

MPhil in Advanced Computer Science

Category Theory for Computer Science

Leaders: Bartosz Klin (course lecturer) and Marcelo Fiore
Timing: Michaelmas
Prerequisites: Basic mathematical background
Structure: 16 Lectures

AIMS

This module aims to provide foundations on the mathematical theory of categories as needed in applications to computer science.

SYLLABUS

1. The language of categories: universal properties; categories; isomorphism; monomorphisms and epimorphisms.
2. Constructions in categories: graphs; diagrams; limits and colimits.
3. Constructions on categories: duality; products; sums; slices.
4. Functors: covariance, contravariance, mixed variance; full and faithfulness; equivalences.
5. Categorical algebra: universal algebra; algebras and coalgebras for a functor; initial algebras; induction and coinduction principles.
6. Functors: variable sets; representable functors.
7. Natural transformations: functor categories; Yoneda lemma and embedding.
8. Adjunctions: free constructions; triangular laws; preservation properties; reflections; cartesian closed categories; quantifiers as adjoints.
9. Monads: monads from adjunctions; algebras and the Eilenberg-Moore category; free algebras and the Kleisli category; free monads.

OBJECTIVES

On completion of this module students should:

- be fluent in the basic language and proof methods of category theory, and thereby
- be able to read and study research papers in computer science that use and/or are based on category theory.

COURSEWORK

Exercise sheets will be provided.

PRACTICAL WORK

N/A

ASSESSMENT

All assessment will be set and marked by the course lecturer, and will take place in two stages.

- Firstly, students will be evaluated on a given list of problems for which written solutions should be provided halfway through the course. This will account for 25% of the total mark for the course.
- Secondly, students will be evaluated on a take home exam at the end of the course. This will account for 75% of the total mark for the course.

RECOMMENDED READING

- [1] M. Barr and C. Wells. *Category Theory for Computing Science*. Centre de Recherches Mathématiques, third edition, 1999.
- [2] R. Crole. *Categories for Types*. Cambridge University Press, 1993.
- [3] F. W. Lawvere and S. Schanuel. *Conceptual Mathematics*. Cambridge University Press, 1997.
- [4] S. Mac Lane. *Categories for the Working Mathematician*. Springer-Verlag, 1971.
- [5] P. Taylor. *Practical Foundations of Mathematics*. Cambridge Studies in Advanced Mathematics 59, Cambridge University Press, 1999.

Last updated: January 2009