Prolog Lecture 3

- Symbolic evaluation of arithmetic
- Controlling backtracking: cut
- Negation

Symbolic Evaluation

Let's write some Prolog rules to evaluate symbolic arithmetic expressions such as plus(1,mult(4,5))

```
eval(plus(A,B),C) :- eval(A,A1),
                     eval(B,B1),
                     C is A1 + B1.

eval(mult(A,B),C) :- eval(A,A1),
                     eval(B,B1),
                     C is A1 * B1.

eval(A,A).
```

Evaluation starts with the first matching clause

Q: How does Prolog evaluate:

```
eval(plus(1,mult(4,5)),Ans)
```

A: Step 1, see if the first matching clause is true

```
eval(plus(A,B),C) :- eval(A,A1),
                     eval(B,B1),
                     C is A1 + B1.
```

In this case the variable bindings are:

- A = 1, B = mult(4,5) and C = Ans

Next it looks at the body of the rule

The body of the clause with head `eval(plus(A,B),C)` and variable bindings

A = 1, B = mult(4,5) and C = Ans is:

```
eval(1,A1),
eval(mult(4,5),B1),
Ans is A1 + B1.
```

This is a conjunction: all parts must be true for the clause to be true

The body is checked term by term from left to right

First part of the body: `eval(1,A1)`

Try: eval(plus(A,B),C) :- eval(A,A1), eval(B,B1), C is A1 + B1.

Fail because 1 does not unify with plus(A,B)

Try: eval(mult(A,B),C) :- eval(A,A1), eval(B,B1), C is A1 * B1.

Fail because 1 does not unify with mult(A,B)

Try: eval(A,A).

Succeed: eval(1,A1) is true if A1 = 1

The body is checked term by term from left to right

From previous slide, `eval(1,A1)` was provable, with the side-effect of binding: `A1=1`.

So continuing through the body (note `A1` is now bound):

```
eval(1,1),
eval(mult(4,5),B1),
Ans is 1 + B1.
```

The body is checked term by term from left to right

So `eval(mult(4,5),B1)` will bind `B1=20`:

```
eval(1,1),
eval(mult(4,5),20),
Ans is 1 + 20.
```

The body is checked term by term from left to right

Ans will be bound to 21, after “is” does its job.

```
eval(1,1),
eval(mult(4,5),20),
21 is 1 + 20.
```

eval(plus(1,mult(4,5)),Ans)

eval(plus(A,B),C) :-
    eval(A,A1), eval(B,B1), C is A1 + B1.

eval(plus(1,mult(4,5)),Ans) :-
    eval(1,T1), eval(mult(4,5),T2), Ans is T1 + T2.

eval(A,A).

```
eval(plus(A,B),C):-
     eval(A,A1),
     eval(B,B1),
     C is A1 + B1.

eval(mult(A,B),C):-
     eval(A,A1),
     eval(B,B1),
     C is A1 * B1.

eval(A,A).
```

First eval/3 clause

Third eval/3 clause

Choice Point

Be sure that you understand why the second eval/3 clause does not appear in this choice point

eval(plus(1,mult(4,5)),Ans)

eval(plus(A,B),C) :-
    eval(A,A1), eval(B,B1), C is A1 + B1.

eval(A,A).

eval(plus(1,mult(4,5)),Ans) :-
    eval(1,T1), eval(mult(4,5),T2), Ans is T1 + T2.

eval(A,A).
eval(1,1).

```
eval(plus(A,B),C):-
    eval(A,A1),
    eval(B,B1),
    C is A1 + B1.

eval(mult(A,B),C):-
    eval(A,A1),
    eval(B,B1),
    C is A1 * B1.

eval(A,A).
```

eval(plus(1,mult(4,5)),Ans)

eval(plus(A,B),C) :-
    eval(A,A1), eval(B,B1), C is A1 + B1.

eval(A,A).

eval(plus(1,mult(4,5)),Ans) :-
    eval(1,T1), eval(mult(4,5),T2), Ans is T1 + T2.

eval(A,A).
eval(1,1).

eval(mult(A,B),C) :-
    eval(A,A1),eval(B,B1), C is A1 * B1.

eval(A,A).

eval(mult(4,5),T2) :-
    eval(4,T3),eval(5,T4), T2 is T3 * T4.

```
eval(plus(A,B),C):-
    eval(A,A1),
    eval(B,B1),
    C is A1 + B1.

eval(mult(A,B),C):-
    eval(A,A1),
    eval(B,B1),
    C is A1 * B1.

eval(A,A).
```

eval(plus(1,mult(4,5)),Ans)

eval(plus(A,B),C) :-
    eval(A,A1), eval(B,B1), C is A1 + B1.

eval(A,A).

eval(plus(1,mult(4,5)),Ans) :-
    eval(1,T1), eval(mult(4,5),T2), Ans is T1 + T2.

eval(A,A).
eval(1,1).

eval(mult(A,B),C) :-
    eval(A,A1),eval(B,B1), C is A1 * B1.

eval(A,A).

eval(mult(4,5),T2) :-
    eval(4,T3),eval(5,T4), T2 is T3 * T4.

eval(A,A).
eval(4,4).

```
eval(plus(A,B),C):-
    eval(A,A1),
    eval(B,B1),
    C is A1 + B1.

eval(mult(A,B),C):-
    eval(A,A1),
    eval(B,B1),
    C is A1 * B1.

eval(A,A).
```

eval(plus(1,mult(4,5)),Ans)

eval(plus(A,B),C) :-
    eval(A,A1), eval(B,B1), C is A1 + B1.

eval(A,A).

eval(plus(1,mult(4,5)),Ans) :-
    eval(1,T1), eval(mult(4,5),T2), Ans is T1 + T2.

eval(A,A).
eval(1,1).

eval(mult(A,B),C) :-
    eval(A,A1),eval(B,B1), C is A1 * B1.

eval(A,A).

eval(mult(4,5),T2) :-
    eval(4,T3),eval(5,T4), T2 is T3 * T4.

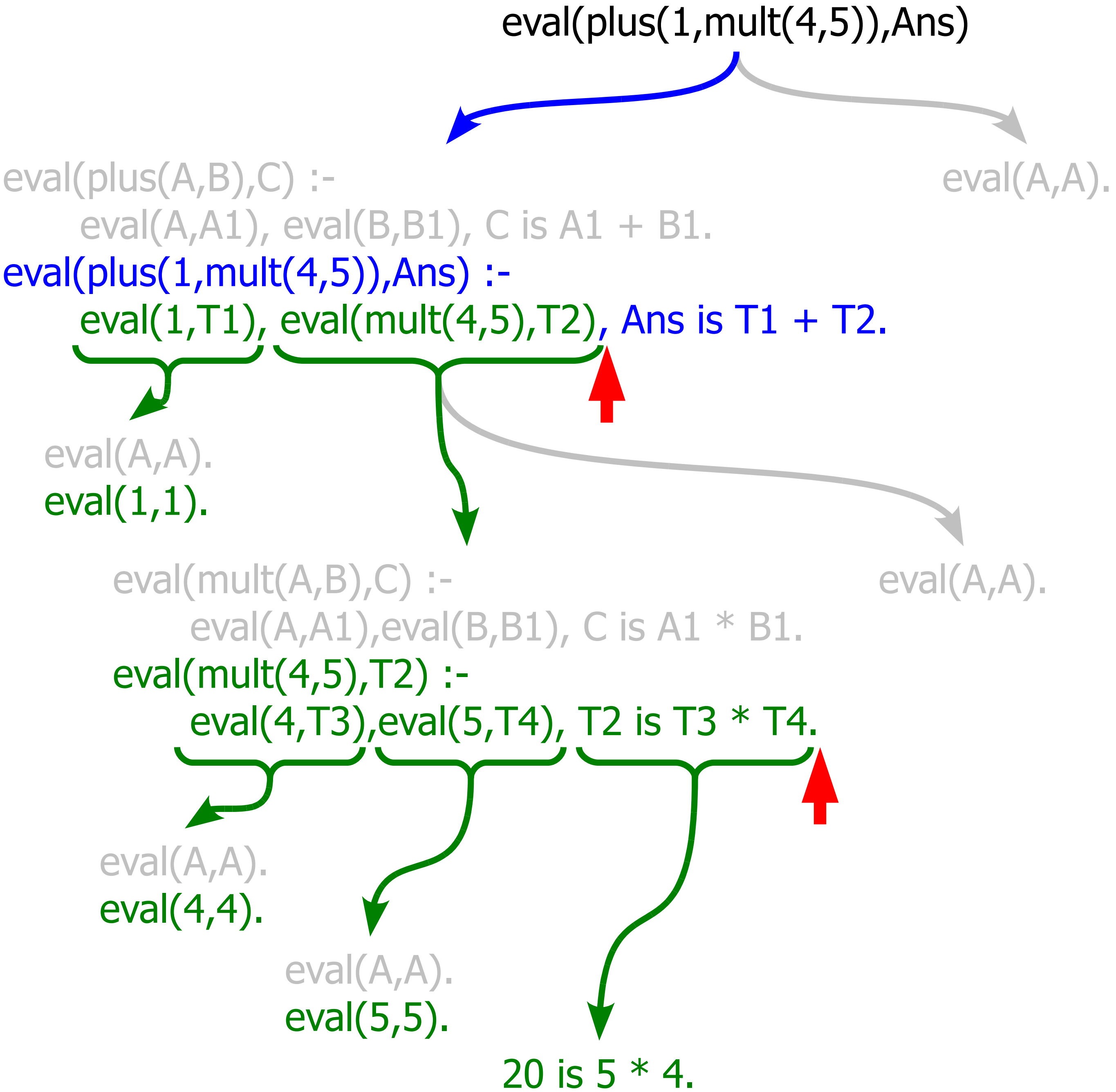
eval(A,A).
eval(4,4).

eval(A,A).
eval(5,5).

```
eval(plus(A,B),C):-
    eval(A,A1),
    eval(B,B1),
    C is A1 + B1.

eval(mult(A,B),C):-
    eval(A,A1),
    eval(B,B1),
    C is A1 * B1.

eval(A,A).
```




```
eval(plus(A,B),C):-
    eval(A,A1),
    eval(B,B1),
    C is A1 + B1.

eval(mult(A,B),C):-
    eval(A,A1),
    eval(B,B1),
    C is A1 * B1.

eval(A,A).
```

eval(plus(1,mult(4,5)),Ans)

eval(plus(A,B),C) :-
    eval(A,A1), eval(B,B1), C is A1 + B1.

eval(A,A).

eval(plus(1,mult(4,5)),Ans) :-
    eval(1,T1), eval(mult(4,5),T2), Ans is T1 + T2.

eval(A,A).
eval(1,1).

eval(mult(A,B),C) :-
    eval(A,A1),eval(B,B1), C is A1 * B1.

eval(A,A).

eval(mult(4,5),T2) :-
    eval(4,T3),eval(5,T4), T2 is T3 * T4.

eval(A,A).
eval(4,4).

eval(A,A).
eval(5,5).

20 is 5 * 4.

21 is 1 + 20.

```
eval(plus(A,B),C):-
    eval(A,A1),
    eval(B,B1),
    C is A1 + B1.

eval(mult(A,B),C):-
    eval(A,A1),
    eval(B,B1),
    C is A1 * B1.

eval(A,A).
```

What happens if we use backtracking and ask Prolog for the next solution?

eval(plus(1,mult(4,5)),Ans)

eval(plus(A,B),C) :-
eval(A,A1), eval(B,B1), C is A1 + B1.

eval(A,A).

eval(plus(1,mult(4,5)),Ans) :-
eval(1,T1), eval(mult(4,5),T2), Ans is T1 + T2.

eval(A,A).
eval(1,1).

eval(mult(A,B),C) :-
eval(A,A1),eval(B,B1), C is A1 * B1.

eval(A,A).

eval(mult(4,5),T2) :-
eval(4,T3),eval(5,T4), T2 is T3 * T4.

eval(A,A).
eval(4,4).

eval(A,A).
eval(5,5).

20 is 5 * 4.

~~21 is 1 + 20.~~

```
eval(plus(A,B),C):-
    eval(A,A1),
    eval(B,B1),
    C is A1 + B1.

eval(mult(A,B),C):-
    eval(A,A1),
    eval(B,B1),
    C is A1 * B1.

eval(A,A).
```

eval(plus(1,mult(4,5)),Ans)

eval(plus(A,B),C) :-
    eval(A,A1), eval(B,B1), C is A1 + B1.

eval(A,A).

eval(plus(1,mult(4,5)),Ans) :-
    eval(1,T1), eval(mult(4,5),T2), Ans is T1 + T2.

eval(A,A).
eval(1,1).

eval(mult(A,B),C) :-
    eval(A,A1),eval(B,B1), C is A1 * B1.

eval(A,A).

eval(mult(4,5),T2) :-
    eval(4,T3),eval(5,T4), T2 is T3 * T4.

eval(A,A).
eval(4,4).

eval(A,A).
eval(5,5).

20 is 5 * 4.

```
eval(plus(A,B),C):-
    eval(A,A1),
    eval(B,B1),
    C is A1 + B1.

eval(mult(A,B),C):-
    eval(A,A1),
    eval(B,B1),
    C is A1 * B1.

eval(A,A).
```

eval(plus(1,mult(4,5)),Ans)

eval(plus(A,B),C) :-
eval(A,A1), eval(B,B1), C is A1 + B1.

eval(A,A).

eval(plus(1,mult(4,5)),Ans) :-
eval(1,T1), eval(mult(4,5),T2), Ans is T1 + T2.

eval(A,A).
eval(1,1).

eval(mult(A,B),C) :-
eval(A,A1),eval(B,B1), C is A1 * B1.

eval(A,A).

eval(mult(4,5),T2) :-
eval(4,T3),eval(5,T4), T2 is T3 * T4.

eval(A,A).
eval(4,4).

eval(A,A).
eval(5,5).

```
eval(plus(A,B),C):-
    eval(A,A1),
    eval(B,B1),
    C is A1 + B1.

eval(mult(A,B),C):-
    eval(A,A1),
    eval(B,B1),
    C is A1 * B1.

eval(A,A).
```

eval(plus(1,mult(4,5)),Ans)

eval(plus(A,B),C) :-
    eval(A,A1), eval(B,B1), C is A1 + B1.

eval(A,A).

eval(plus(1,mult(4,5)),Ans) :-
    eval(1,T1), eval(mult(4,5),T2), Ans is T1 + T2.

eval(A,A).
eval(1,1).

eval(mult(A,B),C) :-
    eval(A,A1),eval(B,B1), C is A1 * B1.

eval(A,A).

eval(mult(4,5),T2) :-
    eval(4,T3),eval(5,T4), T2 is T3 * T4.

eval(A,A).
eval(4,4).

```
eval(plus(A,B),C):-
    eval(A,A1),
    eval(B,B1),
    C is A1 + B1.

eval(mult(A,B),C):-
    eval(A,A1),
    eval(B,B1),
    C is A1 * B1.

eval(A,A).
```

eval(plus(1,mult(4,5)),Ans)

eval(plus(A,B),C) :-
eval(A,A1), eval(B,B1), C is A1 + B1.

eval(A,A).

eval(plus(1,mult(4,5)),Ans) :-
eval(1,T1), eval(mult(4,5),T2), Ans is T1 + T2.

eval(A,A).
eval(1,1).

eval(mult(A,B),C) :-
eval(A,A1),eval(B,B1), C is A1 * B1.

eval(A,A).

eval(mult(4,5),T2) :-
eval(4,T3),eval(5,T4), T2 is T3 * T4.

```
eval(plus(A,B),C):-
    eval(A,A1),
    eval(B,B1),
    C is A1 + B1.

eval(mult(A,B),C):-
    eval(A,A1),
    eval(B,B1),
    C is A1 * B1.

eval(A,A).
```

eval(plus(1,mult(4,5)),Ans)

eval(plus(A,B),C) :-
    eval(A,A1), eval(B,B1), C is A1 + B1.

eval(plus(1,mult(4,5)),Ans) :-
    eval(1,T1), eval(mult(4,5),T2), Ans is T1 + T2.

eval(A,A).

eval(A,A).
eval(1,1).

eval(mult(A,B),C) :-
    eval(A,A1),eval(B,B1), C is A1 * B1.
eval(mult(4,5),T2) :-
    eval(4,T3),eval(5,T4), T2 is T3 * T4.

eval(A,A).
eval(mult(4,5),mult(4,5)).

```
eval(plus(A,B),C):-
    eval(A,A1),
    eval(B,B1),
    C is A1 + B1.

eval(mult(A,B),C):-
    eval(A,A1),
    eval(B,B1),
    C is A1 * B1.

eval(A,A).
```

eval(plus(1,mult(4,5)),Ans)

eval(plus(A,B),C) :-
    eval(A,A1), eval(B,B1), C is A1 + B1.

eval(A,A).

eval(plus(1,mult(4,5)),Ans) :-
    eval(1,T1), eval(mult(4,5),T2), Ans is T1 + T2.

eval(A,A).
eval(1,1).

eval(mult(A,B),C) :-
    eval(A,A1),eval(B,B1), C is A1 * B1.
eval(mult(4,5),T2) :-
    eval(4,T3),eval(5,T4), T2 is T3 * T4.

eval(A,A).
eval(mult(4,5),mult(4,5)).

```
eval(plus(A,B),C):-
    eval(A,A1),
    eval(B,B1),
    C is A1 + B1.

eval(mult(A,B),C):-
    eval(A,A1),
    eval(B,B1),
    C is A1 * B1.

eval(A,A).
```

Ouch... “is” can't handle the mult(4,5) term!

Ans is 1 + mult(4,5)

(a) Eliminate spurious solutions by making your clauses orthogonal

Need to eliminate the (unwanted) choice point

A way to do this: make sure only one clause matches: **eval(A,A)** becomes **eval(gnd(A),A)**.

```
eval(plus(A,B),C) :- eval(A,A1),
                     eval(B,B1),
                     C is A1 + B1.
eval(mult(A,B),C) :- eval(A,A1),
                     eval(B,B1),
                     C is A1 * B1.
eval(gnd(A),A).
```

(b) Eliminate spurious solutions by explicitly discarding choice points

Alternatively we can tell Prolog to commit to its first choice and discard the choice point (p114)

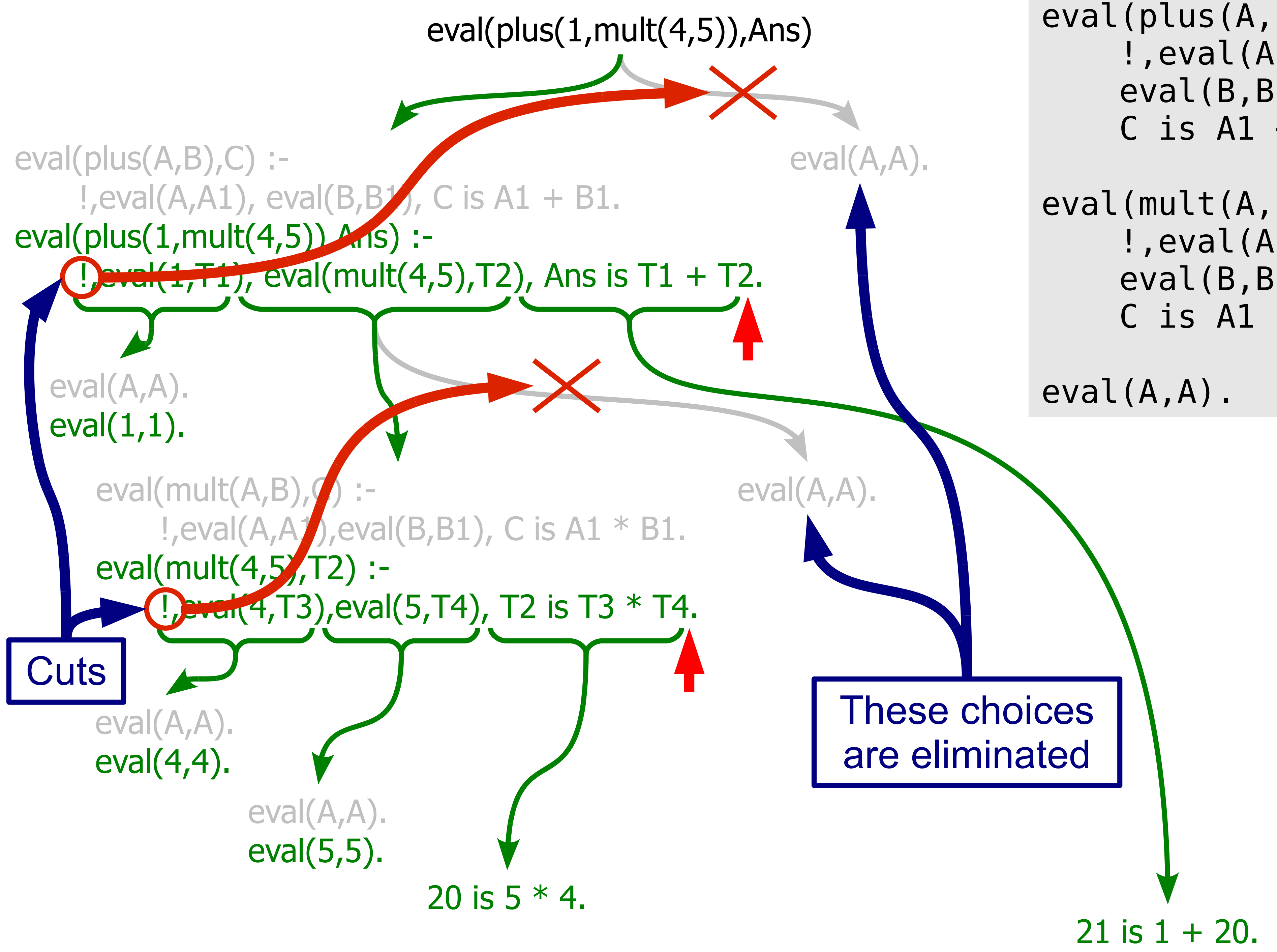
We do this with the cut operator. Written: !

```
eval(plus(A,B),C) :- !,eval(A,A1),
                       eval(B,B1),
                       C is A1 + B1.
eval(mult(A,B),C) :- !,eval(A,A1),
                       eval(B,B1),
                       C is A1 * B1.
eval(A,A).
```

```
eval(plus(A,B),C):-
    !,eval(A,A1),
    eval(B,B1),
    C is A1 + B1.

eval(mult(A,B),C):-
    !,eval(A,A1),
    eval(B,B1),
    C is A1 * B1.

eval(A,A).
```



Cutting out choice

Whenever Prolog evaluates a cut it discards all choice points back to the parent clause

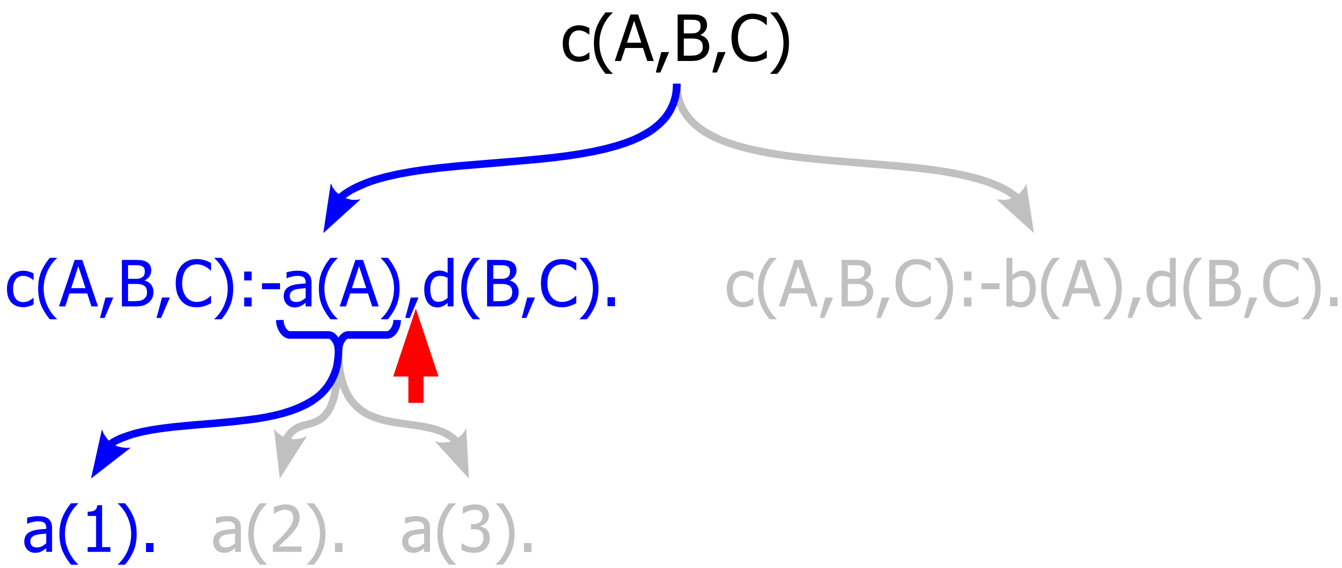
An example:

```
a(1).          c(A,B,C) :- a(A),d(B,C).
a(2).          c(A,B,C) :- b(A),d(B,C).
a(3).          d(B,C) :- a(B),!,a(C).
b(apple).      d(B,_) :- b(B).
b(orange).
```

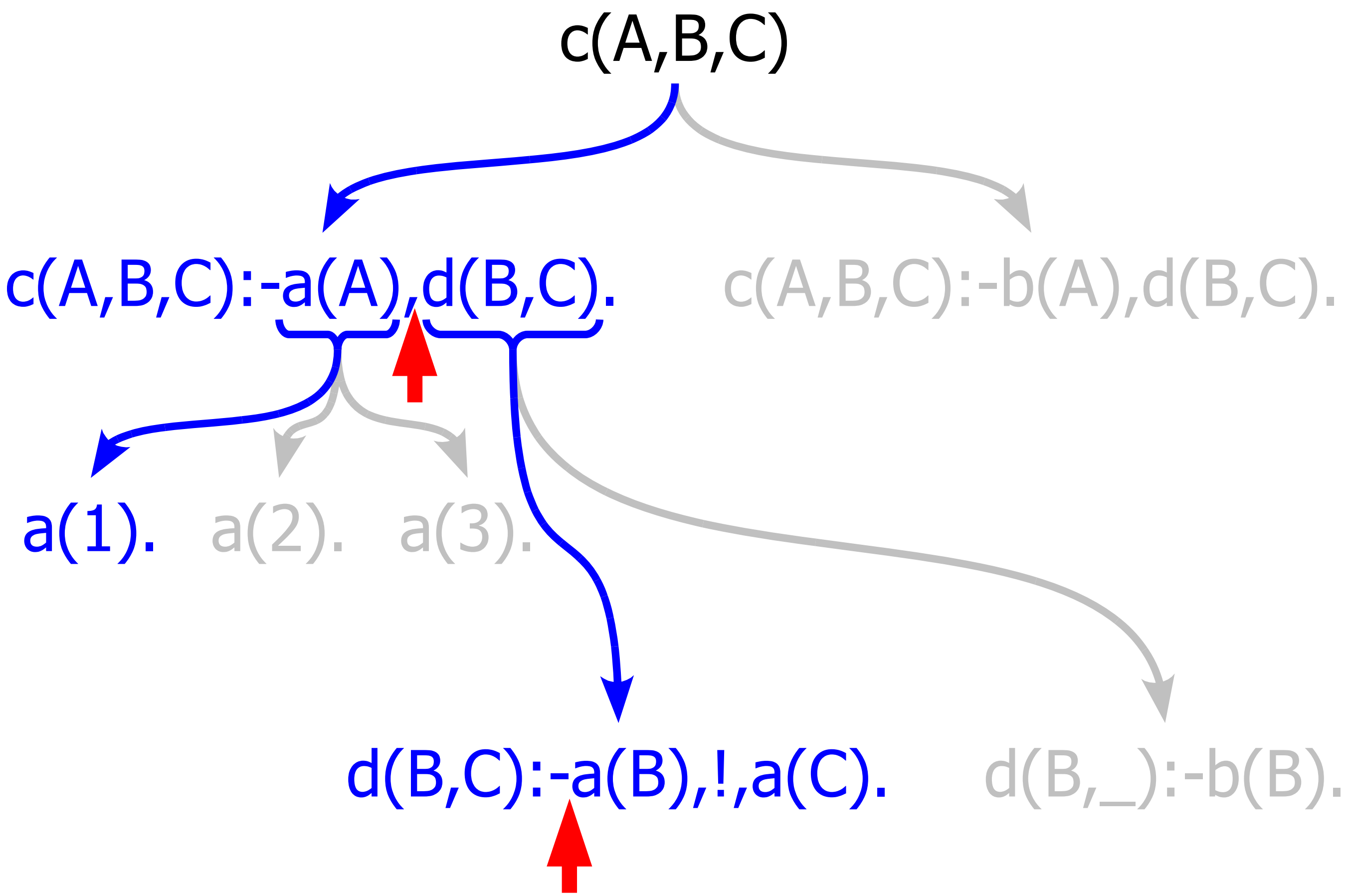
c(A,B,C)

c(A,B,C):-a(A),d(B,C).   c(A,B,C):-b(A),d(B,C).

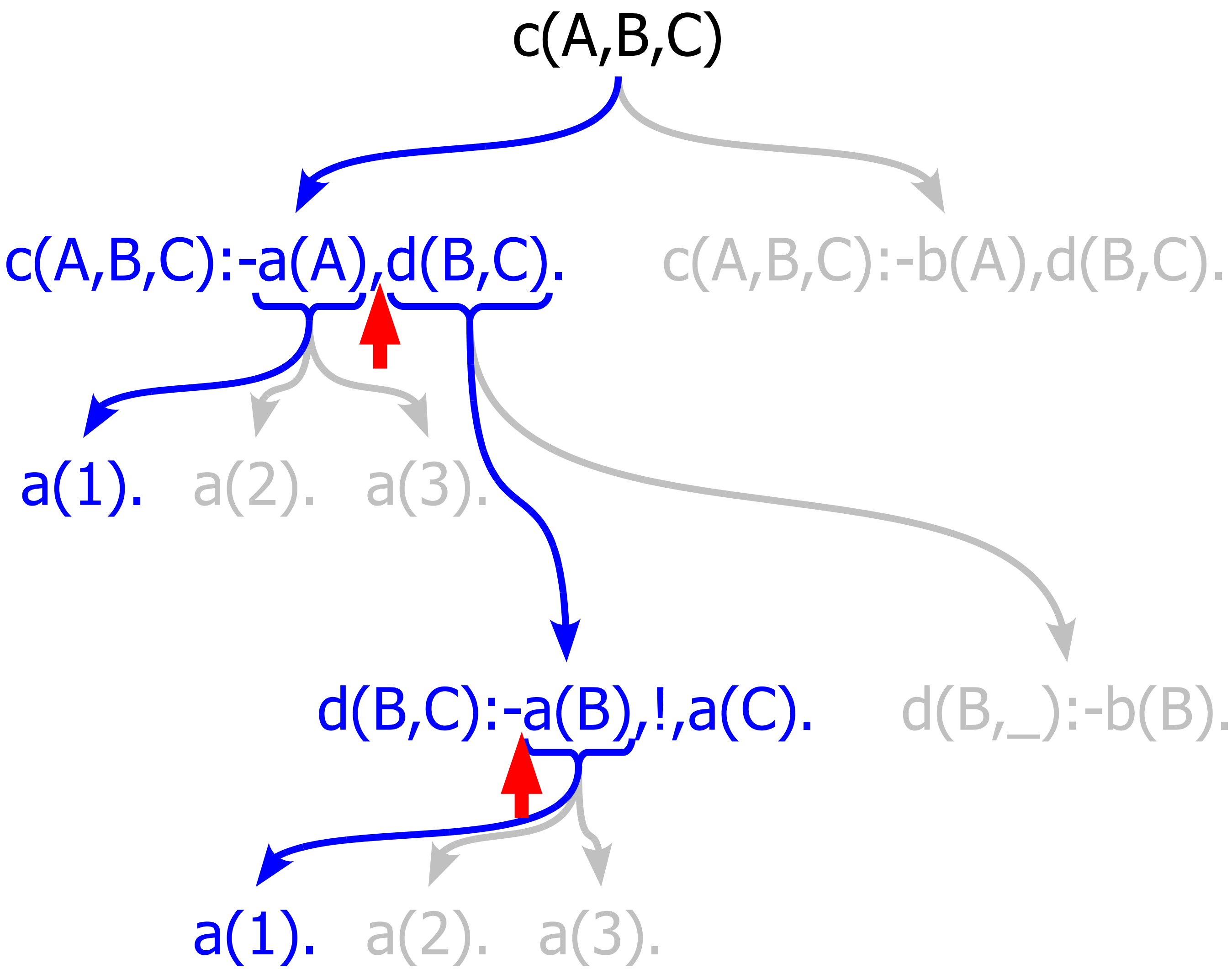
```
a(1).
a(2).
a(3).
b(apple).
b(orange).
c(A,B,C):-a(A),d(B,C).
c(A,B,C):-b(A),d(B,C).
d(B,C):-a(B),!,a(C).
d(B,_):-b(B).
```



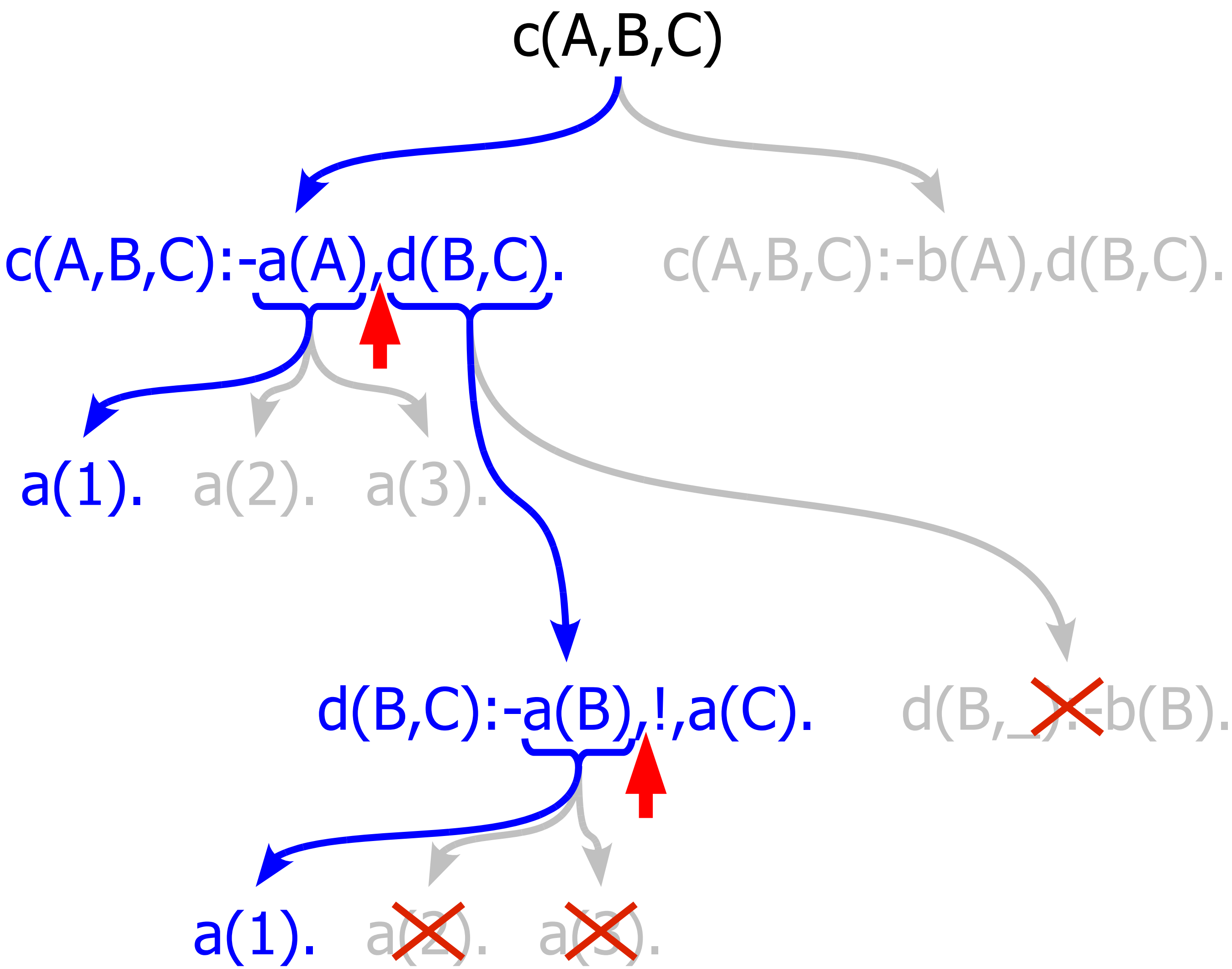

```
a(1).
a(2).
a(3).
b(apple).
b(orange).
c(A,B,C):-a(A),d(B,C).
c(A,B,C):-b(A),d(B,C).
d(B,C):-a(B),!,a(C).
d(B,_):-b(B).
```



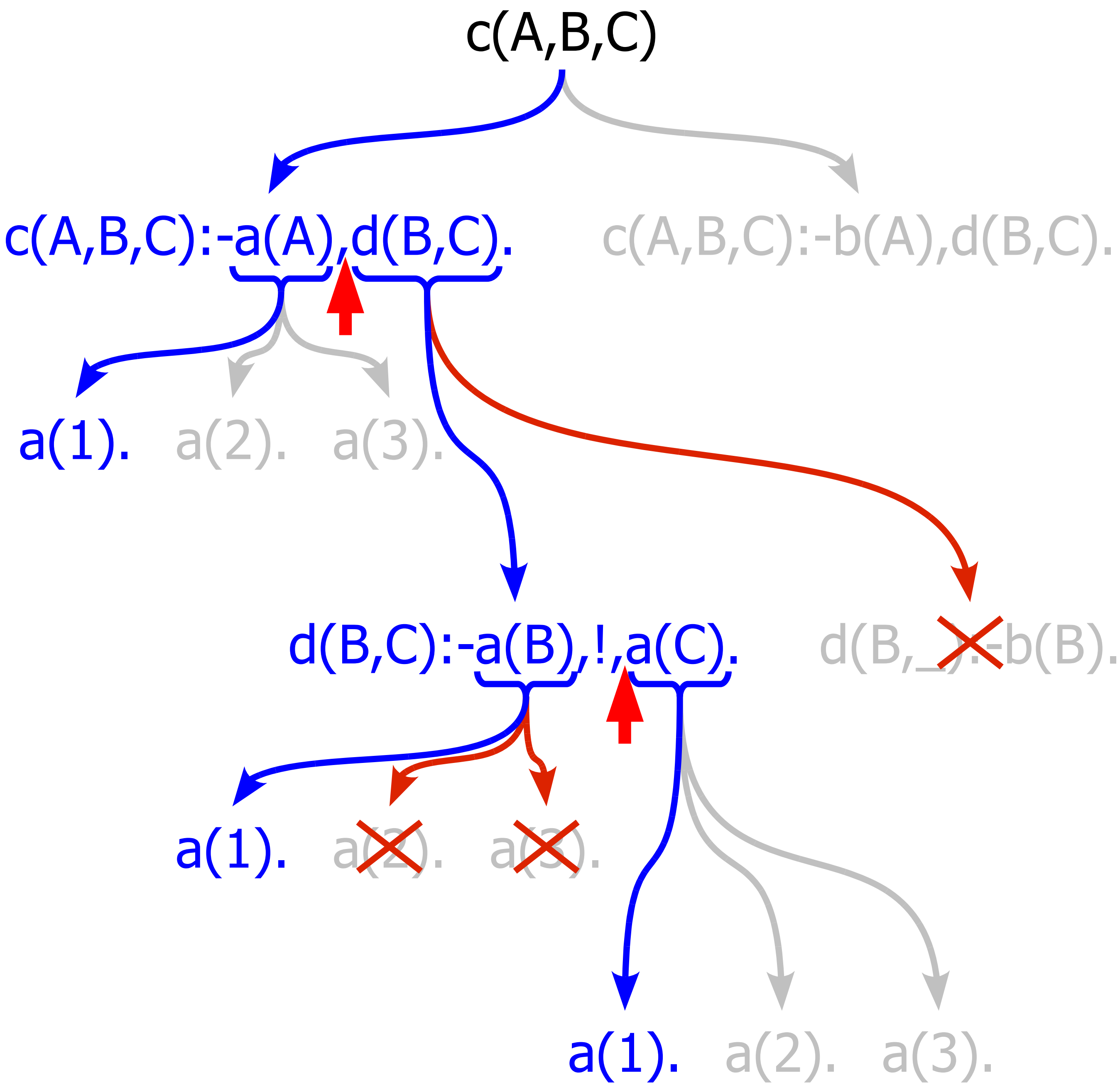

```
a(1).
a(2).
a(3).
b(apple).
b(orange).
c(A,B,C):-a(A),d(B,C).
c(A,B,C):-b(A),d(B,C).
d(B,C):-a(B),!,a(C).
d(B,_):-b(B).
```



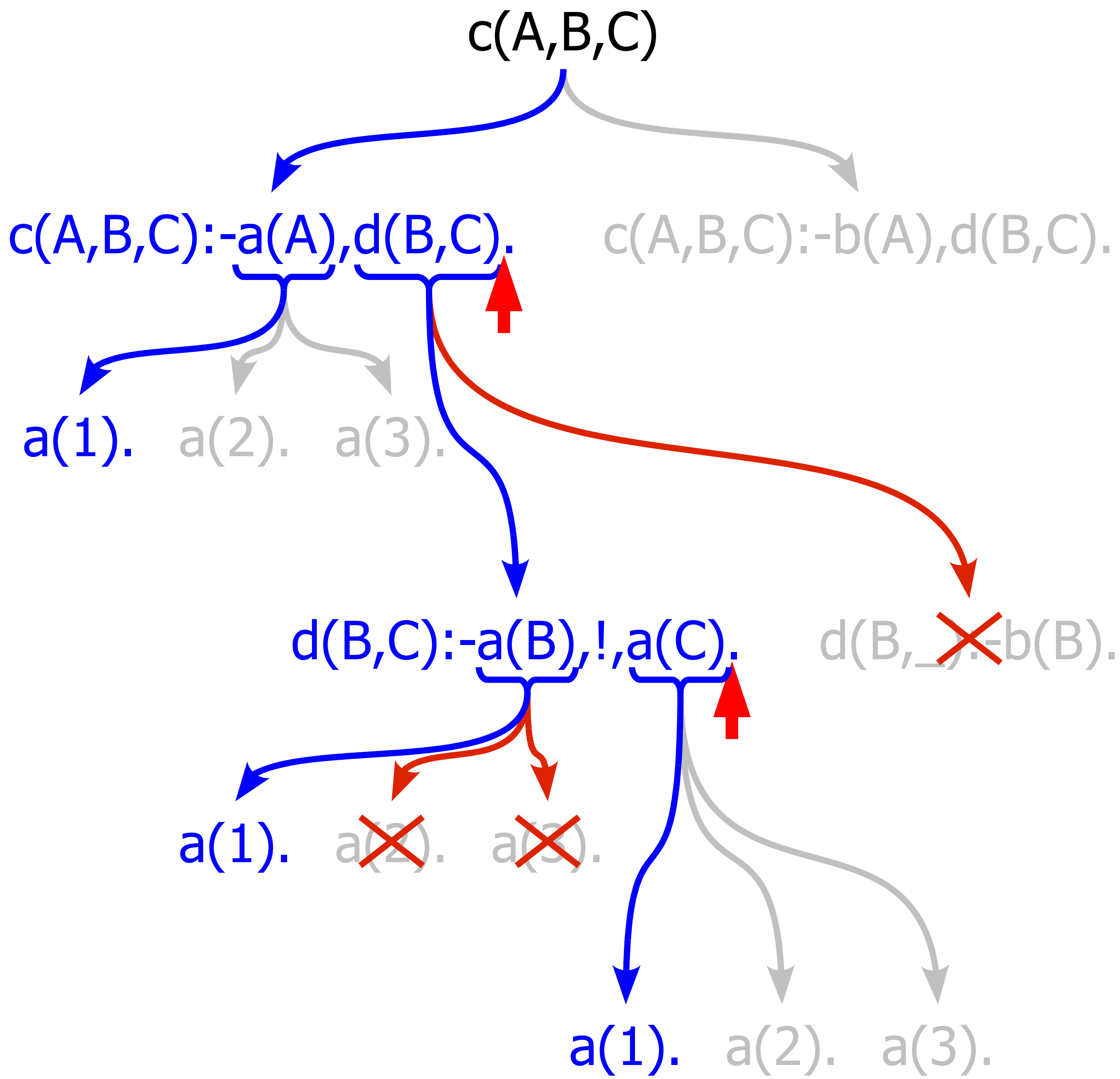

```
a(1).
a(2).
a(3).
b(apple).
b(orange).
c(A,B,C):-a(A),d(B,C).
c(A,B,C):-b(A),d(B,C).
d(B,C):-a(B),!,a(C).
d(B,_):-b(B).
```



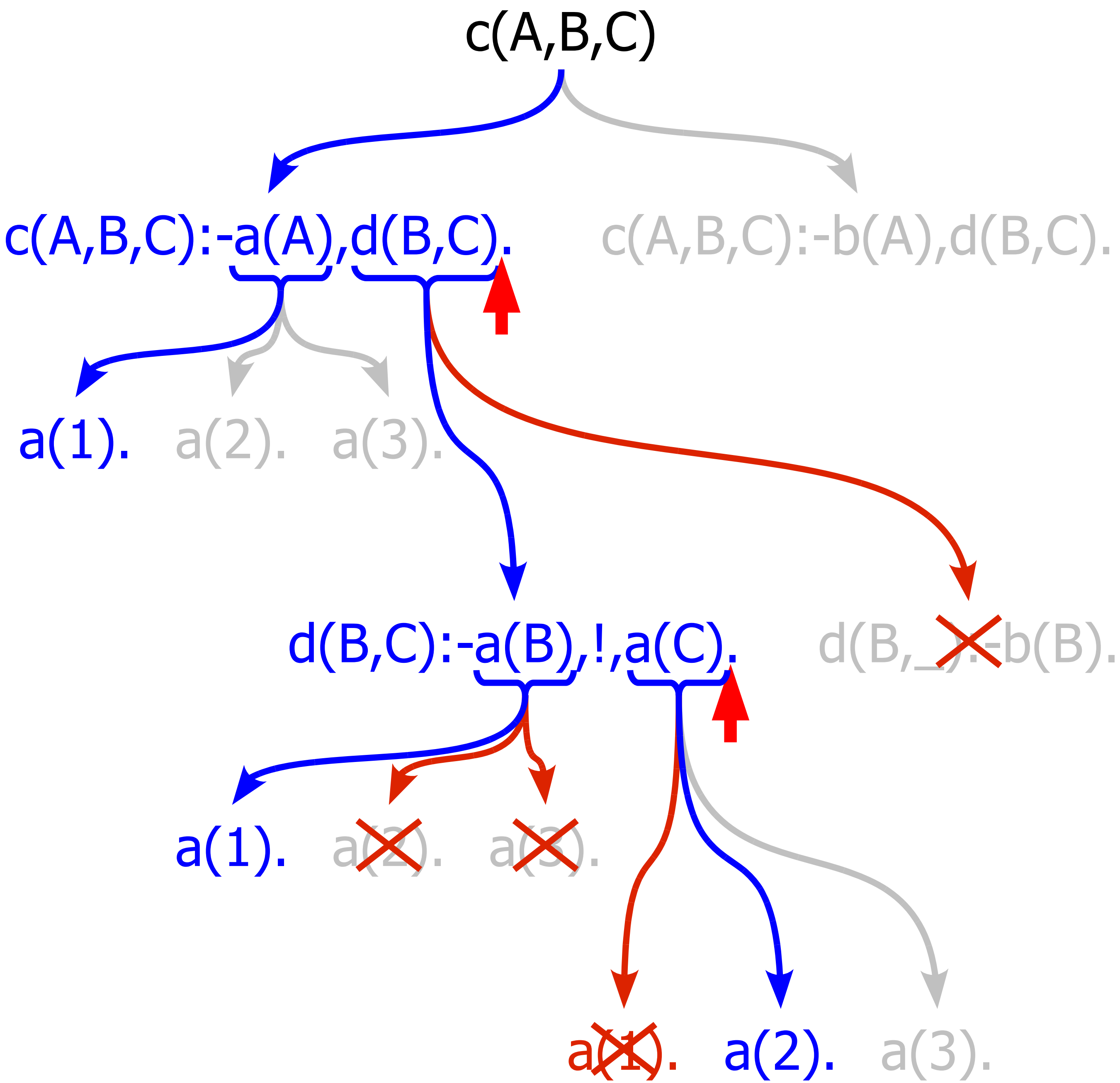

```
a(1).
a(2).
a(3).
b(apple).
b(orange).
c(A,B,C):-a(A),d(B,C).
c(A,B,C):-b(A),d(B,C).
d(B,C):-a(B),!,a(C).
d(B,_):-b(B).
```




```
a(1).
a(2).
a(3).
b(apple).
b(orange).
c(A,B,C):-a(A),d(B,C).
c(A,B,C):-b(A),d(B,C).
d(B,C):-a(B),!,a(C).
d(B,_):-b(B).
```

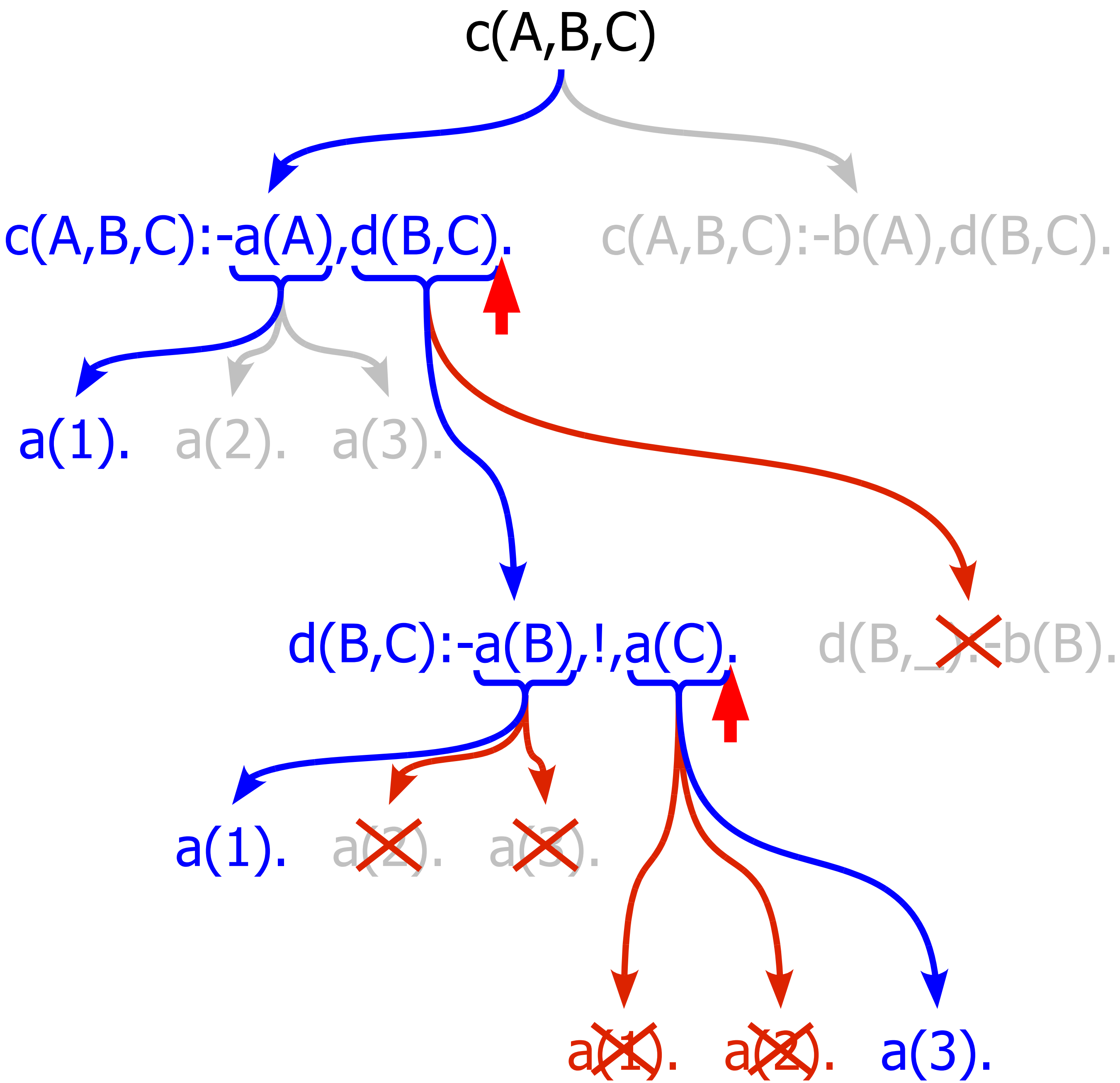



```
a(1).
a(2).
a(3).
b(apple).
b(orange).
c(A,B,C):-a(A),d(B,C).
c(A,B,C):-b(A),d(B,C).
d(B,C):-a(B),!,a(C).
d(B,_):-b(B).
```



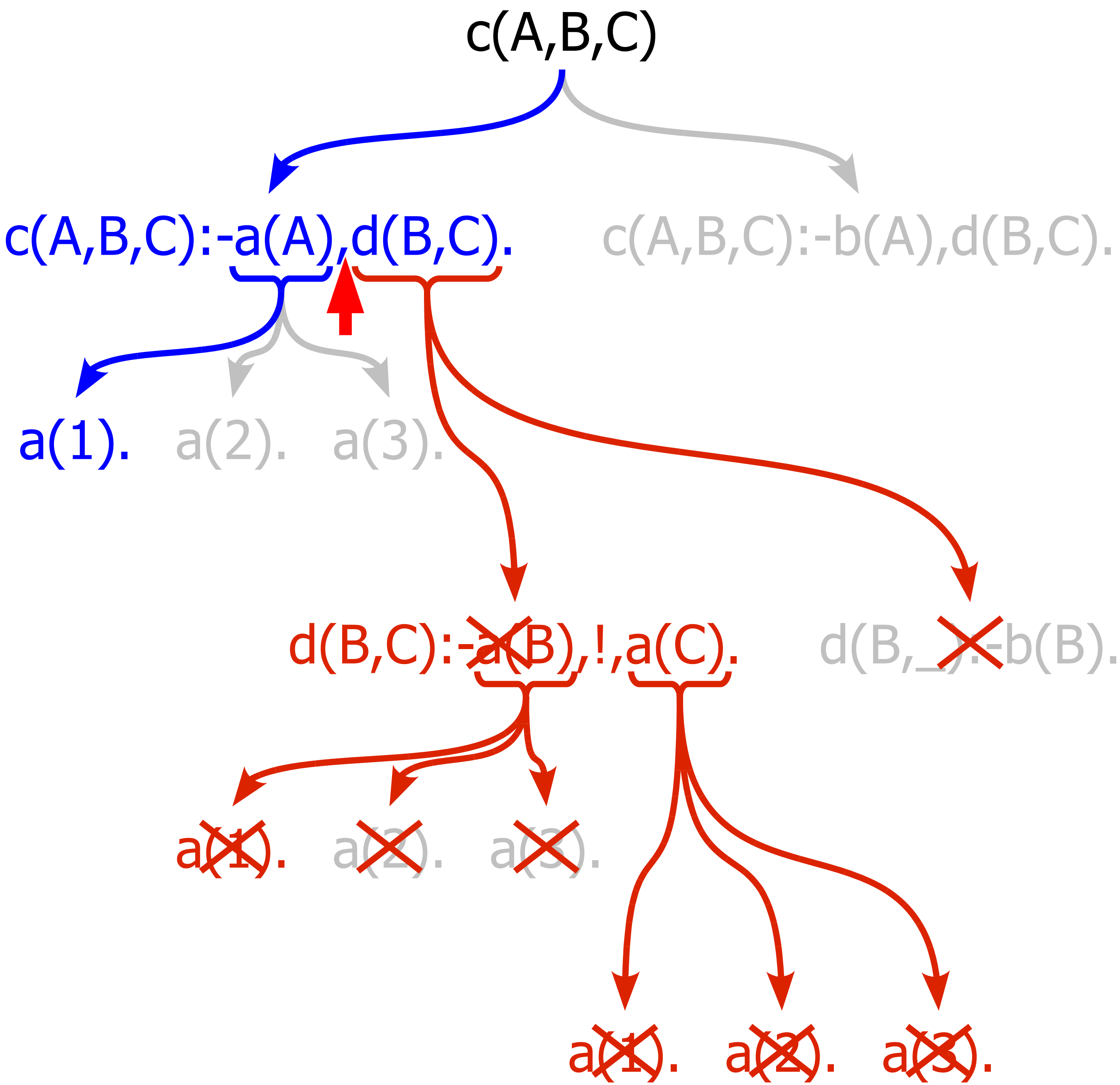

```
a(1).
a(2).
a(3).
b(apple).
b(orange).
c(A,B,C):-a(A),d(B,C).
c(A,B,C):-b(A),d(B,C).
d(B,C):-a(B),!,a(C).
d(B,_):-b(B).
```

Backtrack once



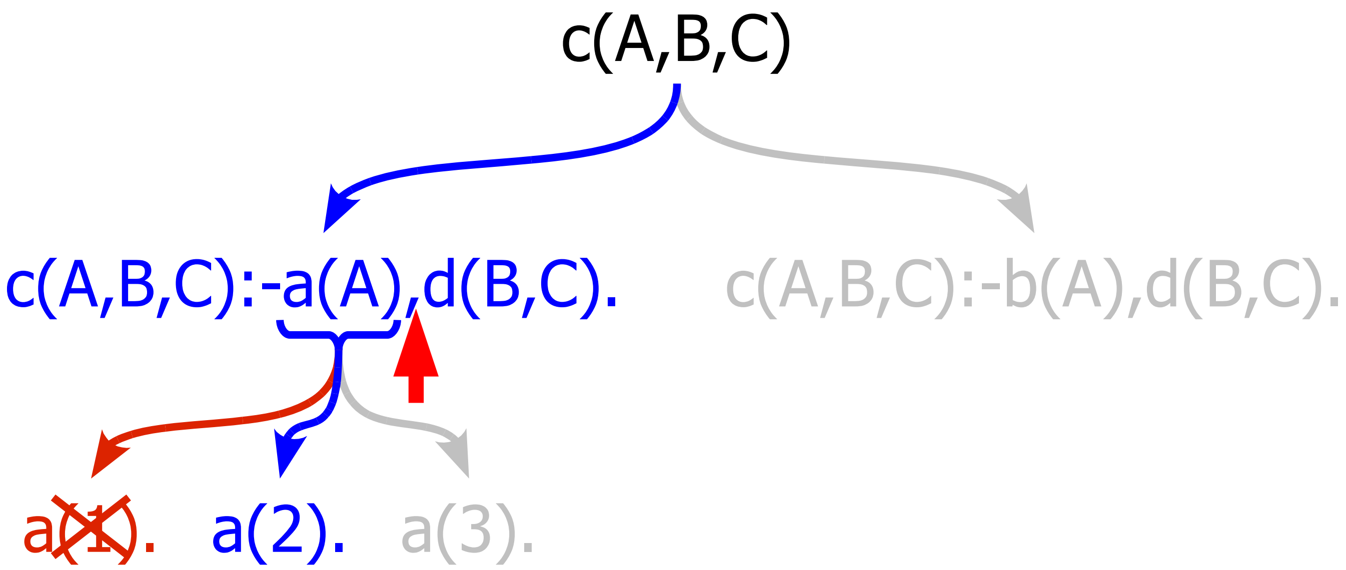

```
a(1).
a(2).
a(3).
b(apple).
b(orange).
c(A,B,C):-a(A),d(B,C).
c(A,B,C):-b(A),d(B,C).
d(B,C):-a(B),!,a(C).
d(B,_):-b(B).
```

Backtrack twice



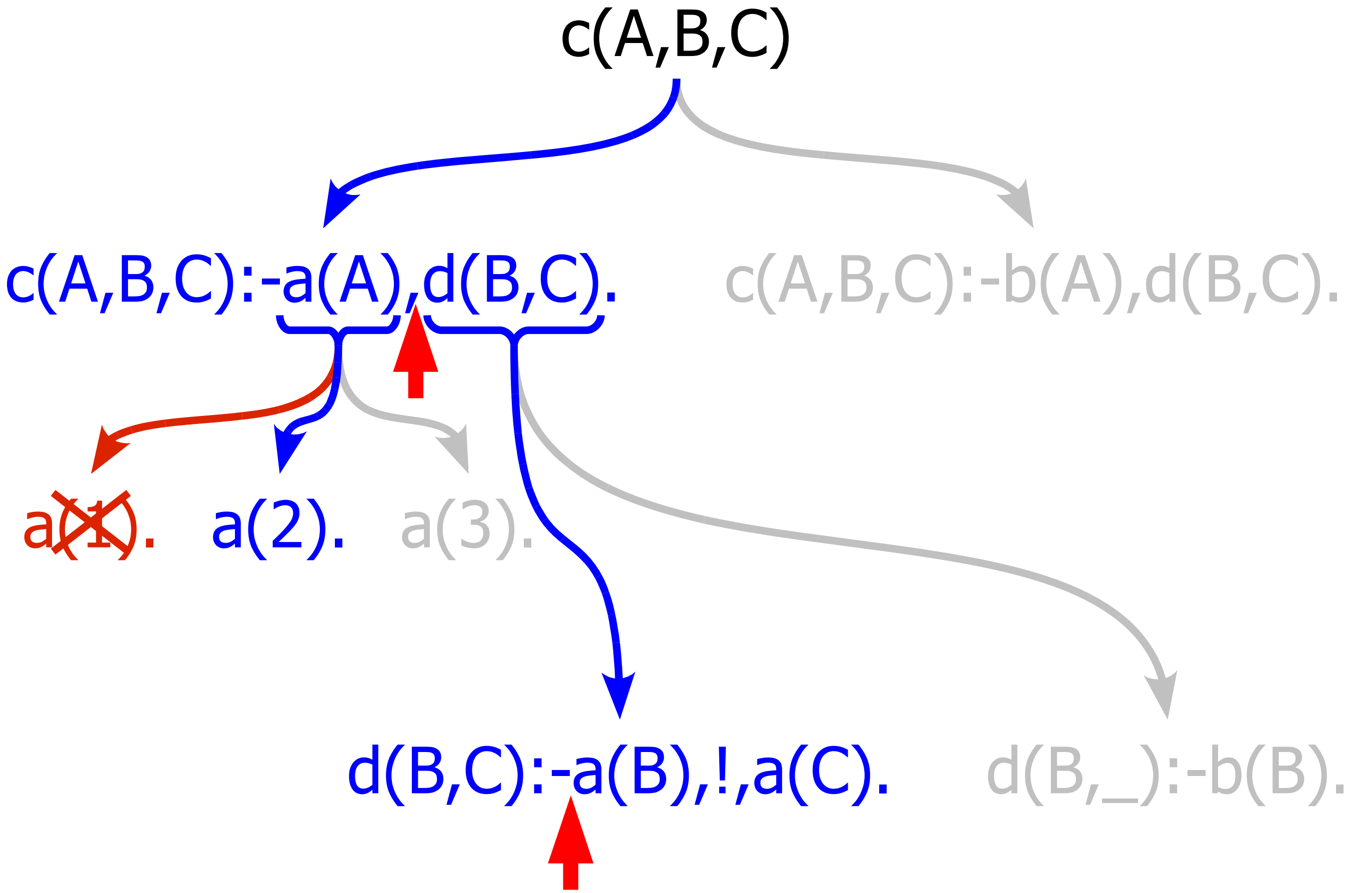

```
a(1).
a(2).
a(3).
b(apple).
b(orange).
c(A,B,C):-a(A),d(B,C).
c(A,B,C):-b(A),d(B,C).
d(B,C):-a(B),!,a(C).
d(B,_):-b(B).
```

Backtrack three times




```
a(1).
a(2).
a(3).
b(apple).
b(orange).
c(A,B,C):-a(A),d(B,C).
c(A,B,C):-b(A),d(B,C).
d(B,C):-a(B),!,a(C).
d(B,_):-b(B).
```

First a/1 has other solutions




```
a(1).
a(2).
a(3).
b(apple).
b(orange).
c(A,B,C):-a(A),d(B,C).
c(A,B,C):-b(A),d(B,C).
d(B,C):-a(B),!,a(C).
d(B,_):-b(B).
```

Can try to derive d/2 afresh...

Cut can change the logical meaning of your program

```
p :- a,b.
p :- c.
```

$p \Leftrightarrow (a \wedge b) \vee c$

```
p :- a,!,b.
p :- c.
```

$p \Leftrightarrow (a \wedge b) \vee (\neg a \wedge c)$

This is a red cut – **DANGER!** (p128)

Cut can be used for efficiency reasons

```
split([],[],[]).
split([H|T],[H|L],R) :- H < 5, split(T,L,R).
split([H|T],L,[H|R]) :- H >= 5, split(T,L,R).
```

If the second clause succeeds the third cannot

- we don't need to keep a choice point
- yet the interpreter cannot infer this on its own

Cut can be used for efficiency reasons

```
split([],[],[]).
split([H|T],[H|L],R) :- H < 5,!, split(T,L,R).
split([H|T],L,[H|R]) :- H >= 5, split(T,L,R).
```

Add a cut to make the orthogonality explicit

- This is a green cut – it just helps program execution go faster

We could go one step further at the expense of readability

```
split([],[],[]).
split([H|T],[H|L],R) :- H < 5,!, split(T,L,R).
split([H|T],L,[H|R]) :- split(T,L,R).
```

The comparison in the third clause is no longer necessary

- but now each clause does not stand on its own
- stylistic preference – I avoid doing this

Programmers new to Prolog often cause determinism errors

Check that your predicates return the correct number of answers!

- e.g. providing a correct answer multiple times is likely to cause bugs that are difficult to find

Below is a predicate you can use in debugging

- (Note that findall is not discussed in lectures)

```
numsol(Predicate,NumberOfSolutions):-
     findall(dummy,Predicate,AnsList),
     length(AnsList,NumberOfSolutions).
```

Testing using the numsol predicate

A warning will be generated about mergesplat/3.

- What is wrong with the mergesplat/3 predicate?

```
mergesplat([],[],[]).
mergesplat(A,[],A).
mergesplat([],B,B).
mergesplat([A|As],[B|Bs],[A,B|Rest]):-
        mergesplat(As,Bs,Rest).

:-numsol(merge([1,2],[a,b],_),1).
:-numsol(mergesplat([1,2],[a,b],_),1).
```

Cut gives us more expressive power

```
isDifferent(A,A) :- !,fail.
isDifferent(_,_).
```

isDifferent(A,B) is true iff A and B do not unify

Questions that you should be able to answer:

- Is this a red or a green cut?
- How can you define the fail/0 predicate?

Using cut, we can implement "not" (Negation by failure)

```
not(A) :- A,!,fail.
not(_).
```

not(A) is true if A cannot be shown to be true

- This is negation by failure (p124)

Negation by failure is based on the closed world assumption: (p129)

Everything that is true in the "world" is stated (or can be derived from) the clauses in the program

Negation Example

```
good_food(theWrestlers).
good_food(theCambridgeLodge).
expensive(theCambridgeLodge).

bargain(R) :- good_food(R),
              not(expensive(R)).
```

we can ask:

- bargain(R)

and Prolog replies:

- R = theWrestlers

Negation Gotcha!

```
good_food(theWrestlers).
good_food(theCambridgeLodge).
expensive(theCambridgeLodge).

bargain(R) :- not(expensive(R)),
              good_food(R).
```

we can ask the same query:

– bargain(R)

and Prolog replies:

– no

Clause body terms have been swapped around!

Why?

```
good_food(theWrestlers).
good_food(theCambridgeLodge).
expensive(theCambridgeLodge).

bargain(R) :- not(expensive(R)),
              good_food(R).
```

Prolog first tries to find an R such that expensive(R) is true.

- therefore not(expensive(R)) will fail if there are any expensive restaurants

We sometimes identify the way to use parameters of a rule

Prolog's non-logical properties can make it important whether or not an argument to a predicate is bound

% indicates a comment to the end of that line

```
% this comment in some hypothetical code is
% describing how to query myrule(+A,+B,-C,-D)
```

The convention for comments about rule parameters:

+X is a ground term

-X is a variable term

?X means it does not matter

Query “myrule” with two ground (input) terms A and B and two variable (output) terms C and D

Prolog variables and quantifiers

When R is not bound, quantifiers need attention

`expensive(R)`

- "There exists an R that is expensive".

`not(expensive(R))`

- "There does not exist an R that is expensive".
- In other words, "for all R, not expensive(R)".

Databases

Information can be stored as tuples in Prolog's internal database

```
tName(dme26,'David Eyers').
tName(awm22,'Andrew Moore').

tGrade(dme26,'IA',2.1).
tGrade(dme26,'IB',1).
tGrade(dme26,'II',1).
tGrade(awm22,'IA',2.1).
tGrade(awm22,'IB',1).
tGrade(awm22,'II',1).
```

Databases

We can now write a program to find all names:

```
qName(N) :- tName(_,N).
```

Or a program to find the full name and all grades for dme26.

```
qGrades(F,C,G) :- tName(I,F), tGrade(I,C,G).
```

Further exercises are in the problem sheet...