
OLAP Online Analytical Processing

- Commonly associated with terms like Decision Support, Data Warehousing, etc.

|  | OLAP | OLTP |
| ---: | ---: | :--- |
| Supports | analysis | day-to-day operations |
| Data is | historical | current |
| Transactions mostly | reads | updates |
| optimized for | query processing | updates |
| Normal Forms | not important | important |

## Example : Data Warehouse (Decision support)



Operational Database
business analysis queries


Data Warehouse

## Example : Embedded databases



Device

FIDO = Fetch Intensive Data Organization

## Example : Hinxton Bio-informatics

Database system design from the European
Bioinformatics Institute (Hinxton UK)
Other archives


## "NoSQL" Movement (subject of Lectures 11, 12)

A few technologies

- Key-value store
- Directed Graph Databases
- Main-memory stores
- Distributed hash tables


## Applications

- Google's Map-Reduce
- Facebook
- Cluster-based computing
- ...

Always remember to ask : What problem am I solving?

## Why do we have different kinds of Databases?

- Relational
- Object-Oriented Databases
- Data Warehouse
- "No SQL" databases


## Recommended Reading

## Textbooks

SKS Silberschatz, A., Korth, H.F. and Sudarshan, S. (2002). Database system concepts. McGraw-Hill (4th edition).
(Adjust accordingly for other editions)
Chapters 1 (DBMSs)
2 (Entity-Relationship Model)
3 (Relational Model)
4.1-4.7 (basic SQL)
6.1 - 6.4 (integrity constraints)

7 (functional dependencies and normal forms)
22 (OLAP)
UW Ullman, J. and Widom, J. (1997). A first course in database systems. Prentice Hall.
CJD Date, C.J. (2004). An introduction to database systems. Addison-Wesley (8th ed.).

## Reading for the fun of it ...

## Research Papers (Google for them)

C1970 E.F. Codd, (1970). "A Relational Model of Data for Large Shared Data Banks". Communications of the ACM.
F1977 Ronald Fagin (1977) Multivalued dependencies and a new normal form for relational databases. TODS 2 (3).
L2003 L. Libkin. Expressive power of SQL. TCS, 296 (2003).
C+1996 L. Colby et al. Algorithms for deferred view maintenance. SIGMOD 199.
G+1997 J. Gray et al. Data cube: A relational aggregation operator generalizing group-by, cross-tab, and sub-totals (1997) Data Mining and Knowledge Discovery.

H2001 A. Halevy. Answering queries using views: A survey. VLDB Journal. December 2001.

## Lecture 02 : The relational data model

- Mathematical relations and relational schema
- Using SQL to implement a relational schema
- Keys
- Database query languages
- The Relational Algebra
- The Relational Calculi (tuple and domain)
- a bit of SQL


## Let's start with mathematical relations

Suppose that $S_{1}$ and $S_{2}$ are sets. The Cartesian product, $S_{1} \times S_{2}$, is the set

$$
S_{1} \times S_{2}=\left\{\left(s_{1}, s_{2}\right) \mid s_{1} \in S_{1}, s_{2} \in S_{2}\right\}
$$

A (binary) relation over $S_{1} \times S_{2}$ is any set $r$ with

$$
r \subseteq S_{1} \times S_{2}
$$

In a similar way, if we have $n$ sets,

$$
S_{1}, S_{2}, \ldots, S_{n}
$$

then an $n$-ary relation $r$ is a set

$$
r \subseteq S_{1} \times S_{2} \times \cdots \times S_{n}=\left\{\left(s_{1}, s_{2}, \ldots, s_{n}\right) \mid s_{i} \in S_{i}\right\}
$$

## Relational Schema

Let $\mathbf{X}$ be a set of $k$ attribute names.

- We will often ignore domains (types) and say that $R(\mathbf{X})$ denotes a relational schema.
- When we write $R(\mathbf{Z}, \mathbf{Y})$ we mean $R(\mathbf{Z} \cup \mathbf{Y})$ and $\mathbf{Z} \cap \mathbf{Y}=\phi$.
- $u .[\mathbf{X}]=v .[\mathbf{X}]$ abbreviates $u . A_{1}=v . A_{1} \wedge \cdots \wedge u . A_{k}=v . A_{k}$.
- $\overrightarrow{\mathbf{X}}$ represents some (unspecified) ordering of the attribute names, $A_{1}, A_{2}, \ldots, A_{k}$


## Mathematical vs. database relations

Suppose we have an $n$-tuple $t \in S_{1} \times S_{2} \times \cdots \times S_{n}$. Extracting the $i$-th component of $t$, say as $\pi_{i}(t)$, feels a bit low-level.

- Solution: (1) Associate a name, $A_{i}$ (called an attribute name) with each domain $S_{i}$. (2) Instead of tuples, use records - sets of pairs each associating an attribute name $A_{i}$ with a value in domain $S_{i}$.

A database relation $R$ over the schema
$A_{1}: S_{1} \times A_{2}: S_{2} \times \cdots \times A_{n}: S_{n}$ is a finite set

$$
R \subseteq\left\{\left\{\left(A_{1}, s_{1}\right),\left(A_{2}, s_{2}\right), \ldots,\left(A_{n}, s_{n}\right)\right\} \mid s_{i} \in S_{i}\right\}
$$

## Example

## A relational schema

Students(name: string, sid: string, age : integer)
A relational instance of this schema

## Students = \{

$\{($ name, Fatima $),($ sid, fm21), (age, 20) $\}$, $\{($ name, Eva), (sid, ev77), (age, 18) $\}$, $\{($ name, James), (sid, jj25), (age, 19)\} \}

A tabular presentation

| name | sid | age |
| :--- | :--- | :--- |
| Fatima | fm21 | 20 |
| Eva | ev77 | 18 |
| James | jj25 | 19 |

## Key Concepts

## Relational Key

Suppose $R(\mathbf{X})$ is a relational schema with $\mathbf{Z} \subseteq \mathbf{X}$. If for any records $u$ and $v$ in any instance of $R$ we have

$$
u \cdot[\mathbf{Z}]=v \cdot[\mathbf{Z}] \Longrightarrow u \cdot[\mathbf{X}]=v \cdot[\mathbf{X}],
$$

then $\mathbf{Z}$ is a superkey for $R$. If no proper subset of $\mathbf{Z}$ is a superkey, then $\mathbf{Z}$ is a key for $R$. We write $R(\underline{\mathbf{Z}}, \mathbf{Y})$ to indicate that $\mathbf{Z}$ is a key for $R(\mathbf{Z} \cup \mathbf{Y})$.

Note that this is a semantic assertion, and that a relation can have multiple keys.

## Creating Tables in SQL

create table Students

$$
\begin{aligned}
& \text { (sid varchar }(10), \\
& \text { name varchar }(50), \\
& \text { age int); }
\end{aligned}
$$

-- insert record with attribute names
insert into Students set

$$
\text { name }=\text { 'Fatima', age }=20, \text { sid }=\text { 'fm21'; }
$$

-- or insert records with values in same order
-- as in create table
insert into Students values

$$
\begin{aligned}
& \left(' j j 25^{\prime}, ~ ' J a m e s ', ~ 19\right), ~ \\
& (' \text { 'ev77' , 'Eva' , 18); }
\end{aligned}
$$

## Listing a Table in SQL

-- list by attribute order of create table mysql> select * from Students;


## Listing a Table in SQL

-- list by specified attribute order mysql> select name, age, sid from Students;


## Keys in SQL

A key is a set of attributes that will uniquely identify any record (row) in a table.

```
-- with this create table
create table Students
    (sid varchar(10),
        name varchar(50),
        age int,
        primary key (sid));
-- if we try to insert this (fourth) student ...
mysql> insert into Students set
    name = 'Flavia', age = 23, sid = 'fm21';
ERROR 1062 (23000): Duplicate
    entry 'fm21' for key 'PRIMARY'
```


## What is a (relational) database query language?

Input : a collection of relation instances

Output : a single relation instance

$$
R_{1}, R_{2}, \cdots, R_{k} \quad \Longrightarrow \quad Q\left(R_{1}, R_{2}, \cdots, R_{k}\right)
$$

How can we express $Q$ ?
In order to meet Codd's goals we want a query language that is high-level and independent of physical data representation.

There are many possibilities ...

## The Relational Algebra (RA)

| $Q$ | $R$ | base relation |
| :---: | :---: | :---: |
|  | $\sigma_{p}(Q)$ | selection |
|  | $\pi_{\mathrm{x}}(Q)$ | projection |
|  | $Q \times Q$ | product |
|  | $Q-Q$ | difference |
|  | $Q \cup Q$ | union |
|  | $Q \cap Q$ | intersection |
|  | $\rho_{M}(Q)$ | renaming |

- $p$ is a simple boolean predicate over attributes values.
- $\mathbf{X}=\left\{A_{1}, A_{2}, \ldots, A_{k}\right\}$ is a set of attributes.
- $M=\left\{A_{1} \mapsto B_{1}, \quad A_{2} \mapsto B_{2}, \ldots, A_{k} \mapsto B_{k}\right\}$ is a renaming map.


## Relational Calculi

The Tuple Relational Calculus (TRC)

$$
Q=\{t \mid P(t)\}
$$

The Domain Relational Calculus (DRC)

$$
Q=\left\{\left(A_{1}=v_{1}, A_{2}=v_{2}, \ldots, A_{k}=v_{k}\right) \mid P\left(v_{1}, v_{2}, \cdots, v_{k}\right)\right\}
$$

## The SQL standard

- Origins at IBM in early 1970's.
- SQL has grown and grown through many rounds of standardization :
- ANSI: SQL-86
- ANSI and ISO : SQL-89, SQL-92, SQL:1999, SQL:2003, SQL:2006, SQL:2008
- SQL is made up of many sub-languages:
- Query Language
- Data Definition Language
- System Administration Language


## Selection

## $R$

$Q(R)$

| $A$ | $B$ | $C$ | $D$ |
| :---: | :---: | :---: | :---: |
| 20 | 10 | 0 | 55 |
| 11 | 10 | 0 | 7 |
| 4 | 99 | 17 | 2 |
| 77 | 25 | 4 | 0 |

RA $Q=\sigma_{A>12}(R)$
TRC $Q=\{t \mid t \in R \wedge t . A>12\}$
DRC $Q=\{\{(A, a),(B, b),(C, c),(D, d)\} \mid$
$\{(A, a),(B, b),(C, c),(D, d)\} \in R \wedge a>12\}$
SQL select * from R where R.A > 12

## Projection

R

$$
\begin{array}{c|c|c|c}
A & B & C & D \\
\hline 20 & 10 & 0 & 55 \\
11 & 10 & 0 & 7 \\
4 & 99 & 17 & 2 \\
77 & 25 & 4 & 0
\end{array} \quad \begin{array}{ccc|c}
B & & C \\
\hline 10 & 0 \\
99 & 17 \\
25 & 4
\end{array}
$$

RA $Q=\pi_{B, C}(R)$
$\operatorname{TRC} Q=\{t \mid \exists u \in R \wedge t .[B, C]=u .[B, C]\}$
DRC $Q=\{\{(B, b),(C, c)\} \mid$
$\exists\{(A, a),(B, b),(C, c),(D, d)\} \in R\}$
SQL select distinct $B, C$ from $R$

## Why the distinct in the SQL?

The SQL query

$$
\text { select } B, C \text { from } R
$$

will produce a bag (multiset)!
$R$

| $A$ | $B$ | $C$ | $D$ |  |  |  |  |
| :---: | :---: | :---: | :---: | :--- | :--- | :---: | :---: | :---: |
| 20 | 10 | 0 | 55 |  |  | $B$ | $C$ |
| 11 | 10 | 0 | 7 |  | 10 | 0 | $\star \star \star$ |
| 4 | 99 | 17 | 2 |  | 10 | 0 | $\star \star \star$ |
| 77 | 25 | 4 | 0 |  | 99 | 17 |  |
|  |  |  | 25 | 4 |  |  |  |

SQL is actually based on multisets, not sets. We will look into this more in Lecture 11.

## Lecture 03 : Entity-Relationship (E/R) modelling

## Outline

- Entities
- Relationships
- Their relational implementations
- n-ary relationships
- Generalization
- On the importance of SCOPE


## Some real-world data ...

... from the Internet Movie Database (IMDb).

| Title | Year | Actor |
| :--- | :---: | :--- |
| Austin Powers: International Man of Mystery | 1997 | Mike Myers |
| Austin Powers: The Spy Who Shagged Me | 1999 | Mike Myers |
| Dude, Where's My Car? | 2000 | Bill Chott |
| Dude, Where's My Car? | 2000 | Marc Lynn |

## Entities diagrams and Relational Schema



These diagrams represent relational schema

> Movie(MovieID, Title, Year)
> Person(PersonID, FirstName, LastName)

Yes, this ignores types ...

## Entity sets (relational instances)

## Movie

| MovielD | Title | Year |
| :--- | :--- | :--- |
| 55871 | Austin Powers: International Man of Mystery | 1997 |
| 55873 | Austin Powers: The Spy Who Shagged Me | 1999 |
| 171771 | Dude, Where's My Car? | 2000 |

(Tim used line number from IMDb raw file movies.list as MovieID.)
Person

| PersonID | FirstName | LastName |
| :--- | :--- | :--- |
| 6902836 | Mike | Myers |
| 1757556 | Bill | Chott |
| 5882058 | Marc | Lynn |

(Tim used line number from IMDb raw file actors.list as PersonID)

## Relationships



## Foreign Keys and Referential Integrity

## Foreign Key

Suppose we have $R(\mathbf{Z}, \mathbf{Y})$. Furthermore, let $S(\mathbf{W})$ be a relational schema with $\mathbf{Z} \subseteq \mathbf{W}$. We say that $\mathbf{Z}$ represents a Foreign Key in $S$ for $R$ if for any instance we have $\pi_{\mathbf{Z}}(S) \subseteq \pi_{\mathbf{Z}}(R)$. This is a semantic assertion.

Referential integrity
A database is said to have referential integrity when all foreign key constraints are satisfied.

## A relational representation

A relational schema

## Acts $\ln ($ MovielD, PersonID)

With referential integrity constraints

$$
\begin{aligned}
& \pi_{\text {MovielD }}(\text { ActsIn }) \subseteq \pi_{\text {MovielD }}(\text { Movie }) \\
& \pi_{\text {PersonID }}(\text { ActsIn }) \subseteq \pi_{\text {PersonID }}(\text { Person })
\end{aligned}
$$

## ActsIn

| PersonID | MovielD |
| :--- | :--- |
| 6902836 | 55871 |
| 6902836 | 55873 |
| 1757556 | 171771 |
| 5882058 | 171771 |

## Foreign Keys in SQL

```
create table ActsIn
( MovieID int not NULL,
    PersonID int not NULL,
    primary key (MovieID, PersonID),
    constraint actsin_movie
        foreign key (MovieID)
        references Movie(MovieID),
    constraint actsin_person
        foreign key (PersonID)
        references Person(PersonID))
```


## Relational representation of relationships, in general?

That depends ...
Mapping Cardinalities for binary relations, $R \subseteq S \times T$
Relation $R$ is
many to many
meaning
one to many $\quad \forall t \in T, s_{1}, s_{2} \in S .\left(R\left(s_{1}, t\right) \wedge R\left(s_{2}, t\right)\right) \Longrightarrow s_{1}=s_{2}$
many to one $\quad \forall s \in S, t_{1}, t_{2} \in T .\left(R\left(s, t_{1}\right) \wedge R\left(s, t_{2}\right)\right) \Longrightarrow t_{1}=t_{2}$
one to one
one to many and many to one
Note that the database terminology differs slightly from standard mathematical terminology.

## Diagrams for Mapping Cardinalities

ER diagram Relation $R$ is

many to many $(M: N)$

one to many ( $1: M$ )
many to one ( $M: 1$ )
one to one (1:1)

## Relationships to Relational Schema



Relation $R$ is
many to many ( $M: N$ )
one to many (1:M)
many to one ( $M: 1$ )
one to one (1:1) $R(\underline{X}, Z, U)$ and/or $R(X, \underline{Z}, U)$ (alternate keys

## "one to one" does not mean a "1-to-1 correspondence"



This database instance is OK

$$
\begin{aligned}
& \\
& \begin{array}{lll} 
& \mathbf{R} & \\
\mathbf{Z} & \mathbf{X} & \mathbf{U} \\
\hline z_{1} & x_{2} & u_{1}
\end{array} \\
& \begin{array}{ll}
\mathbf{x}^{\mathbf{T}} & \mathbf{Y} \\
\mathbf{X}_{1} & y_{1} \\
x_{2} & y_{2} \\
x_{3} & y_{3} \\
x_{4} & y_{4}
\end{array}
\end{aligned}
$$

## Some more real-world data ... (a slight change of SCOPE)

| Title | Year | Actor | Role |
| :--- | :---: | :--- | :--- |
| Austin Powers: International Man of Mystery | 1997 | Mike Myers | Austin Powers |
| Austin Powers: International Man of Mystery | 1997 | Mike Myers | Dr. Evil |
| Austin Powers: The Spy Who Shagged Me | 1999 | Mike Myers | Austin Powers |
| Austin Powers: The Spy Who Shagged Me | 1999 | Mike Myers | Dr. Evil |
| Austin Powers: The Spy Who Shagged Me | 1999 | Mike Myers | Fat Bastard |
| Dude, Where's My Car? | 2000 | Bill Chott | Big Cult Guard 1 |
| Dude, Where's My Car? | 2000 | Marc Lynn | Cop with Whips |

How will this change our model?

## Will ActsIn remain a binary Relationship?



No! An actor can have many roles in the same movie!

## Could ActsIn be modeled as a Ternary Relationship?



Yes, this works!

## Can a ternary relationship be modeled with multiple binary relationships?



The Casting entity seems artificial. What attributes would it have?

## Sometimes ternary to multiple binary makes more sense ...



## Generalization



## Questions

- Is every movie either comedy or a drama?
- Can a movie be a comedy and a drama?

But perhaps this isn't a good model ...

- What attributes would distinguish Drama and Comedy entities?
- What abound Science Fiction?
- Perhaps Genre would make a nice entity, which could have a relationship with Movie.


## Question: What is the right model?

Answer: The question doesn't make sense!

- There is no "right" model ...
- It depends on the intended use of the database.
- What activity will the DBMS support?
- What data is needed to support that activity?

The issue of SCOPE is missing from most textbooks

- Suppose that all databases begin life with beautifully designed schemas.
- Observe that many operational databases are in a sorry state.
- Conclude that the scope and goals of a database continually change, and that schema evolution is a difficult problem to solve, in practice.


## Another change of SCOPE ...

## Movies with detailed release dates

| Title | Country | Day | Month | Year |
| :--- | :--- | :--- | :--- | :--- |
| Austin Powers: International Man of Mystery | USA | 02 | 05 | 1997 |
| Austin Powers: International Man of Mystery | Iceland | 24 | 10 | 1997 |
| Austin Powers: International Man of Mystery | UK | 05 | 09 | 1997 |
| Austin Powers: International Man of Mystery | Brazil | 13 | 02 | 1998 |
| Austin Powers: The Spy Who Shagged Me | USA | 08 | 06 | 1999 |
| Austin Powers: The Spy Who Shagged Me | Iceland | 02 | 07 | 1999 |
| Austin Powers: The Spy Who Shagged Me | UK | 30 | 07 | 1999 |
| Austin Powers: The Spy Who Shagged Me | Brazil | 08 | 10 | 1999 |
| Dude, Where's My Car? | USA | 10 | 12 | 2000 |
| Dude, Where's My Car? | Iceland | 9 | 02 | 2001 |
| Dude, Where's My Car? | UK | 9 | 02 | 2001 |
| Dude, Where's My Car? | Brazil | 9 | 03 | 2001 |
| Dude, Where's My Car? | Russia | 18 | 09 | 2001 |

## ... and an attribute becomes an entity with a connecting relation.



## Lecture 04 : Relational algebra and relational calculus

## Outline

- Constructing new tuples!
- Joins
- Limitations of Relational Algebra


## Renaming

$R$

| $A$ | $B$ | $C$ | $D$ |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :--- | :--- | :---: | :---: | :---: | :---: |
| 20 | 10 | 0 | 55 |  |  | $A$ | $E$ | $C$ | $F$ |
| 11 | 10 | 0 | 7 |  | 20 | 10 | 0 | 55 |  |
| 4 | 99 | 17 | 2 |  | 11 | 10 | 0 | 7 |  |
| 77 | 25 | 4 | 0 |  | 4 | 99 | 17 | 2 |  |
|  |  |  |  | 25 | 4 | 0 |  |  |  |

RA $Q=\rho_{\{B \mapsto E, D \mapsto F\}}(R)$
TRC $Q=\{t \mid \exists u \in R \wedge t . A=u . A \wedge t . E=u . E \wedge t . C=$ $u . C \wedge t . F=u . D\}$
DRC $Q=\{\{(A, a),(E, b),(C, c),(F, d)\} \mid$

$$
\exists\{(A, a),(B, b),(C, c),(D, d)\} \in R\}
$$

SQL select A, B as E, C, D as F from R

## Union

$$
R \quad S \quad Q(R, S)
$$

| $A$ | $B$ |
| :---: | :---: |
| 20 | 10 |
| 11 | 10 |
| 4 | 99 |

$$
\begin{array}{c|c}
A & B \\
\hline 20 & 10 \\
77 & 1000
\end{array}
$$


$R \mathrm{RA} Q=R \cup S$
TRC $Q=\{t \mid t \in R \vee t \in S\}$
$\operatorname{DRC} Q=\{\{(A, a),(B, b)\} \mid\{(A, a),(B, b)\} \in$ $R \vee\{(A, a),(B, b)\} \in S\}$
SQL (select * from R) union (select * from S)

## Intersection

R

$$
S \quad Q(R)
$$



RA $Q=R \cap S$
$\operatorname{TRC} Q=\{t \mid t \in R \wedge t \in S\}$
$\operatorname{DRC} Q=\{\{(A, a),(B, b)\} \mid\{(A, a),(B, b)\} \in$ $R \wedge\{(A, a),(B, b)\} \in S\}$
SQL

```
(select * from R) intersect (select * from
```


## Difference

$R$

| $A$ | $B$ |  |  |
| :---: | :---: | :---: | :---: |
| 20 | 10 |  |  |
| 11 | 10 |  | $A$ |
| 20 | 10 |  |  |
| 4 | 99 |  |  |$\quad$| 77 | 1000 |
| :--- | :--- |$\quad \Longrightarrow \quad$| $A$ | $B$ |
| :---: | :---: |
| 11 | 10 |
| 4 |  |

RA $Q=R-S$
$\operatorname{TRC} Q=\{t \mid t \in R \wedge t \notin S\}$
$\operatorname{DRC} Q=\{\{(A, a),(B, b)\} \mid\{(A, a),(B, b)\} \in$ $R \wedge\{(A, a),(B, b)\} \notin S\}$
SQL (select * from R) except (select * from S)

## Wait, are we missing something?

Suppose we want to add information about college membership to our Student database. We could add an additional attribute for the college.


## Put logically independent data in distinct tables?

Students


Colleges :



$$
+-----+---------------+
$$

| $\left\|\begin{array}{ll}\text { k } & \text { King's } \\ \mid & \mathrm{cl} \\ \mid & \text { Clare } \\ \mid & \text { sid } \\ \text { q } & \text { Sidney Sussex } \\ \text { \| } & \text { Queens' }\end{array}\right\|$ |
| :--- | :--- |

## Product

| $R$ |  | $S$ |  |  |
| :---: | :---: | :---: | :---: | :---: |
| $A$ | $B$ | $C$ | $D$ |  |
| 20 | 10 |  | 14 | 99 |
| 11 | 10 |  | 77 | 100 |
| 4 | 99 |  |  |  |


| $Q(R, S)$ |  |  |  |
| :---: | :---: | :---: | :---: |
| $A$ | $B$ | $C$ | $D$ |
| 20 | 10 | 14 | 99 |
| 20 | 10 | 77 | 100 |
| 11 | 10 | 14 | 99 |
| 11 | 10 | 77 | 100 |
| 4 | 99 | 14 | 99 |
| 4 | 99 | 77 | 100 |

Note the automatic flattening
RA $Q=R \times S$
$\operatorname{TRC} Q=\{t \mid \exists u \in R, v \in S, t .[A, B]=u \cdot[A, B] \wedge t .[C, D]=$ $v .[C, D]\}$
$\operatorname{DRC} Q=\{\{(A, a),(B, b),(C, c),(D, d)\} \mid$
$\{(A, a),(B, b)\} \in R \wedge\{(C, c),(D, d)\} \in S\}$
SQL select $A, B, C, D$ from $R, S$

## Product is special!

$$
\left. \quad \Longrightarrow \quad \begin{array}{c|c|c|c} 
\\
A
\end{array} \quad \begin{array}{c}
A \\
A \mapsto C, B \mapsto D
\end{array}\right)
$$

- $x$ is the only operation in the Relational Algebra that created new records (ignoring renaming),
- But $\times$ usually creates too many records!
- Joins are the typical way of using products in a constrained manner.


## Natural Join

## Natural Join

Given $R(\mathbf{X}, \mathbf{Y})$ and $S(\mathbf{Y}, \mathbf{Z})$, we define the natural join, denoted $R \bowtie S$, as a relation over attributes $\mathbf{X}, \mathbf{Y}, \mathbf{Z}$ defined as

$$
R \bowtie S \equiv\{t \mid \exists u \in R, v \in S, u \cdot[\mathbf{Y}]=v .[\mathbf{Y}] \wedge t=u .[\mathbf{X}] \cup u .[\mathbf{Y}] \cup v .[\mathbf{Z}]\}
$$

In the Relational Algebra:

$$
R \bowtie S=\pi_{\mathbf{x}, \mathbf{Y} \mathbf{Z}}\left(\sigma_{\mathbf{Y}=\mathbf{Y}^{\prime}}\left(R \times \rho_{\overrightarrow{\mathbf{Y}}_{\boldsymbol{Y}} \rightarrow \overrightarrow{\mathbf{Y}}^{\prime}}(S)\right)\right)
$$

## Join example

Students

| name | sid | age | cid |
| :--- | :--- | :--- | :--- |
| Fatima | fm21 | 20 | cl |
| Eva | ev77 | 18 | k |
| James | jj25 | 19 | cl |

## Colleges



$\Longrightarrow \quad$| name | cname |
| :---: | :---: |
|  | Clare |
| Eva | King's |
|  | James |
|  | Clare |

## The same in SQL

```
select name, cname
from Students, Colleges
where Students.cid = Colleges.cid
```

| name | cname |
| :---: | :---: |
| Eva | King's |
| Fatima | Clare |
| James | Clare |

## Division

Given $R(\mathbf{X}, \mathbf{Y})$ and $S(\mathbf{Y})$, the division of $R$ by $S$, denoted $R \div S$, is the relation over attributes $\mathbf{X}$ defined as (in the TRC)

$$
R \div S \equiv\{x \mid \forall s \in S, x \cup s \in R\} .
$$

| name | award | $\div$ | award |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Fatima | writing |  |  |  |  |
| Fatima | music |  | music |  | name |
| Eva | music |  | writing |  | Eva |
| Eva | writing |  | dance |  |  |
| Eva | dance |  |  |  |  |
| James | dance |  |  |  |  |

## Division in the Relational Algebra?

Clearly, $R \div S \subseteq \pi_{\mathbf{x}}(R)$. So $R \div S=\pi_{\mathbf{x}}(R)-C$, where $C$ represents counter examples to the division condition. That is, in the TRC,

$$
C=\{x \mid \exists s \in S, x \cup s \notin R\} .
$$

- $U=\pi_{\mathbf{X}}(R) \times S$ represents all possible $x \cup s$ for $x \in \mathbf{X}(R)$ and $s \in S$,
- so $T=U-R$ represents all those $x \cup s$ that are not in $R$,
- so $C=\pi_{\mathbf{x}}(T)$ represents those records $x$ that are counter examples.


## Division in RA

$$
R \div S \equiv \pi_{\mathbf{x}}(R)-\pi_{\mathbf{x}}\left(\left(\pi_{\mathbf{x}}(R) \times S\right)-R\right)
$$

## Query Safety

A query like $Q=\{t \mid t \in R \wedge t \notin S\}$ raises some interesting questions. Should we allow the following query?

$$
Q=\{t \mid t \notin S\}
$$

We want our relations to be finite!

## Safety

A (TRC) query

$$
Q=\{t \mid P(t)\}
$$

is safe if it is always finite for any database instance.

- Problem : query safety is not decidable!
- Solution : define a restricted syntax that guarantees safety. Safe queries can be represented in the Relational Algebra.


## Limitations of simple relational query languages

- The expressive power of RA, TRC, and DRC are essentially the same.
- None can express the transitive closure of a relation.
- We could extend RA to more powerful languages (like Datalog).
- SQL has been extended with many features beyond the Relational Algebra.
- stored procedures
- recursive queries
- ability to embed SQL in standard procedural languages


## Lecture 05 : SQL and integrity constraints

Outline

- NULL in SQL
- three-valued logic
- Multisets and aggregation in SQL


## What is nULL in SQL?

## What if you don't know Kim's age?

```
mysql> select * from students;
+------+--------+------+
    | sid | name | age |
+------+--------+------+
| ev77 | Eva | 18 |
| fm21 | Fatima | 20 |
| jj25 | James | 19 |
| ks87 | Kim | NULL |
+------+--------+------+
```


## What is NULL?

- NULL is a place-holder, not a value!
- NULL is not a member of any domain (type),
- For records with NULL for age, an expression like age > 20 must unknown!
- This means we need (at least) three-valued logic.

Let $\perp$ represent We don't know!

$$
\begin{array}{c|ccc}
\wedge & \mathbf{T} & \mathbf{F} & \perp \\
\hline \mathbf{T} & \mathbf{T} & \mathbf{F} & \perp \\
\mathbf{F} & \mathbf{F} & \mathbf{F} & \mathbf{F} \\
\perp & \perp & \mathbf{F} & \perp
\end{array}
$$



| $v$ | $\neg V$ |
| :---: | :---: |
| $\mathbf{T}$ | $\mathbf{F}$ |
| $\mathbf{F}$ | $\mathbf{T}$ |
| $\perp$ | $\perp$ |

## NULL can lead to unexpected results

```
mysql> select * from students;
+------+--------+------+
    | sid | name | age |
+------+--------+------+
| ev77 | Eva | 18 |
| fm21 | Fatima | 20 |
| jj25 | James | 19 |
| ks87 | Kim | NULL |
+------+--------+------+
mysql> select * from students where age <> 19;
+------+--------+------+
| sid | name | age |
+------+--------+------+
| ev77 | Eva 
+------+--------+------+
```


## The ambiguity of NULL

## Possible interpretations of NULL

- There is a value, but we don't know what it is.
- No value is applicable.
- The value is known, but you are not allowed to see it.

A great deal of semantic muddle is created by conflating all of these interpretations into one non-value.

On the other hand, introducing distinct NULLs for each possible interpretation leads to very complex logics ...

## Not everyone approves of NULL

## C. J. Date [D2004], Chapter 19

"Before we go any further, we should make it very clear that in our opinion (and in that of many other writers too, we hasten to add), NULLs and 3VL are and always were a serious mistake and have no place in the relational model."

## age is not a good attribute ...

The age column is guaranteed to go out of date! Let's record dates of birth instead!

```
create table Students
    ( sid varchar(10) not NULL,
    name varchar(50) not NULL,
    birth_date date,
    cid varchar(3) not NULL,
    primary key (sid),
    constraint student_college foreign key (cid)
    references Colleges(cid) )
```


## age is not a good attribute ...

| sid | name | \| birth_date | cid | |
| :---: | :---: | :---: |
| ev77 | Eva | \| 1990-01-26 | k |
| fm21 | \| Fatima | \| 1988-07-20 | cl |
| jj25 | James | \| 1989-03-14 | cl |

## Use a view to recover original table

(Note : the age calculation here is not correct!)

```
create view StudentsWithAge as
select sid, name,
        (year(current_date()) - year(birth_date)) as age,
        cid
    from Students;
```



Views are simply identifiers that represent a query. The view's name can be used as if it were a stored table.

## But that calculation is not correct ...

Clearly the calculation of age does not take into account the day and month of year.

From 2010 Database Contest (winner : Sebastian Probst Eide)

```
SELECT year(CURRENT_DATE()) - year(birth_date) -
    CASE WHEN month(CURRENT_DATE()) < month(birth_date)
    THEN 1
    ELSE
    CASE WHEN month(CURRENT_DATE()) = month(birth_date)
    THEN
        CASE WHEN day(CURRENT_DATE()) < day(birth_date)
        THEN 1
        ELSE 0
        END
    ELSE 0
    END
    END
AS age FROM Students
```


## An Example ...



## ... of duplicates



## Why Multisets?

## Duplicates are important for aggregate functions.

```
mysql> select min(mark),
                    max(mark),
                    sum(mark),
                    avg (mark)
    from marks;
+-----------+-----------+-----------+---------------
| min(mark) | max(mark) | sum(mark) | avg(mark) |
+-----------+-----------+-----------+---------------
```


## The group by clause

```
mysql> select course,
            min(mark),
            max(mark),
            avg(mark)
            from marks
                group by course;
```



```
+-----------+-----------+-----------+---------------
```


## Visualizing group by

| sid | course | mark |
| :---: | :---: | :---: |
| ev77 | databases | 92 |
| ev77 | spelling | 99 |
| tgg22 | spelling | 3 |
| tgg22 | databases | 100 |
| fm21 | databases | 92 |
| fm21 | spelling | 100 |
| jj25 | databases | 88 |
| jj25 | spelling | 92 |


| course mark <br> spelling <br> spelling 99 <br> spelling 100 <br> spelling 92 <br> course mark <br> databases 92 <br> databases 100 <br> databases 92 <br> databases 88  |  |
| :--- | :---: |

## Visualizing group by

|  |  |
| :--- | :---: |
| course mark <br> spelling <br> spelling 99 <br> spelling 100 <br> spelling 92 <br>   <br> course mark <br> databases 92 <br> databases 100 <br> databases 92 <br> databases 88 |  |


| $\min (\mathbf{\text { mark }}$ | course | $\min ($ mark $)$ |
| :---: | :---: | :---: |
|  | spelling <br> databases | 3 |
|  | 88 |  |
|  |  |  |

## The having clause

## How can we select on the aggregated columns?

```
mysql> select course,
                    min(mark),
    max(mark),
    avg(mark)
        from marks
        group by course
        having min(mark) > 60;
+-----------+-----------+------------+----------------
| course | min(mark) | max(mark) | avg(mark) |
+-----------+-----------+-----------+----------------
| databases | 88 | 100 | 93.0000 |
```


## Use renaming to make things nicer ...

mysql> select course,
min(mark) as minimum,
max (mark) as maximum,
avg (mark) as average
from marks
group by course
having minimum > 60;
$+-----------+---------+---------+--------+$
$\mid$ course | minimum | maximum | average |
+----------+-----------------+-------+
$\mid$ databases | 88 | $100 \mid 93.0000$ |
+-----------+---------+---------+------------+

## Lecture 06 : Database updates

## Outline

- General integrity constraints
- ACID transactions
- A simple language for transactions
- Reasoning about transactions.
- Update anomalies
- Problems with data redundancy


## General integrity constraints

- Suppose that $C$ is some constraint we would like to enforce on our database.
- Let $Q_{\neg C}$ be a query that captures all violations of $C$.
- Enforce (somehow) that the assertion that is always $Q_{\neg C}$ empty.

```
create view C_violations as ....
create assertion check_C
    check not (exists C_violations)
```


## A more direct notation will be useful ...

Just write predicates with quantifiers $\forall x \in Q, P(x)$ and $\exists x \in Q, P(x)$, where $Q$ is a query in a relational calculus.
Example. A key constraint.

$$
\forall x \in R, \forall y \in R, x \cdot k=y \cdot k \Longrightarrow x=y
$$

Example. A foreign key constraint.

$$
\forall x \in R_{1}, \exists y \in R_{2}, x \cdot k=y . k
$$

For a database assertion $P$, the notation $D B \models P$ means that $P$ holds in the database instance $D B$.

## Transactions - ACID properties

The an ACID transaction is an abstraction implemented by most relational Database systems.

## ACID

Atomicity Either all actions are carried out, or none are

- logs needed to undo operations, if needed

Consistency If each transaction is consistent, and the database is initially consistent, then it is left consistent

- This is very much a part of applications design.

Isolation Transactions are isolated, or protected, from the effects of other scheduled transactions

- Serializability, 2-phase commit protocol

Durability If a transactions completes successfully, then its effects persist

- Logging and crash recovery

Should be review from Concurrent and Distributed Systems so we will

## A simple language for transactions?

Although the relational algebra or relational calculi are widely used, there seems to be no analogous formalism for database updates and transactions. So we will invent one!

Transactions will have the form

$$
\operatorname{transaction} f\left(x_{1}, x_{2}, \ldots, x_{k}\right)=E
$$

where

| $E::=$ | $\operatorname{skip}$ | (do nothing) |
| :--- | :--- | :--- |
|  | abort | (abort transaction) |
|  | insert $(R, t)$ | (raw insert) |
|  | $\operatorname{delete}(R, p)$ | (raw delete, $p$ quantifier-free) |
|  | $E_{1} ; E_{2}$ | (sequence ) |
|  | if $P$ then $E_{1}$ else $E_{2}$ | ( $P$ an assertion) |

## Hoare Logic for Database updates

We write

$$
\{P\} E\{Q\}
$$

to mean that if $D B \models P$ then $E(D B) \models Q$.
One way to think about an integrity constraint $C$
For all transactions

$$
\operatorname{transaction} f\left(x_{1}, x_{2}, \ldots, x_{k}\right)=E
$$

and all values $v_{1}, \ldots v_{k}$ we want

$$
\{C\} f\left(v_{1}, v_{2}, \ldots, v_{k}\right)\{C\}
$$

That is, constraint $C$ is an invariant of for all transactions.

## The weakest precondition

We defined he weakest precondition, $\operatorname{wpc}(E, Q)$, so that if $P \Longrightarrow \operatorname{wpc}(E, Q)$, then $\{P\} E\{Q\}$.

We ignore abort for simplicity ...

$$
\begin{aligned}
\operatorname{wpc}(\operatorname{skip}, Q)= & Q \\
\operatorname{wpc}(\operatorname{INS}(R, t), Q)= & Q[R \cup\{t\} / R] \\
\operatorname{wpc}(\operatorname{DEL}(R, p), Q)= & Q[\{t \in R \mid \neg p(t)\} / R] \\
\operatorname{wpc}\left(E_{1} ; E_{2}, Q\right)= & \operatorname{wpc}\left(E_{1}, \operatorname{wpc}\left(E_{2}, Q\right)\right) \\
\operatorname{wpc}\left(\text { if } T \text { then } E_{1} \text { else } E_{2}, Q\right)= & \left(T \Longrightarrow \operatorname{wpc}\left(E_{1}, Q\right)\right) \wedge \\
& \left(\neg T \Longrightarrow \operatorname{wpc}\left(E_{2}, Q\right)\right)
\end{aligned}
$$

(Gosh, for some reason this reminds me of Continuation Passing Style (CPS) transformation .... )

So, for $C$ to be an invariant of $f$ we want for all $v_{1}, v_{2}, \ldots, v_{k}$,

$$
C \Longrightarrow \operatorname{wpc}\left(f\left(v_{1}, v_{2}, \ldots, v_{k}\right), C\right) .
$$

## Example : key constraint

$$
R_{k}=\forall x \in R, \forall y \in R, x \cdot k=y \cdot k \Longrightarrow x=y
$$

transactioninsert $(R, t)=$ if $\forall z \in R, z . k \neq t . k$ then $\operatorname{INS}(R, t)$ else skip
We want

$$
R_{k} \Longrightarrow \operatorname{wpc}\left(\operatorname{insert}(R, t), R_{k}\right)
$$

Left as an exercise.

## Problem : internal vs external modeling

- Integrity constraints only make assertions about the internal consistency a database.
- That is useful only when we know that our database is a good model of that aspect of the world we want to capture.
- What if our consistent database is useless?


## Update anomalies

Big Table

| sid | name | college | course | part | term_name |
| :---: | :---: | :---: | :---: | :---: | :---: |
| yy88 | Yoni | New Hall | Algorithms I | IA | Easter |
| uu99 | Uri | King's | Algorithms I | IA | Easter |
| bb44 | Bin | New Hall | Databases | IB | Lent |
| bb44 | Bin | New Hall | Algorithms II | IB | Michaelmas |
| zz70 | Zip | Trinity | Databases | IB | Lent |
| zz70 | Zip | Trinity | Algorithms II | IB | Michaelmas |

- How can we tell if an insert record is consistent with current records?
- Can we record data about a course before students enroll?
- Will we wipe out information about a college when last student associated with the college is deleted?


## Redundancy implies more locking ...

... at least for correct transactions!

## Big Table

| sid | name | college | course | part | term_name |
| :---: | :---: | :---: | :---: | :---: | :---: |
| yy88 | Yoni | New Hall | Algorithms I | IA | Easter |
| uu99 | Uri | King's | Algorithms I | IA | Easter |
| bb44 | Bin | New Hall | Databases | IB | Lent |
| bb44 | Bin | New Hall | Algorithms II | IB | Michaelmas |
| zz70 | Zip | Trinity | Databases | IB | Lent |
| zz70 | Zip | Trinity | Algorithms II | IB | Michaelmas |

- Change New Hall to Murray Edwards College
- Conceptually simple update
- May require locking entire table.


## Redundancy is the root of (almost) all database evils

- It may not be obvious, but redundancy is also the cause of update anomalies.
- By redundancy we do not mean that some values occur many times in the database!
- A foreign key value may be have millions of copies!
- But then, what do we mean?
- We will model logical redundancy with functional dependencies (next lecture).


## Lecture 07 : Schema refinement I



## Outline

- ER is for top-down and informal (but rigorous) design
- FDs are used for bottom-up and formal design and analysis
- update anomalies
- Reasoning about Functional Dependencies
- Heath's rule


## Functional Dependency

## Functional Dependency (FD)

Let $R(\mathbf{X})$ be a relational schema and $\mathbf{Y} \subseteq \mathbf{X}, \mathbf{Z} \subseteq \mathbf{X}$ be two attribute sets. We say $\mathbf{Y}$ functionally determines $\mathbf{Z}$, written $\mathbf{Y} \rightarrow \mathbf{Z}$, if for any two tuples $u$ and $v$ in an instance of $R(\mathbf{X})$ we have

$$
u . \mathbf{Y}=v . \mathbf{Y} \rightarrow u . \mathbf{Z}=v . \mathbf{Z} .
$$

We call $\mathbf{Y} \rightarrow \mathbf{Z}$ a functional dependency.
A functional dependency is a semantic assertion. It represents a rule that should always hold in any instance of schema $R(\mathbf{X})$.

## Example FDs

Big Table

| sid | name | college | course | part | term_name |
| :---: | :---: | :---: | :---: | :---: | :---: |
| yy88 | Yoni | New Hall | Algorithms I | IA | Easter |
| uu99 | Uri | King's | Algorithms I | IA | Easter |
| bb44 | Bin | New Hall | Databases | IB | Lent |
| bb44 | Bin | New Hall | Algorithms II | IB | Michaelmas |
| zz70 | Zip | Trinity | Databases | IB | Lent |
| zz70 | Zip | Trinity | Algorithms II | IB | Michaelmas |

- sid $\rightarrow$ name
- sid $\rightarrow$ college
- course $\rightarrow$ part
- course $\rightarrow$ term_name


## Keys, revisited

## Candidate Key

Let $R(\mathbf{X})$ be a relational schema and $\mathbf{Y} \subseteq \mathbf{X}$. $\mathbf{Y}$ is a candidate key if
(1) The FD $\mathbf{Y} \rightarrow \mathbf{X}$ holds, and
(2) for no proper subset $\mathbf{Z} \subset \mathbf{Y}$ does $\mathbf{Z} \rightarrow \mathbf{X}$ hold.

Prime and Non-prime attributes
An attribute $A$ is prime for $R(\mathbf{X})$ if it is a member of some candidate key for $R$. Otherwise, $A$ is non-prime.

Database redundancy roughly means the existence of non-key functional dependencies!

## Semantic Closure

Notation

$$
F \models \mathbf{Y} \rightarrow \mathbf{Z}
$$

means that any database instance that that satisfies every FD of $F$, must also satisfy $\mathbf{Y} \rightarrow \mathbf{Z}$.

The semantic closure of $F$, denoted $F^{+}$, is defined to be

$$
F^{+}=\{\mathbf{Y} \rightarrow \mathbf{Z} \mid \mathbf{Y} \cup \mathbf{Z} \subseteq \operatorname{atts}(F) \text { and } \wedge F \models \mathbf{Y} \rightarrow \mathbf{Z}\}
$$

The membership problem is to determine if $\mathbf{Y} \rightarrow \mathbf{Z} \in F^{+}$.

## Reasoning about Functional Dependencies

We write $F \vdash \mathbf{Y} \rightarrow \mathbf{Z}$ when $\mathbf{Y} \rightarrow \mathbf{Z}$ can be derived from $F$ via the following rules.

Armstrong's Axioms
Reflexivity If $\mathbf{Z} \subseteq \mathbf{Y}$, then $F \vdash \mathbf{Y} \rightarrow \mathbf{Z}$.
Augmentation If $F \vdash \mathbf{Y} \rightarrow \mathbf{Z}$ then $F \vdash \mathbf{Y}, \mathbf{W} \rightarrow \mathbf{Z}, \mathbf{W}$.
Transitivity If $F \vdash \mathbf{Y} \rightarrow \mathbf{Z}$ and $F \models \mathbf{Z} \rightarrow \mathbf{W}$, then $F \vdash \mathbf{Y} \rightarrow \mathbf{W}$.

## Logical Closure (of a set of attributes)

Notation

$$
\operatorname{closure}(F, \mathbf{X})=\{A \mid F \vdash \mathbf{X} \rightarrow A\}
$$

## Claim 1

If $\mathbf{Y} \rightarrow \mathbf{W} \in F$ and $\mathbf{Y} \subseteq \operatorname{closure}(F, \mathbf{X})$, then $\mathbf{W} \subseteq \operatorname{closure}(F, \mathbf{X})$.
Claim 2
$\mathbf{Y} \rightarrow \mathbf{W} \in F^{+}$if and only if $\mathbf{W} \subseteq$ closure $(F, \mathbf{Y})$.

## Soundness and Completeness

## Soundness

$$
F \vdash f \Longrightarrow f \in F^{+}
$$

Completeness

$$
f \in F^{+} \Longrightarrow F \vdash f
$$

## Proof of Completeness (soundness left as an exercise)

Show $\neg(F \vdash f) \Longrightarrow \neg(F \models f)$ :

- Suppose $\neg(F \vdash \mathbf{Y} \rightarrow \mathbf{Z})$ for $R(\mathbf{X})$.

- $\exists B \in \mathbf{Z}$, with $B \notin \mathbf{Y}^{+}$.
- Construct an instance of $R$ with just two records, $u$ and $v$, that agree on $\mathbf{Y}^{+}$but not on $\mathbf{X}-\mathbf{Y}^{+}$.
- By construction, this instance does not satisfy $\mathbf{Y} \rightarrow \mathbf{Z}$.
- But it does satisfy F! Why?
- let $\mathbf{S} \rightarrow \mathbf{T}$ be any FD in $F$, with $u .[\mathbf{S}]=v$.[S].
- So $\mathbf{S} \subseteq \mathbf{Y}+$. and so $\mathbf{T} \subseteq \mathbf{Y}+$ by claim 1,
- and so $u .[T]=v .[T]$


## Closure

By soundness and completeness

$$
\operatorname{closure}(F, \mathbf{X})=\{\boldsymbol{A} \mid F \vdash \mathbf{X} \rightarrow \boldsymbol{A}\}=\left\{\boldsymbol{A} \mid \mathbf{X} \rightarrow \boldsymbol{A} \in \boldsymbol{F}^{+}\right\}
$$

Claim 2 (from previous lecture)
$\mathbf{Y} \rightarrow \mathbf{W} \in F^{+}$if and only if $\mathbf{W} \subseteq$ closure $(F, \mathbf{Y})$.
If we had an algorithm for closure $(F, \mathbf{X})$, then we would have a (brute force!) algorithm for enumerating $F^{+}$:

## $F^{+}$

- for every subset $\mathbf{Y} \subseteq \operatorname{atts}(F)$
for every subset $\mathbf{Z} \subseteq$ closure( $F, \mathbf{Y}$ ), output $\mathbf{Y} \rightarrow \mathbf{Z}$


## Attribute Closure Algorithm

- Input : a set of FDs $F$ and a set of attributes $\mathbf{X}$.
- Output : $\mathbf{Y}=\operatorname{closure}(F, \mathbf{X})$
(1) $\mathbf{Y}:=\mathbf{X}$
(2) while there is some $\mathbf{S} \rightarrow \mathbf{T} \in F$ with $\mathbf{S} \subseteq \mathbf{Y}$ and $\mathbf{T} \nsubseteq \mathbf{Y}$, then $\mathbf{Y}:=\mathbf{Y} \cup \mathbf{T}$.


## An Example (UW1997, Exercise 3.6.1)

$R(A, B, C, D)$ with $F$ made up of the FDs

$$
\begin{aligned}
& A, B \rightarrow C \\
& C \rightarrow D \\
& D \rightarrow A
\end{aligned}
$$

What is $F^{+}$?
Brute force!
Let's just consider all possible nonempty sets $\mathbf{X}$ — there are only 15 ...

## Example (cont.)

$$
F=\{A, B \rightarrow C, C \rightarrow D, \quad D \rightarrow A\}
$$

For the single attributes we have

- $\{A\}^{+}=\{A\}$,
- $\{B\}^{+}=\{B\}$,
- $\{C\}^{+}=\{A, C, D\}$,

$$
\{C\} \stackrel{C \rightarrow D}{\Longrightarrow}\{C, D\} \xrightarrow{D \rightarrow A}\{A, C, D\}
$$

- $\{D\}^{+}=\{A, D\}$

$$
\{D\} \xrightarrow{D \rightarrow A}\{A, D\}
$$

The only new dependency we get with a single attribute on the left is $C \rightarrow A$.

## Example (cont.)

$$
F=\{A, B \rightarrow C, C \rightarrow D, \quad D \rightarrow A\}
$$

Now consider pairs of attributes.

- $\{A, B\}^{+}=\{A, B, C, D\}$,
so $A, B \rightarrow D$ is a new dependency
- $\{A, C\}^{+}=\{A, C, D\}$, so $A, C \rightarrow D$ is a new dependency
- $\{A, D\}^{+}=\{A, D\}$,
so nothing new.
- $\{B, C\}^{+}=\{A, B, C, D\}$, so $B, C \rightarrow A, D$ is a new dependency
- $\{B, D\}^{+}=\{A, B, C, D\}$,
so $B, D \rightarrow A, C$ is a new dependency
- $\{C, D\}^{+}=\{A, C, D\}$,
so $C, D \rightarrow A$ is a new dependency


## Example (cont.)

$$
F=\{A, B \rightarrow C, C \rightarrow D, D \rightarrow A\}
$$

For the triples of attributes:

- $\{A, C, D\}^{+}=\{A, C, D\}$,
- $\{A, B, D\}^{+}=\{A, B, C, D\}$, so $A, B, D \rightarrow C$ is a new dependency
- $\{A, B, C\}^{+}=\{A, B, C, D\}$, so $A, B, C \rightarrow D$ is a new dependency
- $\{B, C, D\}^{+}=\{A, B, C, D\}$, so $B, C, D \rightarrow A$ is a new dependency

And since $\{A, B, C, D\}+=\{A, B, C, D\}$, we get no new dependencies with four attributes.

## Example (cont.)

We generated 11 new FDs:

$$
\begin{array}{ccccc}
C & \rightarrow A & A, B & \rightarrow & D \\
A, C & \rightarrow D & B, C & \rightarrow & A \\
B, C & \rightarrow D & B, D & \rightarrow & A \\
B, D & \rightarrow C & C, D & \rightarrow & A \\
A, B, C & \rightarrow D & A, B, D & \rightarrow & C \\
B, C, D & \rightarrow A & & &
\end{array}
$$

Can you see the Key?
$\{A, B\},\{B, C\}$, and $\{B, D\}$ are keys.
Note: this schema is already in 3NF! Why?

## Consequences of Armstrong's Axioms

Union If $F \models \mathbf{Y} \rightarrow \mathbf{Z}$ and $F \models \mathbf{Y} \rightarrow \mathbf{W}$, then $F \models \mathbf{Y} \rightarrow \mathbf{W}, \mathbf{Z}$.
Pseudo-transitivity If $F \models \mathbf{Y} \rightarrow \mathbf{Z}$ and $F \models \mathbf{U}, \mathbf{Z} \rightarrow \mathbf{W}$, then

$$
F \models \mathbf{Y}, \mathbf{U} \rightarrow \mathbf{W}
$$

Decomposition If $F \models \mathbf{Y} \rightarrow \mathbf{Z}$ and $\mathbf{W} \subseteq \mathbf{Z}$, then $F \models \mathbf{Y} \rightarrow \mathbf{W}$.
Exercise : Prove these using Armstrong's axioms!

## Proof of the Union Rule

Suppose we have

$$
\begin{aligned}
& F \equiv \mathbf{Y} \rightarrow \mathbf{Z}, \\
& F \not \equiv \mathbf{Y} \rightarrow \mathbf{W} .
\end{aligned}
$$

By augmentation we have

$$
F \models \mathbf{Y}, \mathbf{Y} \rightarrow \mathbf{Y}, \mathbf{Z}
$$

that is,

$$
F \models \mathbf{Y} \rightarrow \mathbf{Y}, \mathbf{Z}
$$

Also using augmentation we obtain

$$
F \models \mathbf{Y}, \mathbf{Z} \rightarrow \mathbf{W}, \mathbf{Z} .
$$

Therefore, by transitivity we obtain

$$
F \models \mathbf{Y} \rightarrow \mathbf{W}, \mathbf{Z} .
$$

## Example application of functional reasoning.

## Heath's Rule

Suppose $R(A, B, C)$ is a relational schema with functional dependency $A \rightarrow B$, then

$$
R=\pi_{A, B}(R) \bowtie_{A} \pi_{A, C}(R) .
$$

## Proof of Heath's Rule

We first show that $R \subseteq \pi_{A, B}(R) \bowtie_{A} \pi_{A, C}(R)$.

- If $u=(a, b, c) \in R$, then $u_{1}=(a, b) \in \pi_{A, B}(R)$ and

$$
u_{2}=(a, c) \in \pi_{A, C}(R)
$$

- Since $\{(a, b)\} \bowtie_{A}\{(a, c)\}=\{(a, b, c)\}$ we know $u \in \pi_{A, B}(R) \bowtie_{A} \pi_{A, C}(R)$.

In the other direction we must show $R^{\prime}=\pi_{A, B}(R) \bowtie_{A} \pi_{A, C}(R) \subseteq R$.

- If $u=(a, b, c) \in R^{\prime}$, then there must exist tuples
$u_{1}=(a, b) \in \pi_{A, B}(R)$ and $u_{2}=(a, c) \in \pi_{A, C}(R)$.
- This means that there must exist a $u^{\prime}=\left(a, b^{\prime}, c\right) \in R$ such that $u_{2}=\pi_{A, C}\left(\left\{\left(a, b^{\prime}, c\right)\right\}\right)$.
- However, the functional dependency tells us that $b=b^{\prime}$, so $u=(a, b, c) \in R$.


## Closure Example

$R(A, B, C, D, E, F)$ with

$$
\begin{aligned}
& A, B \rightarrow C \\
& B, C \rightarrow D \\
& D \rightarrow E \\
& C, F \rightarrow B
\end{aligned}
$$

What is the closure of $\{A, B\}$ ?

$$
\left.\begin{array}{rl}
\{A, B\} & \xrightarrow{A, B \rightarrow C}\{ \\
& \xlongequal[B, C \rightarrow D]{\Longrightarrow}\{A, B, C\} \\
& \stackrel{D \rightarrow E}{\Longrightarrow}
\end{array}\{A, B, C, D\}, D, E\right\},
$$

So $\{A, B\}^{+}=\{A, B, C, D, E\}$ and $A, B \rightarrow C, D, E$.

## Lecture 08 : Normal Forms

## Outline

- First Normal Form (1NF)
- Second Normal Form (2NF)
- 3NF and BCNF
- Multi-valued dependencies (MVDs)
- Fourth Normal Form


## The Plan

Given a relational schema $R(\mathbf{X})$ with FDs $F$ :

- Reason about FDs

Is $F$ missing FDs that are logically implied by those in $F$ ?

- Decompose each $R(\mathbf{X})$ into smaller $R_{1}\left(\mathbf{X}_{1}\right), R_{2}\left(\mathbf{X}_{2}\right), \cdots R_{k}\left(\mathbf{X}_{k}\right)$, where each $R_{i}\left(\mathbf{X}_{i}\right)$ is in the desired Normal Form.

Are some decompositions better than others?

## Desired properties of any decomposition

Lossless-join decomposition
A decomposition of schema $R(\mathbf{X})$ to $S(\mathbf{Y} \cup \mathbf{Z})$ and $T(\mathbf{Y} \cup(\mathbf{X}-\mathbf{Z}))$ is a lossless-join decomposition if for every database instances we have $R=S \bowtie T$.

## Dependency preserving decomposition

A decomposition of schema $R(\mathbf{X})$ to $S(\mathbf{Y} \cup \mathbf{Z})$ and $T(\mathbf{Y} \cup(\mathbf{X}-\mathbf{Z}))$ is dependency preserving, if enforcing FDs on $S$ and $T$ individually has the same effect as enforcing all FDs on $S \bowtie T$.

We will see that it is not always possible to achieve both of these goals.

## First Normal Form (1NF)

We will assume every schema is in 1NF.

## 1NF

A schema $R\left(A_{1}: S_{1}, A_{2}: S_{2}, \cdots, A_{n}: S_{n}\right)$ is in First Normal Form (1NF) if the domains $S_{1}$ are elementary - their values are atomic.
name
Timothy George Griffin

| first_name | middle_name | last_name |
| :---: | :---: | :---: |
| Timothy | George | Griffin |

## Second Normal Form (2NF)

## Second Normal Form (2NF)

A relational schema $R$ is in 2NF if for every functional dependency $\mathbf{X} \rightarrow A$ either

- $A \in \mathbf{X}$, or
- $\mathbf{X}$ is a superkey for $R$, or
- $A$ is a member of some key, or
- $\mathbf{X}$ is not a proper subset of any key.


## 3NF and BCNF

## Third Normal Form (3NF)

A relational schema $R$ is in 3NF if for every functional dependency $\mathbf{X} \rightarrow \boldsymbol{A}$ either

- $A \in \mathbf{X}$, or
- $\mathbf{X}$ is a superkey for $R$, or
- $A$ is a member of some key.


## Boyce-Codd Normal Form (BCNF)

A relational schema $R$ is in BCNF if for every functional dependency $\mathbf{X} \rightarrow \boldsymbol{A}$ either

- $A \in \mathbf{X}$, or
- $\mathbf{X}$ is a superkey for $R$.

Is something missing?

## Another look at Heath's Rule

## Given $R(\mathbf{Z}, \mathbf{W}, \mathbf{Y})$ with FDs $F$

If $\mathbf{Z} \rightarrow \mathbf{W} \in F^{+}$, the

$$
R=\pi_{\mathbf{Z}, \mathbf{W}}(R) \bowtie \pi_{\mathbf{Z}, \mathbf{Y}}(R)
$$

What about an implication in the other direction? That is, suppose we have

$$
R=\pi_{\mathbf{Z}, \mathbf{w}}(R) \bowtie \pi_{\mathbf{Z}, \mathbf{Y}}(R)
$$

Q Can we conclude anything about FDs on $R$ ? In particular, is it true that $\mathbf{Z} \rightarrow \mathbf{W}$ holds?
A No!

## We just need one counter example ...

| $R$ |  | $=$ | $\pi_{A, B}(R)$ | $\bowtie$ | $\pi_{A, C}(R)$ |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $A$ | $B$ | $C$ |  | $A$ | $B$ | $A$ |
| $a$ | $b_{1}$ | $c_{1}$ |  | $a$ | $b_{1}$ |  |
| $a$ | $b_{2}$ | $c_{2}$ |  | $a$ | $b_{2}$ |  |
| $a$ | $b_{1}$ | $c_{2}$ |  |  |  | $c_{1}$ |
| $a$ | $b_{2}$ | $c_{1}$ |  |  |  | $c_{2}$ |
|  |  |  |  |  |  |  |

Clearly $A \rightarrow B$ is not an FD of $R$.

## A concrete example

| course_name | lecturer | text |
| :---: | :---: | :---: |
| Databases | Tim | Ullman and Widom |
| Databases | Fatima | Date |
| Databases | Tim | Date |
| Databases | Fatima | Ullman and Widom |

Assuming that texts and lecturers are assigned to courses independently, then a better representation would in two tables:

| course_name | lecturer |
| :---: | :---: |
| Databases | Tim |
| Databases | Fatima |


| course_name | text |
| :---: | :---: |
| Databases | Ullman and Widom |
| Databases | Date |

## Time for a definition! MVDs

## Multivalued Dependencies (MVDs)

Let $R(\mathbf{Z}, \mathbf{W}, \mathbf{Y})$ be a relational schema. A multivalued dependency, denoted $\mathbf{Z} \rightarrow \mathbf{W}$, holds if whenever $t$ and $u$ are two records that agree on the attributes of $\mathbf{Z}$, then there must be some tuple $v$ such that
(1) $v$ agrees with both $t$ and $u$ on the attributes of $\mathbf{Z}$,
(2) $v$ agrees with $t$ on the attributes of $\mathbf{W}$,
( $v$ agrees with $u$ on the attributes of $\mathbf{Y}$.

## A few observations

## Note 1

Every functional dependency is multivalued dependency,

$$
(\mathbf{Z} \rightarrow \mathbf{W}) \Longrightarrow(\mathbf{Z} \rightarrow \mathbf{W}) .
$$

To see this, just let $v=u$ in the above definition.
Note 2
Let $R(\mathbf{Z}, \mathbf{W}, \mathbf{Y})$ be a relational schema, then

$$
(\mathbf{Z} \rightarrow \mathbf{W}) \Longleftrightarrow(\mathbf{Z} \rightarrow \mathbf{Y}),
$$

by symmetry of the definition.

## MVDs and lossless-join decompositions

## Fun Fun Fact

Let $R(\mathbf{Z}, \mathbf{W}, \mathbf{Y})$ be a relational schema. The decomposition $R_{1}(\mathbf{Z}, \mathbf{W})$, $R_{2}(\mathbf{Z}, \mathbf{Y})$ is a lossless-join decomposition of $R$ if and only if the MVD $\mathbf{Z} \rightarrow \mathbf{W}$ holds.

## Proof of Fun Fun Fact

## Proof of $(\mathbf{Z} \rightarrow \mathbf{W}) \Longrightarrow R=\pi_{\mathbf{z}, \mathbf{w}}(R) \bowtie \pi_{\mathbf{z}, \mathbf{Y}}(R)$

- Suppose $\mathbf{Z} \rightarrow \mathbf{W}$.
- We know (from proof of Heath's rule) that $R \subseteq \pi_{\mathbf{Z}, \mathbf{w}}(R) \bowtie \pi_{\mathbf{Z}, \mathbf{Y}}(R)$. So we only need to show $\pi_{\mathbf{z}, \mathbf{w}}(R) \bowtie \pi_{\mathbf{z}, \mathbf{y}}(R) \subseteq R$.
- Suppose $r \in \pi_{\mathbf{Z}, \mathbf{w}}(R) \bowtie \pi_{\mathbf{Z}, \mathbf{y}}(R)$.
- So there must be a $t \in R$ and $u \in R$ with $\{r\}=\pi_{\mathbf{Z}, \mathbf{w}}(\{t\}) \bowtie \pi_{\mathbf{z}, \mathbf{r}}(\{u\})$.
- In other words, there must be a $t \in R$ and $u \in R$ with $t . \mathbf{Z}=u . \mathbf{Z}$.
- So the MVD tells us that then there must be some tuple $v \in R$ such that
(1) $v$ agrees with both $t$ and $u$ on the attributes of $\mathbf{Z}$,
(3) $v$ agrees with $t$ on the attributes of $\mathbf{W}$,
(3) $v$ agrees with $u$ on the attributes of $\mathbf{Y}$.
- This $v$ must be the same as $r$, so $r \in R$.


## Proof of Fun Fun Fact (cont.)

## Proof of $R=\pi_{\mathbf{Z}, \mathbf{w}}(R) \bowtie \pi_{\mathbf{z}, \mathbf{y}}(R) \Longrightarrow(\mathbf{Z} \rightarrow \mathbf{W})$

- Suppose $R=\pi_{\mathbf{z}, \mathbf{w}}(R) \bowtie \pi_{\mathbf{z}, \mathbf{y}}(R)$.
- Let $t$ and $u$ be any records in $R$ with $t . Z=u . Z$.
- Let $v$ be defined by $\{v\}=\pi_{\mathbf{Z}, \mathbf{W}}(\{t\}) \bowtie \pi_{\mathbf{Z}, \mathbf{Y}}(\{u\})$ (and we know $v \in R$ by the assumption).
- Note that by construction we have
(1) $v . \mathbf{Z}=t . \mathbf{Z}=u . \mathbf{Z}$,
(2) $v \cdot \mathbf{W}=t \cdot \mathbf{W}$,
(3) $v . \mathbf{Y}=u . \mathbf{Y}$.
- Therefore, $\mathbf{Z} \rightarrow \mathbf{W}$ holds.


## Fourth Normal Form

## Trivial MVD

The MVD $\mathbf{Z} \rightarrow \mathbf{W}$ is trivial for relational schema $R(\mathbf{Z}, \mathbf{W}, \mathbf{Y})$ if
(1) $\mathbf{Z} \cap \mathbf{W} \neq\{ \}$, or
(2) $\mathbf{Y}=\{ \}$.

## 4NF

A relational schema $R(\mathbf{Z}, \mathbf{W}, \mathbf{Y})$ is in 4NF if for every MVD $\mathbf{Z} \rightarrow \mathbf{W}$ either

- $\mathbf{Z} \rightarrow \mathbf{W}$ is a trivial MVD, or
- $\mathbf{Z}$ is a superkey for $R$.

Note : 4NF $\subset B C N F \subset 3 N F \subset 2 N F$

## Summary

We always want the lossless-join property. What are our options?

|  | $3 N F$ | BCNF | 4NF |
| ---: | :---: | :---: | :---: |
| Preserves FDs | Yes | Maybe | Maybe |
| Preserves MVDs | Maybe | Maybe | Maybe |
| Eliminates FD-redundancy | Maybe | Yes | Yes |
| Eliminates MVD-redundancy | No | No | Yes |

## Inclusions

Clearly $\mathrm{BCNF} \subseteq 3 N F \subseteq 2 N F$. These are proper inclusions:
In 2NF, but not 3NF
$R(A, B, C)$, with $F=\{A \rightarrow B, B \rightarrow C\}$.
In 3NF, but not BCNF
$R(A, B, C)$, with $F=\{A, B \rightarrow C, C \rightarrow B\}$.

- This is in 3NF since $A B$ and $A C$ are keys, so there are no non-prime attributes
- But not in BCNF since $C$ is not a key and we have $C \rightarrow B$.


## Lecture 09 : Guest Lecture

## Grant Allen (Google)

Technology Program Manager for Google's Site Reliability Engineering team.

## Lectire 10 : Schema refinement III and advanced design

## Outline

- General Decomposition Method (GDM)
- The lossless-join condition is guaranteed by GDM
- The GDM does not always preserve dependencies!
- FDs vs ER models?
- Weak entities
- Using FDs and MVDs to refine ER models
- Another look at ternary relationships


## General Decomposition Method (GDM)

## GDM

(1) Understand your FDs $F$ (compute $F^{+}$),
(2) find $R(\mathbf{X})=R(\mathbf{Z}, \mathbf{W}, \mathbf{Y})$ (sets $\mathbf{Z}, \mathbf{W}$ and $\mathbf{Y}$ are disjoint) with FD
$\mathbf{Z} \rightarrow \mathbf{W} \in F^{+}$violating a condition of desired $N F$,
(0) split $R$ into two tables $R_{1}(\mathbf{Z}, \mathbf{W})$ and $R_{2}(\mathbf{Z}, \mathbf{Y})$
(9) wash, rinse, repeat

## Reminder

For $\mathbf{Z} \rightarrow \mathbf{W}$, if we assume $\mathbf{Z} \cap \mathbf{W}=\{ \}$, then the conditions are
© $\mathbf{Z}$ is a superkey for $R$ (2NF, 3NF, BCNF)
(2) $\mathbf{W}$ is a subset of some key (2NF, 3NF)
( $\mathbf{Z}$ is not a proper subset of any key (2NF)

## The lossless-join condition is guaranteed by GDM

- This method will produce a lossless-join decomposition because of (repeated applications of) Heath's Rule!
- That is, each time we replace an $S$ by $S_{1}$ and $S_{2}$, we will always be able to recover $S$ as $S_{1} \bowtie S_{2}$.
- Note that in GDM step 3, the FD $\mathbf{Z} \rightarrow \mathbf{W}$ may represent a key constraint for $R_{1}$.

But does the method always terminate? Please think about this ....

## General Decomposition Method Revisited

## GDM++

(1) Understand your FDs and MVDs $F$ (compute $F^{+}$),
(2) find $R(\mathbf{X})=R(\mathbf{Z}, \mathbf{W}, \mathbf{Y})$ (sets $\mathbf{Z}, \mathbf{W}$ and $\mathbf{Y}$ are disjoint) with either FD $\mathbf{Z} \rightarrow \mathbf{W} \in F^{+}$or MVD $\mathbf{Z} \rightarrow \mathbf{W} \in F^{+}$violating a condition of desired NF,
(3) split $R$ into two tables $R_{1}(\mathbf{Z}, \mathbf{W})$ and $R_{2}(\mathbf{Z}, \mathbf{Y})$
(9) wash, rinse, repeat

## Return to Example — Decompose to BCNF

$R(A, B, C, D)$

$$
F=\{A, B \rightarrow C, C \rightarrow D, \quad D \rightarrow A\}
$$

## Which FDs in $F^{+}$violate BCNF?

$$
\begin{array}{ccc}
C & \rightarrow A \\
C & \rightarrow & D \\
D & \rightarrow & A \\
A, C & \rightarrow & D \\
C, D & \rightarrow & A
\end{array}
$$

## Return to Example — Decompose to BCNF

## Decompose $R(A, B, C, D)$ to BCNF

Use $C \rightarrow D$ to obtain

- $R_{1}(C, D)$. This is in BCNF. Done.
- $R_{2}(A, B, C)$ This is not in BCNF. Why? $A, B$ and $B, C$ are the only keys, and $C \rightarrow A$ is a FD for $R_{1}$. So use $C \rightarrow A$ to obtain
$R_{2.1}(A, C)$. This is in BCNF. Done.
$R_{2.2}(B, C)$. This is in BCNF. Done.
Exercise : Try starting with any of the other BCNF violations and see where you end up.

The GDM does not always preserve dependencies!
$R(A, B, C, D, E)$

$$
\begin{aligned}
A, B & \rightarrow C \\
D, E & \rightarrow C \\
B & \rightarrow D
\end{aligned}
$$

- $\{A, B\}^{+}=\{A, B, C, D\}$,
- so $A, B \rightarrow C, D$,
- and $\{A, B, E\}$ is a key.
- $\{B, E\}^{+}=\{B, C, D, E\}$,
- so $B, E \rightarrow C, D$,
- and $\{A, B, E\}$ is a key (again)

Let's try for a BCNF decomposition ...

## Decomposition 1

Decompose $R(A, B, C, D, E)$ using $A, B \rightarrow C, D$ :

- $R_{1}(A, B, C, D)$. Decompose this using $B \rightarrow D$ :
$R_{1.1}(B, D)$. Done.
$R_{1.2}(A, B, C)$. Done.
- $R_{2}(A, B, E)$. Done.

But in this decomposition, how will we enforce this dependency?

$$
D, E \rightarrow C
$$

## Decomposition 2

Decompose $R(A, B, C, D, E)$ using $B, E \rightarrow C, D$ :

- $R_{3}(B, C, D, E)$. Decompose this using $D, E \rightarrow C$
$R_{3.1}(C, D, E)$. Done.
$R_{3.2}(B, D, E)$. Decompose this using $B \rightarrow D$ :

$$
\begin{aligned}
& R_{3.2 .1}(B, D) \text {. Done. } \\
& R_{3.2 .2}(B, E) \text {. Done. }
\end{aligned}
$$

- $R_{4}(A, B, E)$. Done.

But in this decomposition, how will we enforce this dependency?
$A, B \rightarrow C$

## Summary

- It is always possible to obtain BCNF that has the lossless-join property (using GDM)

But the result may not preserve all dependencies.

- It is always possible to obtain 3NF that preserves dependencies and has the lossless-join property.

Using methods based on "minimal covers" (for example, see EN2000).

## Recall : a small change of scope ...

... changed this entity

into two entities and a relationship :


But is there something odd about the MovieRelease entity?

## MovieRelease represents a Weak entity set



## Definition

- Weak entity sets do not have a primary key.
- The existence of a weak entity depends on an identifying entity set through an identifying relationship.
- The primary key of the identifying entity together with the weak entities discriminators (dashed underline in diagram) identify each weak entity element.


## Can FDs help us think about implementation?

$$
\begin{aligned}
& \begin{array}{l}
R(I, T, D, C) \\
I \\
\rightarrow \\
I
\end{array} \\
& =\text { MovielD } \\
& T=\text { Title } \\
& D=\text { Date } \\
& C=\text { Country }
\end{aligned}
$$

Turn the decomposition crank to obtain

$$
\begin{gathered}
R_{1}(I, T) \quad R_{2}(I, D, C) \\
\pi_{l}\left(R_{2}\right) \subseteq \pi_{l}\left(R_{1}\right)
\end{gathered}
$$

## Movie Ratings example

Scope $=$ UK

| Titte | Year | Rating |
| :--- | :--- | :--- |
| Austin Powers: International Man of Mystery | 1997 | 15 |
| Austin Powers: The Spy Who Shagged Me | 1999 | 12 |
| Dude, Where's My Car? | 2000 | 15 |

## Scope = Earth

| Title | Year | Country | Rating |
| :--- | :--- | :--- | :--- |
| Austin Powers: International Man of Mystery | 1997 | UK | 15 |
| Austin Powers: International Man of Mystery | 1997 | Malaysia | 18 SX |
| Austin Powers: International Man of Mystery | 1997 | Portugal | M/12 |
| Austin Powers: International Man of Mystery | 1997 | USA | PG-13 |
| Austin Powers: The Spy Who Shagged Me | 1999 | UK | 12 |
| Austin Powers: The Spy Who Shagged Me | 1999 | Portugal | M/12 |
| Austin Powers: The Spy Who Shagged Me | 1999 | USA | PG-13 |
| Dude, Where's My Car? | 2000 | UK | 15 |
| Dude, Where's My Car? | 2000 | USA | PG-13 |
| Dude, Where's My Car? | 2000 | Malaysia | 18PL |

## Example of attribute migrating to strong entity set

From single-country scope,

to multi-country scope:


Note that relation Rated has an attribute!

## Beware of FFDs = Faux Functional Dependencies

## (US ratings)

| Title | Year | Rating | RatingReason |
| :--- | :--- | :--- | :--- |
| Stoned | 2005 | R | drug use |
| Wasted | 2006 | R | drug use |
| High Life | 2009 | R | drug use |
| Poppies: Odyssey of an opium eater | 2009 | R | drug use |

But

## Title $\rightarrow$ \{Rating, RatingReason $\}$

is not a functional dependency.
This is a mildly amusing illustration of a real and pervasive problem deriving a functional dependency after the examination of a limited set of data (or after talking to only a few domain experts).

## Oh, but the real world is such a bother!

```
from IMDb raw data file certificates.list
2 ~ F a s t ~ 2 ~ F u r i o u s ~ ( 2 0 0 3 ) ~ S w i t z e r l a n d : 1 4 ~ ( c a n t o n ~ o f ~ V a u d ) ~
2 ~ F a s t ~ 2 ~ F u r i o u s ~ ( 2 0 0 3 ) ~ S w i t z e r l a n d : 1 6 ~ ( c a n t o n ~ o f ~ Z u r i c h )
28 Days (2000) Canada:13+ (Quebec)
2 8 \text { Days (2000) Canada:14 (Nova Scotia)}
2 8 ~ D a y s ~ ( 2 0 0 0 ) ~ C a n a d a : 1 4 A ~ ( A l b e r t a ) ~
2 8 \text { Days (2000) Canada:AA (Ontario)}
2 8 ~ D a y s ~ ( 2 0 0 0 ) ~ C a n a d a : P A ~ ( M a n i t o b a )
28 Days (2000) Canada:PG (British Columbia)
```


## Ternary or multiple binary relationships?



## Ternary or multiple binary relationships?



## Look again at ER Demo Diagram²

 How might this be refined using FDs or MVDs?
${ }^{2}$ By Pável Calado,
http://www.texample.net/tikz/examples/entity-relationship-diagram

## Lecture 11 : On-line Analytical Processing (OLAP)

## Outline

- Limits of SQL aggregation
- OLAP : Online Analytic Processing
- Data cubes
- Star schema


## Limits of SQL aggregation

| sale | prodld | storeld | amt |
| :--- | :---: | :---: | :---: |
|  | p 1 | c 1 | 12 |
|  | p 2 | c 1 | 11 |
|  | p 1 | c 3 | 50 |
|  | p 2 | c 2 | 8 | L

- Flat tables are great for processing, but hard for people to read and understand.
- Pivot tables and cross tabulations (spreadsheet terminology) are very useful for presenting data in ways that people can understand.
- SQL does not handle pivot tables and cross tabulations well.


## OLAP vs. OLTP

- OLTP : Online Transaction Processing (traditional databases)

Data is normalized for the sake of updates.

- OLAP : Online Analytic Processing

These are (almost) read-only databases.
Data is de-normalized for the sake of queries!
Multi-dimensional data cube emerging as common data model.
This can be seen as a generalization of SQL's group by

## OLAP Databases : Data Models and Design

## The big question

Is the relational model and its associated query language (SQL) well suited for OLAP databases?

- Aggregation (sums, averages, totals, ...) are very common in OLAP queries
- Problem : SQL aggregation quickly runs out of steam.
- Solution : Data Cube and associated operations (spreadsheets on steroids)
- Relational design is obsessed with normalization
- Problem : Need to organize data well since all analysis queries cannot be anticipated in advance.
- Solution : Multi-dimensional fact tables, with hierarchy in dimensions, star-schema design.


## A very influential paper [G+1997]

Data Mining and Knowledge Discovery 1, 29-53 (1997)
(C) 1997 Kluwer Academic Publishers. Manufactured in The Netherlands.

## Data Cube: A Relational Aggregation Operator Generalizing Group-By, Cross-Tab, and Sub-Totals*

JIM GRAY<br>SURAJIT CHAUDHURI<br>Gray@Microsoft.com<br>ADAM BOSWORTH<br>SurajitC@Microsoft.com<br>ANDREW LAYMAN<br>AdamB@Microsoft.com<br>DON REICHART<br>AndrewL@Microsoft.com<br>MURALI VENKATRAO<br>DonRei@Microsoft.com<br>MuraliV@Microsoft.com<br>Microsoft Research, Advanced Technology Division, Microsoft Corporation, One Microsoft Way, Redmond, WA 98052

FRANK PELLOW
HAMID PIRAHESH
Pellow@ vnet.IBM.com
Pirahesh@Almaden.IBM.com
IBM Research, 500 Harry Road, San Jose, CA 95120

## From aggregates to data cubes



## The Data Cube



## Dimensions:

Product,
Location,
Time

- Data modeled as an n-dimensional (hyper-) cube
- Each dimension is associated with a hierarchy
- Each "point" records facts
- Aggregation and cross-tabulation possible along all dimensions


## Hierarchy for Location Dimension



## Cube Operations



## The Star Schema as a design tool



## Lecture 12 : Beyond ACID/Relational framework

- XML or JSON as a data exchange language
- Not all applications require ACID
- "NoSQL" Movement
- Rise of Web and cluster-based computing
- CAP = Consistency, Availability, and Partition tolerance
- The CAP theorem (pick any two!)
- Eventual consistency
- Relationships vs. Aggregates
- Aggregate data models?
- Key-value store
- Can a database really be "schemaless"?


## The End

| HI, THIS IS | OH, DEAR - DID HE | DID YOU REALLY | WELL, WEVE LOST THIS |
| :---: | :---: | :---: | :---: |
| YOUR SON'S SCHOOL. | BREAK SOMETHING? | NAME YOUR SON | YEAR'S STUDENT RECORDS. |
| WE'RE HAVING SOME | IN A WAY-) | Robert'); DROP <br> TABIE Students:- ? | I HOPE YOU'RE HAPPY. |
|  | $k_{0}$ |  | AND I |
|  | $4_{n}$ | $\text { OH. YES. } \angle I T L E$ | W YOUVE LEARNED |
| $7$ | $11$ | BOBBY TABLES, WE CALI HIM | 1 <br> TO SANITIZE YOUR DATABASE INPUT5. |
| M V | TVN |  | 1 DATABASE NPUTS. |

(http://xkcd.com/327)

