MPhil in Advanced Computer Science
Advanced Category Theory in Computer Science

Leader:	Marcelo Fiore (course lecturer)
Timing:	Lent
Prerequisites:	Category Theory in Computer Science
Structure:	8 Lectures

AIMS

This module aims to train students at the forefront of research in the application of category theory to computer science.

SYLLABUS

A range of topics for the course follows.

1. Algebraic theories: universal algebra; equational logic; completeness; theory translations and constructions.

2. Algebras: initial algebras; induction principle; recursive domain equations; free algebras.

3. Presheaves: cartesian closure; essential geometric morphisms; free cocompletions; Kan extensions; coends.

4. Simply typed lambda calculus: λ-definability; Kripke logical relations; glueing; normalisation by evaluation; conservative extensions.

5. Monoidal categories: Day's convolution tensor product; substitution tensor products; operads.

6. Second-order algebraic theories: variable binding and α-equivalence; capture-avoiding substitution; metavariables; meta-substitution.

OBJECTIVES

On completion of this module students should:

- be able to start research in theoretical computer science involving category theory.

COURSEWORK

N/A

PRACTICAL WORK

N/A

ASSESSMENT

The course will be assessed by means of an essay on one or more research papers related to the syllabus. Papers will be chosen by students on their own or from a given list of papers in accordance with the lecturer.

The essays will be marked by the lecturer and returned to the students. Subsequently, a technical discussion with each student on the material of their essay will take place. The mark for the course will be that of the essay, with an upgrade for those students that give evidence of mastering the subject during the discussion.

RECOMMENDED READING

[1] F. Borceux. *Handbook of Categorical Algebra.* Cambridge University Press, 1994.

[2] R. Crole. *Categories for types.* Cambridge University Press, 1993.

[3] B. Day. On closed categories of functors. In *Reports of the Midwest Category Seminar IV*, volume 137 of *Lecture Notes in Mathematics*, pages 1–38. Springer-Verlag, 1970.

[4] M. Fiore. Rough notes on presheaves. Notes, 2001.

[5] M. Fiore. Semantic analysis of normalisation by evaluation for typed lambda calculus. In *4th International Conference on Principles and Practice of Declarative Programming* (PPDP 2002). ACM Press, 2002.

[6] M. Fiore. Algebraic theories and equational logics. Notes, 2008.

[7] M. Fiore, G. Plotkin and D. Turi. Abstract syntax and variable binding. In *14th Logic in Computer Science Conf.* (LICS'99), pages 193-202. IEEE, Computer Society Press, 1999.

[8] G. M. Kelly. On the operads of J. P. May. Reprints in Theory and Applications of Categories, No. 13 (2005) pp. 1–13.

[9] S. Mac Lane. *Categories for the Working Mathematician.* Springer-Verlag, 1971.

[10] D. Scott. Relating theories of the λ-calculus. In *To H. B. Curry: Essays in Combinatory Logic, Lambda Calculus and Formalisms.* Academic Press, 1980.

[11] M. Smyth and G. Plotkin. The category-theoretic solution of recursive domain equations. SIAM Journal of Computing 11 (1982), pp. 761–783.

[12] G. Wraith. Algebraic theories. Lecture Notes Series No. 22, Aarhus Universitet Matematisk Institut, 1975.

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