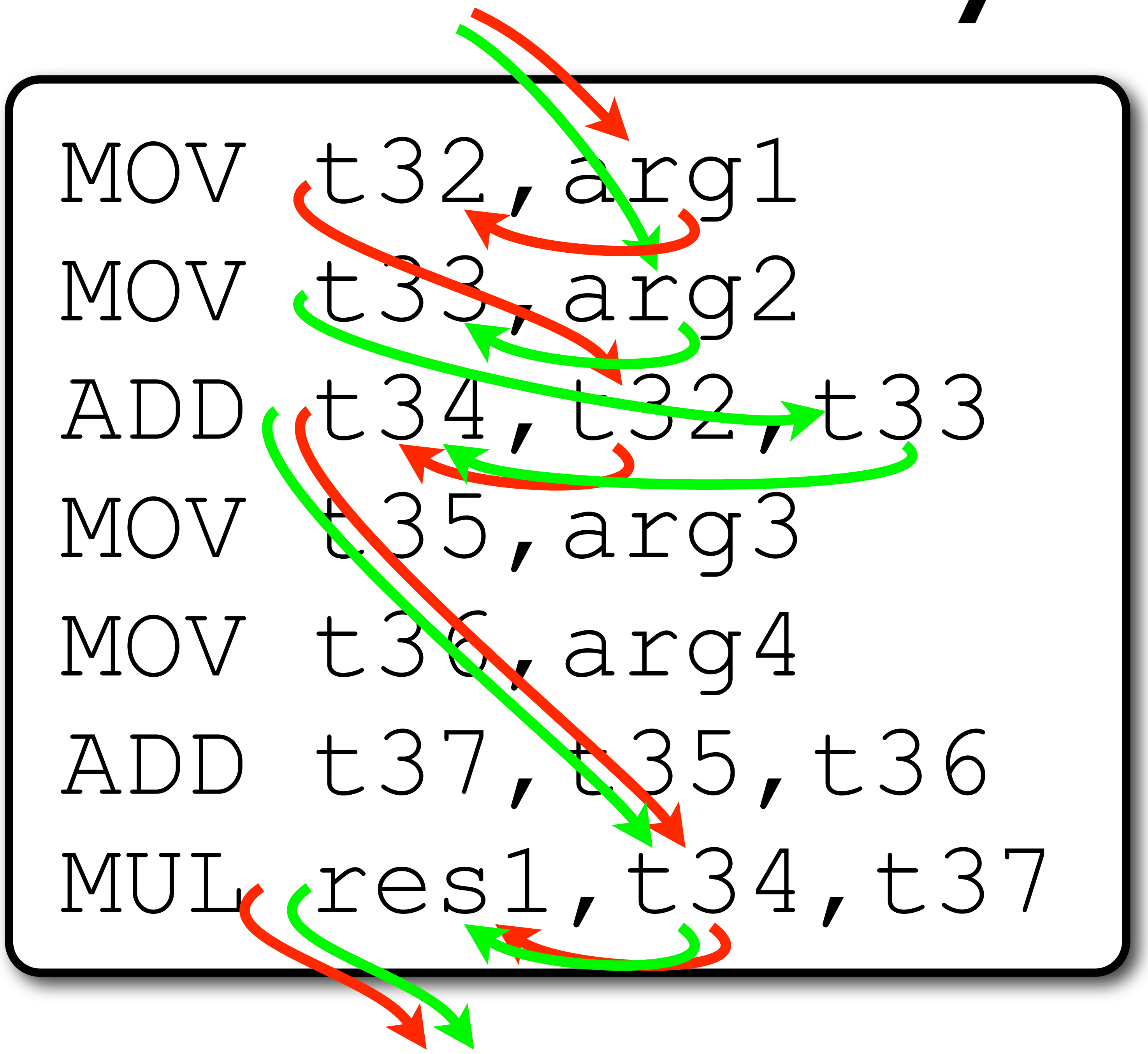
Data-flow analysis



```
MOV t32,arg1
MOV t33,arg2
ADD t34,t32,t33
MOV t35,arg3
MOV t36,arg4
ADD t37,t35,t36
MUL res1,t34,t37
```

Discovering information about how *data* (i.e. variables and their values) may move through a program.

Motivation

Programs may contain

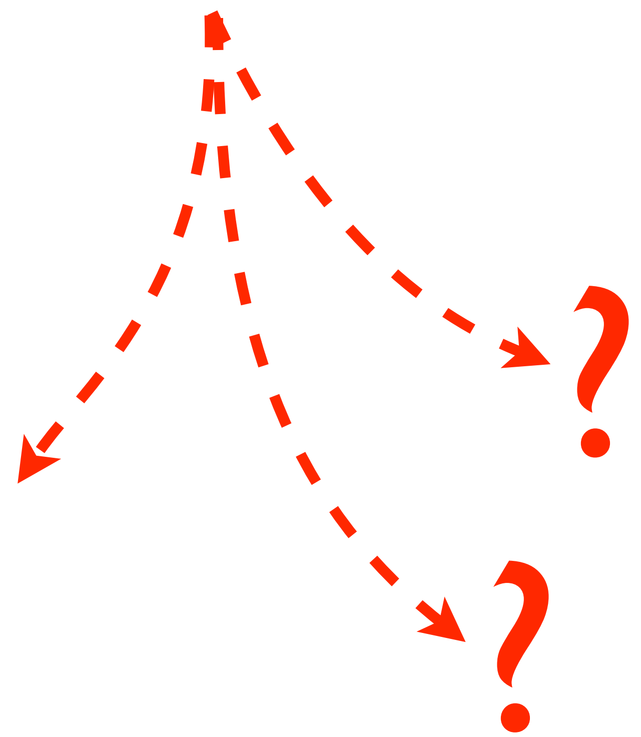
- code which gets executed but which has no useful effect on the program's overall result;
- occurrences of variables being used before they are defined; and
- many variables which need to be allocated registers and/or memory locations for compilation.

The concept of *variable liveness* is useful in dealing with all three of these situations.

Liveness

Liveness is a data-flow property of variables:
"Is the value of this variable needed?" (cf. dead code)

```
int f(int x, int y) {
  int z = x * y;
   ⋮
```



Liveness

At each instruction, each variable in the program is either live or dead.

We therefore usually consider liveness from an instruction's perspective: each instruction (or node of the flowgraph) has an associated *set* of live variables.

```
  ⋮
n: int z = x * y;
   return s + t;
```

live(*n*) = { `s`, `t`, `x`, `y` }

Semantic vs. syntactic

There are two kinds of variable liveness:

- Semantic liveness
- Syntactic liveness

Semantic vs. syntactic

A variable *x* is *semantically* live at a node *n* if there is *some execution sequence* starting at *n* whose (externally observable) behaviour can be affected by changing the value of *x*.

```
int x = y * z;     x LIVE
 ⋮
return x;
```

Semantic vs. syntactic

A variable *x* is *semantically* live at a node *n* if there is *some execution sequence* starting at *n* whose (externally observable) behaviour can be affected by changing the value of *x*.

```
int x = y * z;    x DEAD
 ⋮
x = a + b;
 ⋮
return x;
```

Semantic vs. syntactic

Semantic liveness is concerned with
the *execution behaviour* of the program.

This is undecidable in general.
(e.g. Control flow may depend upon arithmetic.)

Semantic vs. syntactic

A variable is *syntactically* live at a node if there is a path to the exit of the flowgraph along which its value may be used before it is redefined.

Syntactic liveness is concerned with properties of the *syntactic structure* of the program.

Of course, this *is* decidable.

So what's the difference?

Semantic vs. syntactic

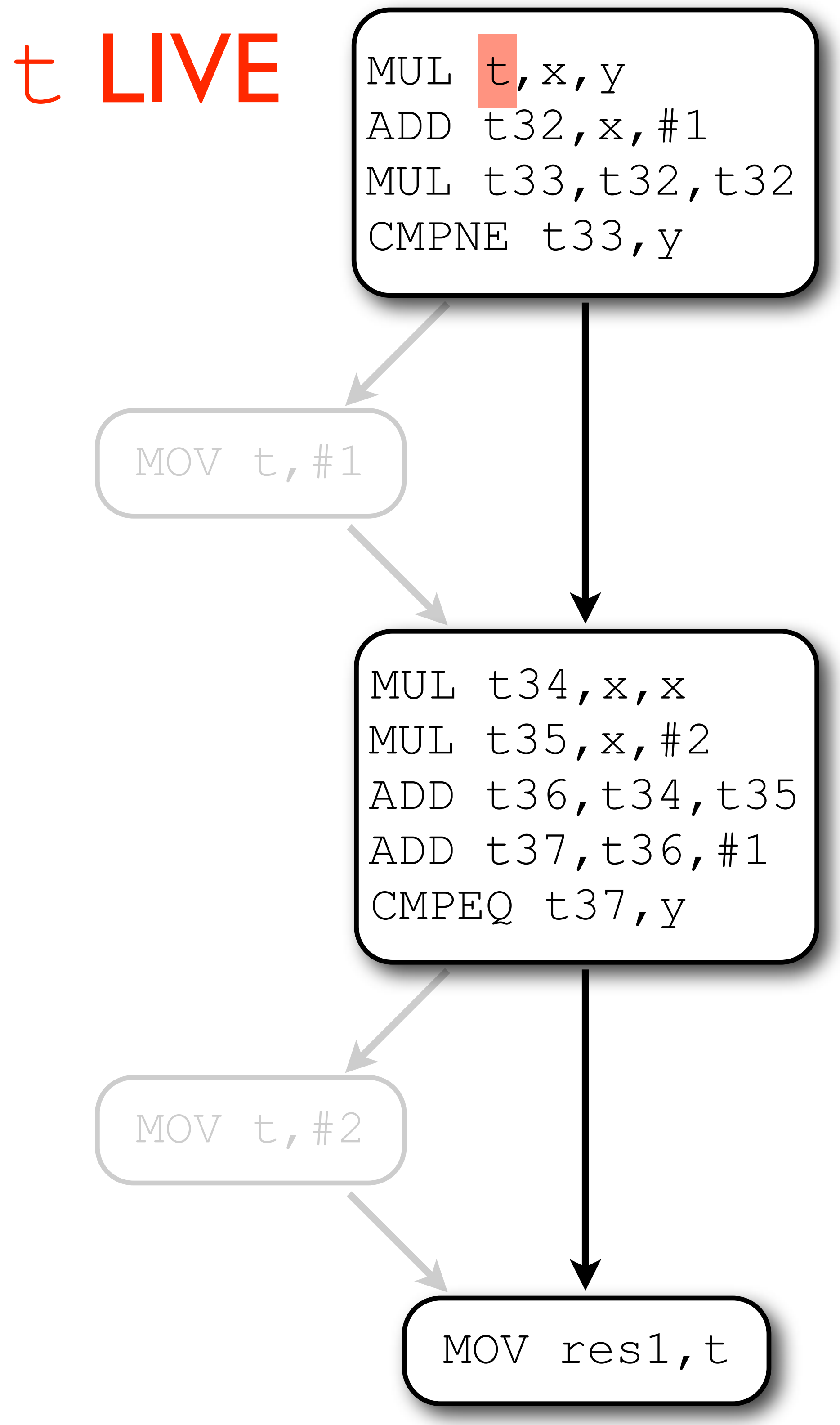
```
int t = x * y;  t DEAD
if ((x+1)*(x+1) == y) {
  t = 1;
}
if (x*x + 2*x + 1 != y) {
  t = 2;
}
return t;
```

Semantically: one of the conditions will be true, so on every execution path `t` is redefined before it is returned. The value assigned by the first instruction is never used.

Semantic vs. syntactic

```
       MUL t,x,y
       ADD t32,x,#1
       MUL t33,t32,t32
       CMPNE t33,y,lab1
       MOV t,#1
lab1:  MUL t34,x,x
       MUL t35,x,#2
       ADD t36,t34,t35
       ADD t37,t36,#1
       CMPEQ t37,y,lab2
       MOV t,#2
lab2:  MOV res1,t
```

Semantic vs. syntactic



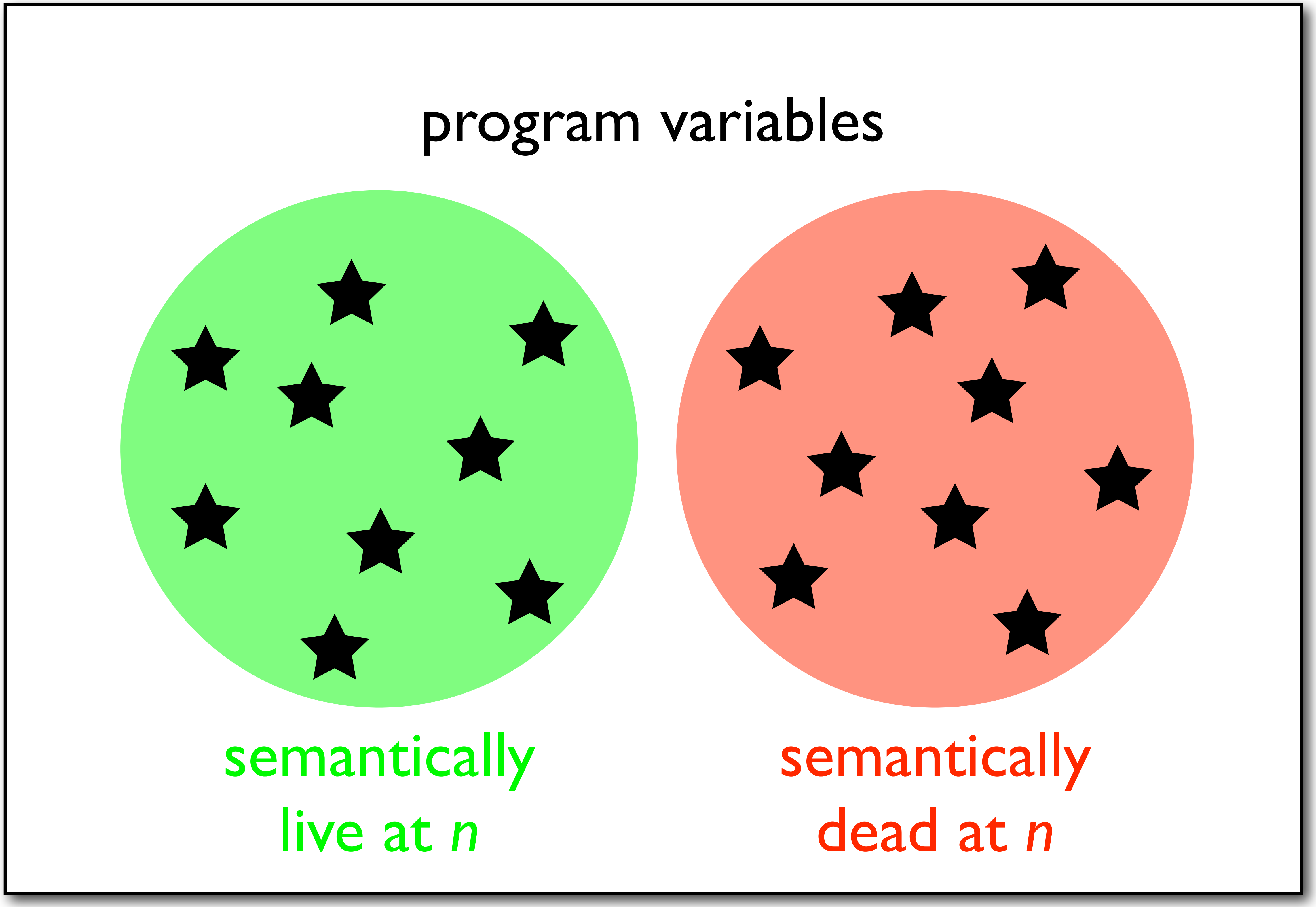
On *this* path through the flowgraph, `t` is not redefined before it's used, so `t` is *syntactically* live at the first instruction.

Note that this path never actually occurs during execution.

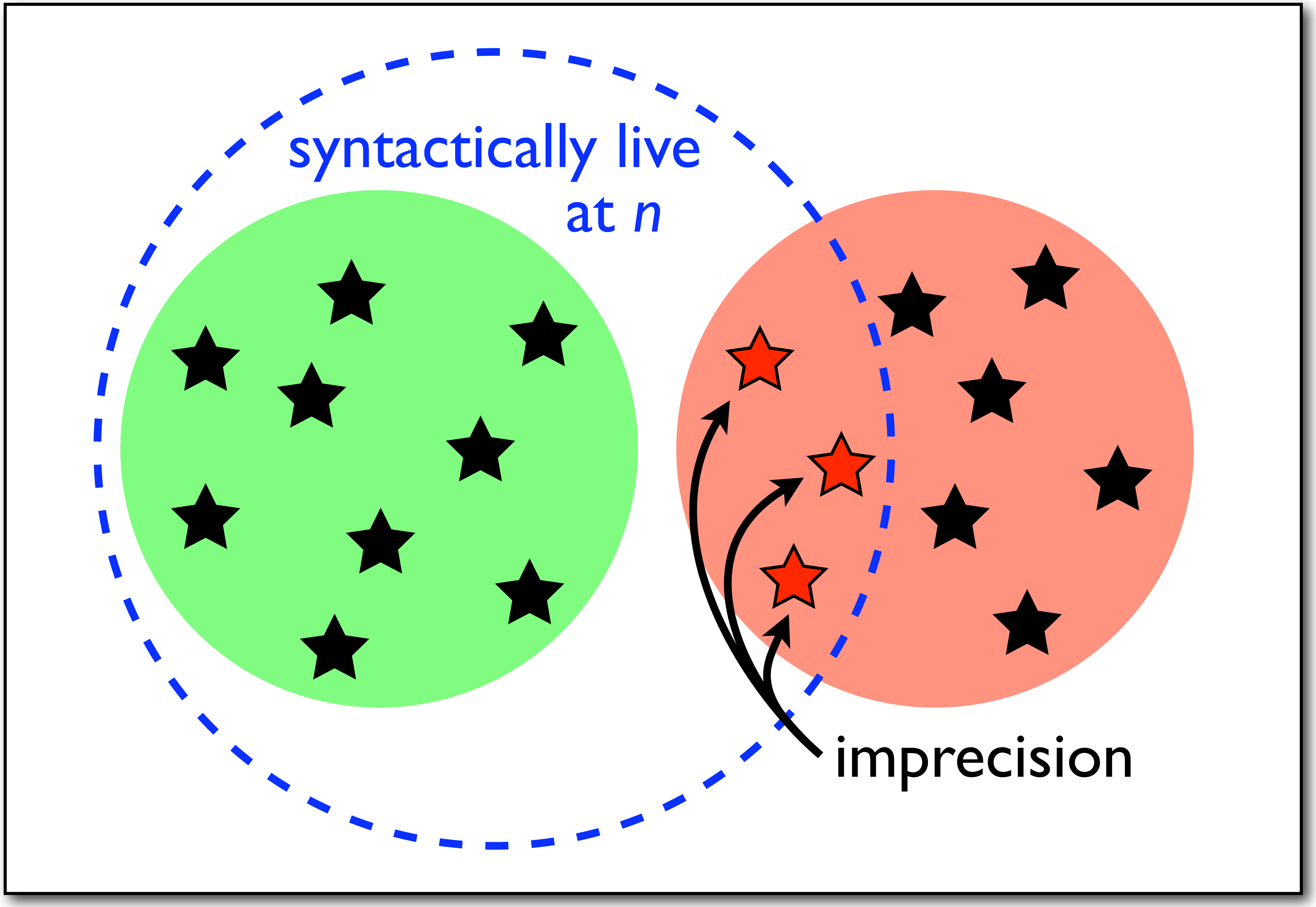
Semantic vs. syntactic

So, as we've seen before, *syntactic* liveness is a computable approximation of *semantic* liveness.

Semantic vs. syntactic



Semantic vs. syntactic



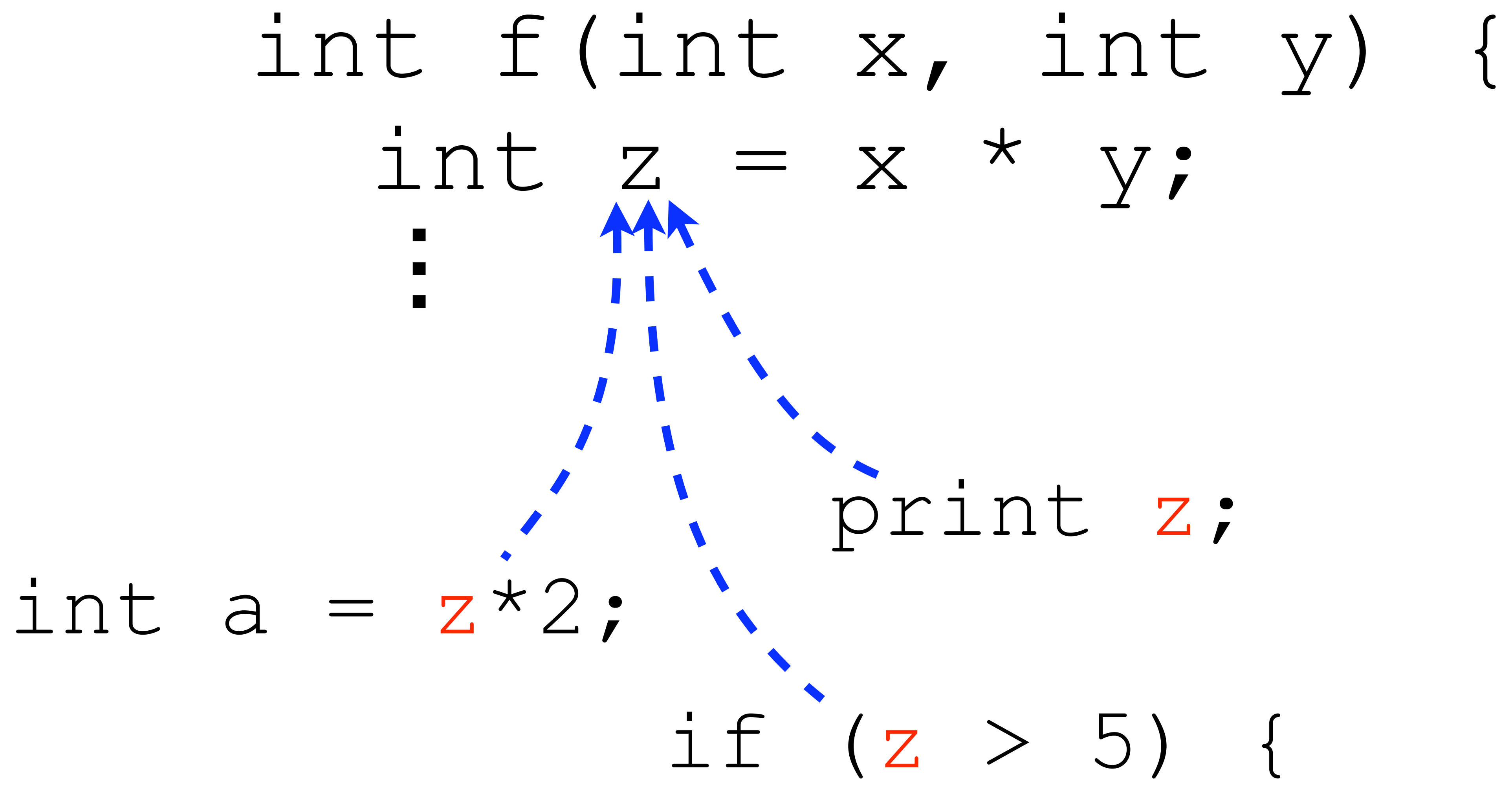
Semantic vs. syntactic

$$sem\text{-}live(n) \subseteq syn\text{-}live(n)$$

Using syntactic methods, we *safely overestimate* liveness.

Live variable analysis

LVA is a *backwards* data-flow analysis: usage information from *future* instructions must be propagated backwards through the program to discover which variables are live.



```
int f(int x, int y) {
  int z = x * y;
  ⋮
                    print z;
int a = z*2;
                 if (z > 5) {
```

Live variable analysis

Variable liveness flows (backwards) through the program in a continuous stream.

Each instruction has an effect on the liveness information as it flows past.

Live variable analysis

An instruction makes a variable live when it *references* (uses) it.

Live variable analysis

{ b, c, e, f }

`a = b * c;` REFERENCE b, c

{ e, f }

`d = e + 1;` REFERENCE e

{ f }

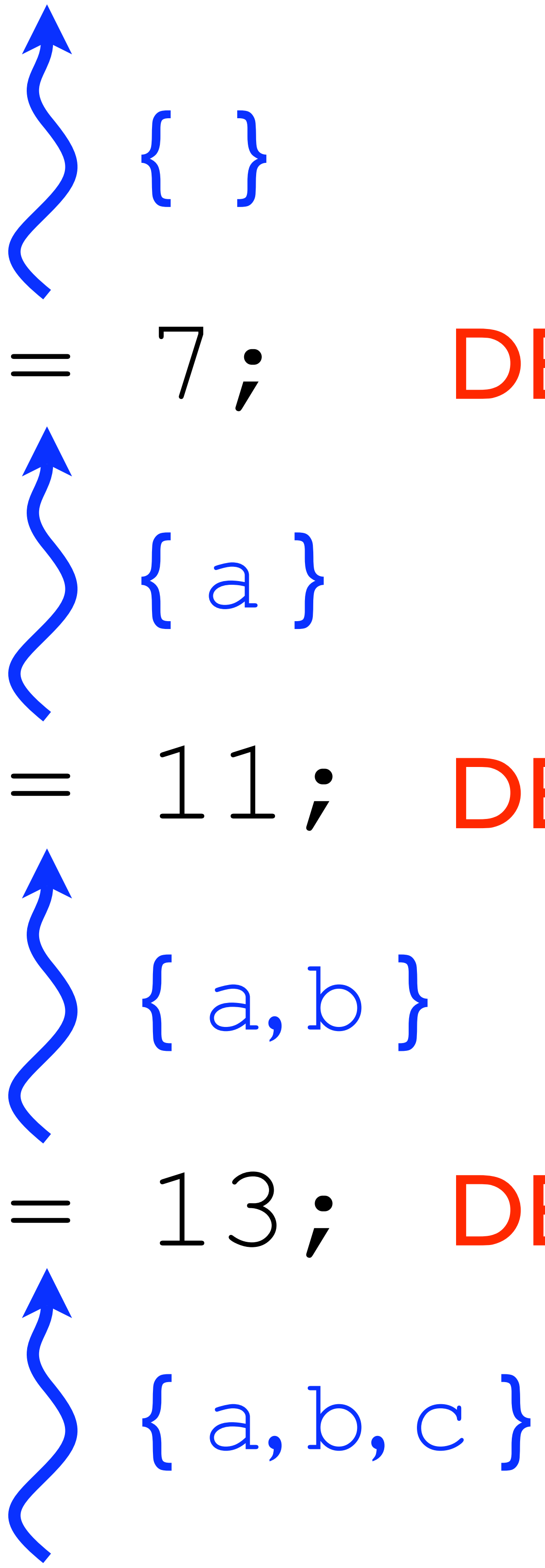
`print f;` REFERENCE f

{ }

Live variable analysis

An instruction makes a variable dead when it *defines* (assigns to) it.

Live variable analysis



Live variable analysis

We can devise functions *ref*(*n*) and *def*(*n*) which give the sets of variables referenced and defined by the instruction at node *n*.

ref(`x = 3`) = { }

def(`x = 3`) = { `x` }

ref(`print x`) = { `x` }

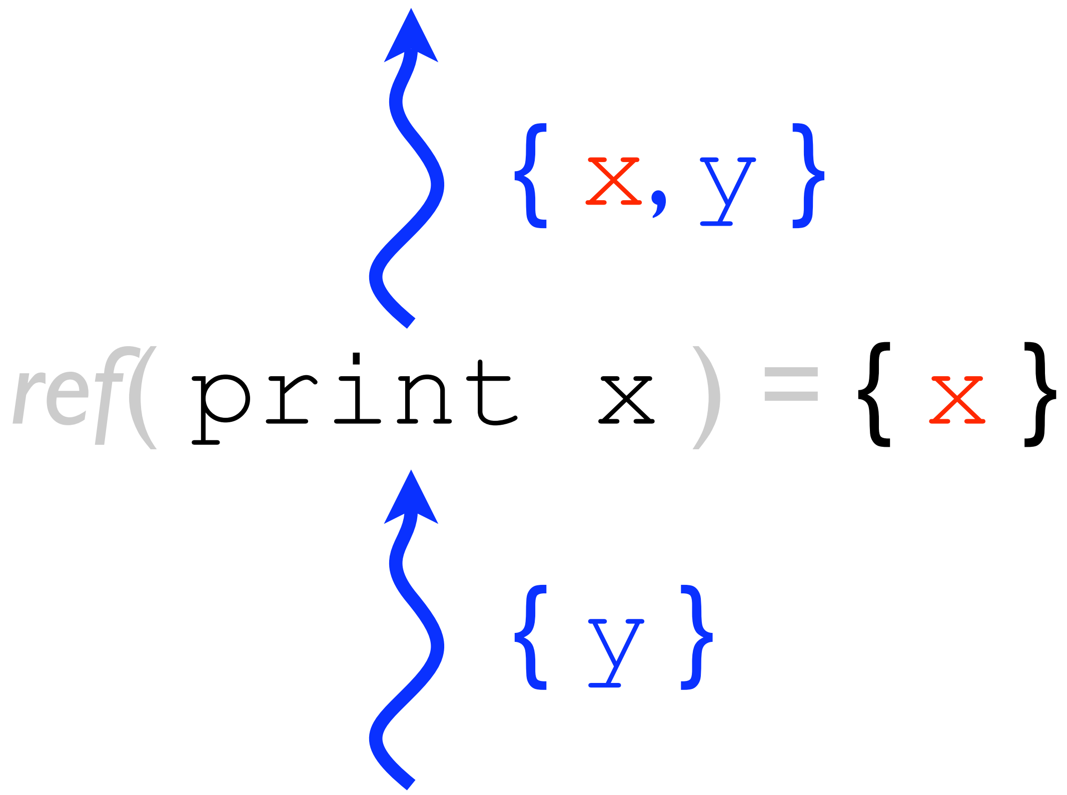
def(`print x`) = { }

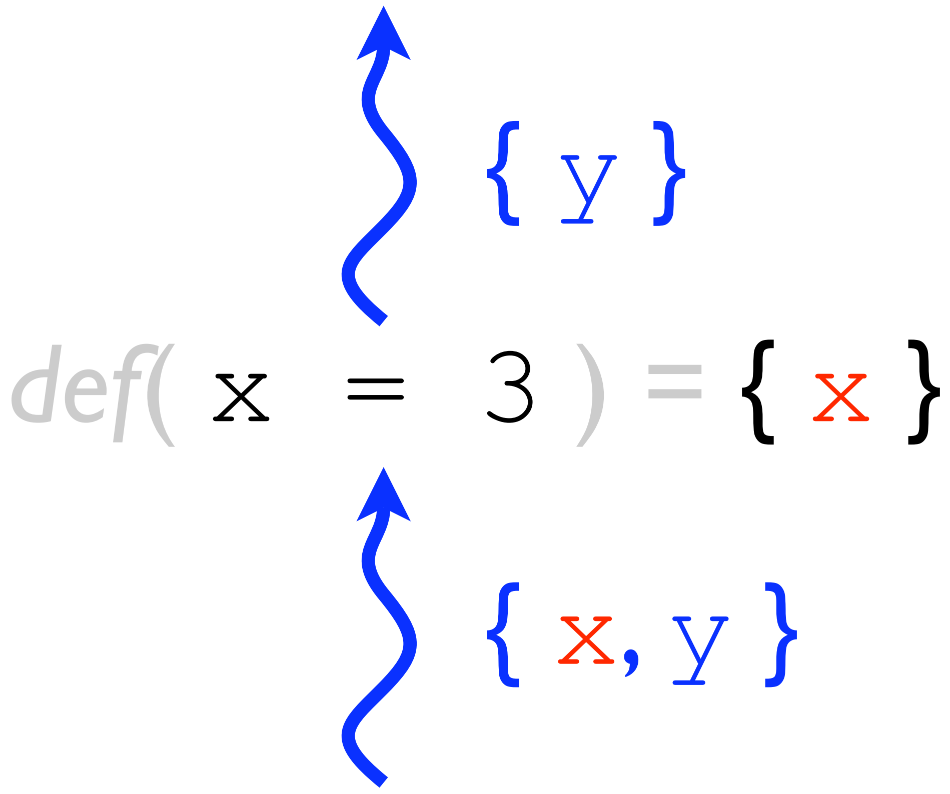
ref(`x = x + y`) = { `x`, `y` }

def(`x = x + y`) = { `x` }

Live variable analysis

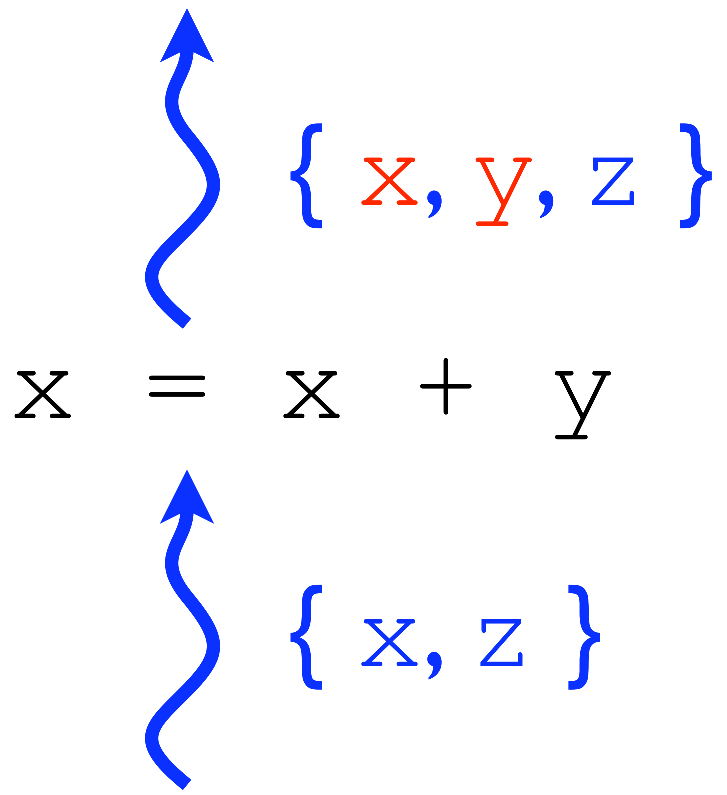
As liveness flows backwards past an instruction, we want to modify the liveness information by *adding* any variables which it references (they become live) and *removing* any which it defines (they become dead).





Live variable analysis

If an instruction both references and defines variables, we must remove the defined variables *before* adding the referenced ones.



def(`x = x + y`) = { `x` }

ref(`x = x + y`) = { `x`, `y` }

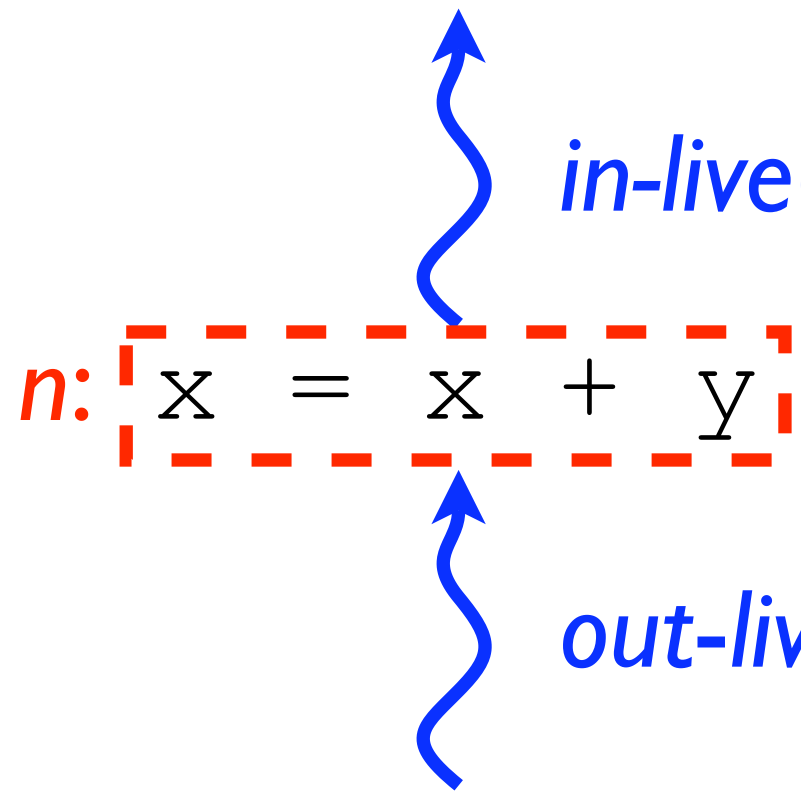
Live variable analysis

So, if we consider *in-live*(*n*) and *out-live*(*n*), the sets of variables which are live immediately *before* and immediately *after* a node, the following equation must hold:

$$\mathit{in\text{-}live}(n) = \Big(\mathit{out\text{-}live}(n) \setminus \mathit{def}(n)\Big) \cup \mathit{ref}(n)$$

Live variable analysis

$$\textit{in-live}(n) = \Big(\textit{out-live}(n) \setminus \textit{def}(n)\Big) \cup \textit{ref}(n)$$



n:
```
x = x + y
```

$\textit{in-live}(n) = \big(\textit{out-live}(n) \setminus \textit{def}(n)\big) \cup \textit{ref}(n)$

$= (\{\, \mathtt{x}, \mathtt{z} \,\} \setminus \{\, \mathtt{x} \,\}) \cup \{\, \mathtt{x}, \mathtt{y} \,\}$

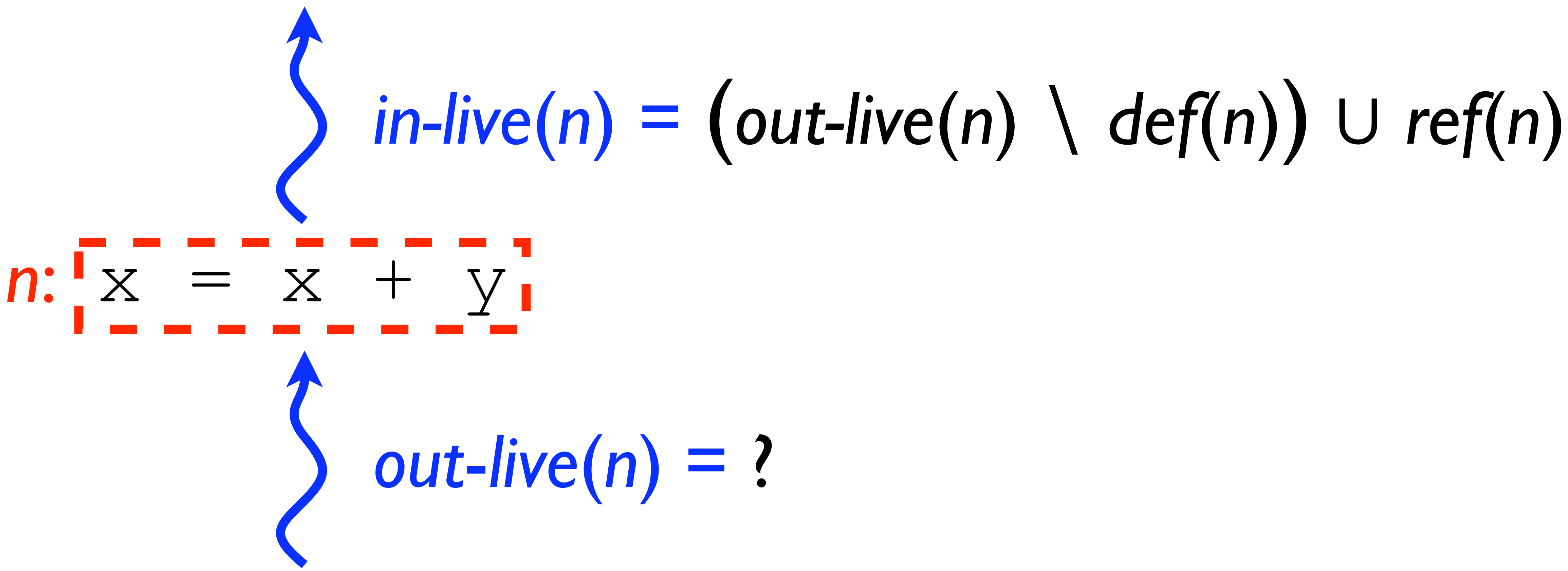
$= \{\, \mathtt{z} \,\} \cup \{\, \mathtt{x}, \mathtt{y} \,\} = \{\, \mathtt{x}, \mathtt{y}, \mathtt{z} \,\}$

$\textit{out-live}(n) = \{\, \mathtt{x}, \mathtt{z} \,\}$

$\textit{def}(n) = \{\, \mathtt{x} \,\}$ $\qquad$ $\textit{ref}(n) = \{\, \mathtt{x}, \mathtt{y} \,\}$

Live variable analysis

So we know how to calculate *in-live*(n) from the values of *def*(n), *ref*(n) and *out-live*(n). But how do we calculate *out-live*(n)?


$$\textit{in-live}(n) = \big(\textit{out-live}(n) \setminus \textit{def}(n)\big) \cup \textit{ref}(n)$$

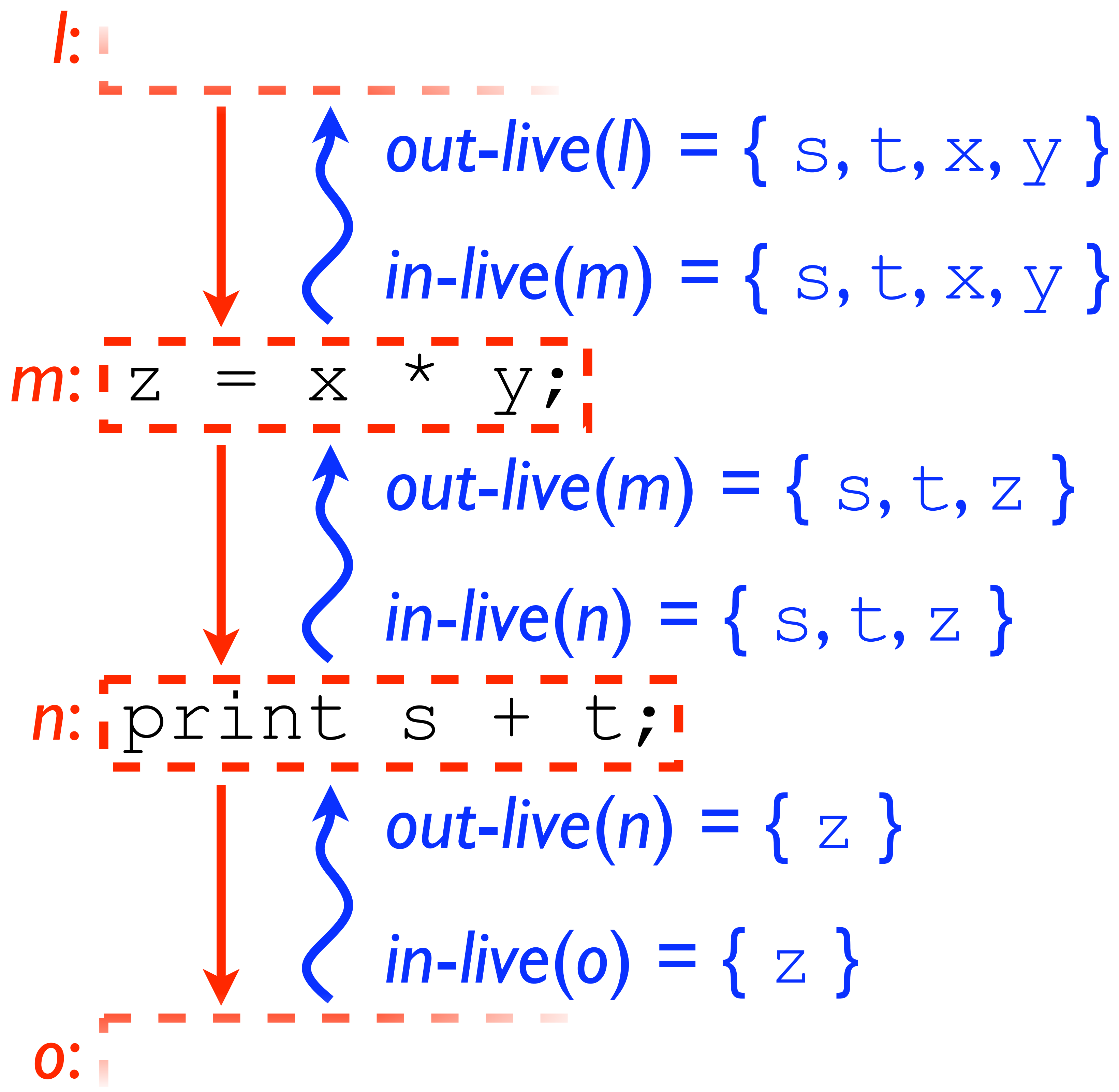
n: `x = x + y`

$$\textit{out-live}(n) = ?$$

Live variable analysis

In straight-line code each node has a unique successor, and the variables live at the exit of a node are exactly those variables live at the entry of its successor.

Live variable analysis



out-live(*l*) = { `s`, `t`, `x`, `y` }

in-live(*m*) = { `s`, `t`, `x`, `y` }

```
z = x * y;
```

out-live(*m*) = { `s`, `t`, `z` }

in-live(*n*) = { `s`, `t`, `z` }

```
print s + t;
```

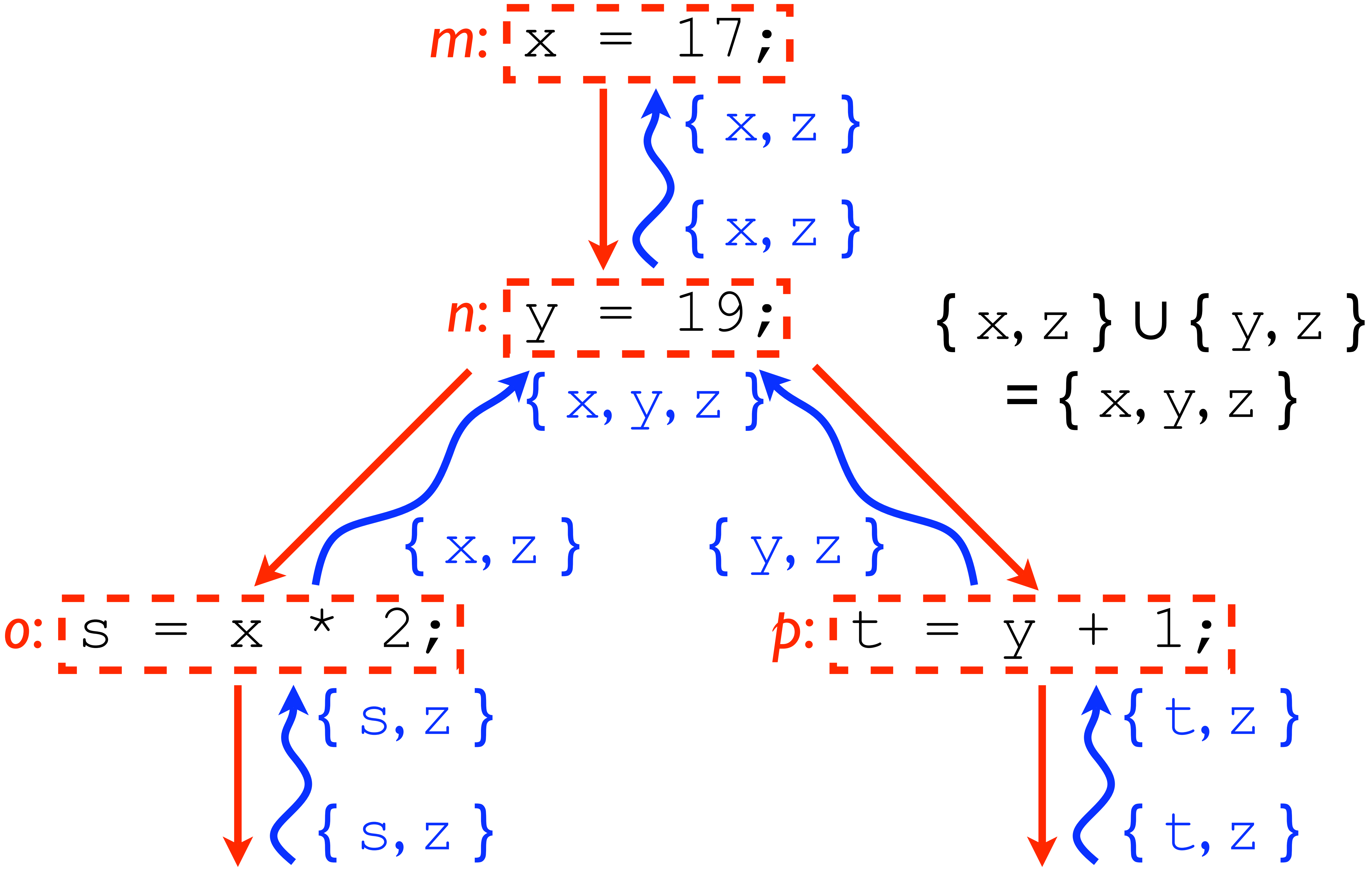
out-live(*n*) = { `z` }

in-live(*o*) = { `z` }

Live variable analysis

In general, however, each node has an arbitrary number of successors, and the variables live at the exit of a node are exactly those variables live at the entry of *any* of its successors.

Live variable analysis



$\{ x, z \} \cup \{ y, z \}$
$= \{ x, y, z \}$

Live variable analysis

So the following equation must also hold:

$$out\text{-}live(n) = \bigcup_{s \in succ(n)} in\text{-}live(s)$$

Data-flow equations

These are the *data-flow equations* for live variable analysis, and together they tell us everything we need to know about how to propagate liveness information through a program.

$$in\text{-}live(n) = \Big(out\text{-}live(n) \setminus def(n)\Big) \cup ref(n)$$

$$out\text{-}live(n) = \bigcup_{s \in succ(n)} in\text{-}live(s)$$

Data-flow equations

Each is expressed in terms of the other, so we can combine them to create one overall liveness equation.

$$live(n) = \left(\left(\bigcup_{s \in succ(n)} live(s) \right) \setminus def(n) \right) \cup ref(n)$$

Algorithm

We now have a formal description of liveness, but we need an actual algorithm in order to do the analysis.

Algorithm

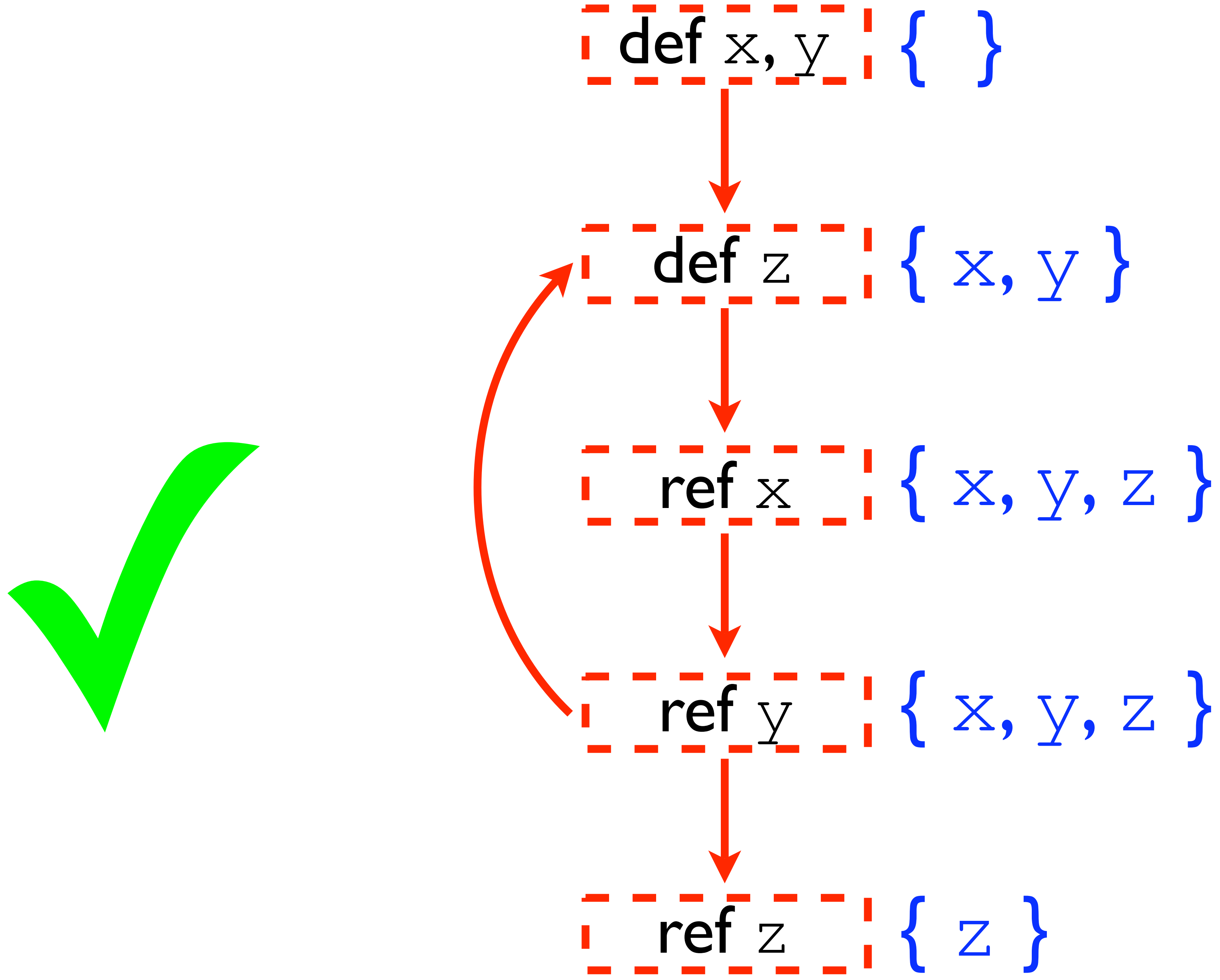
"Doing the analysis" consists of computing a value *live(n)* for each node *n* in a flowgraph such that the liveness data-flow equations are satisfied.

A simple way to solve the data-flow equations is to adopt an iterative strategy.

Algorithm

$$\begin{tikzcd}
 & \boxed{\text{def } \mathtt{x}, \mathtt{y}} \arrow[d] & \{\ \} \\
 & \boxed{\text{def } \mathtt{z}} \arrow[d] & \{\ \mathtt{x}, \mathtt{y}\ \} \\
\text{✗} & \boxed{\text{ref } \mathtt{x}} \arrow[d] & \{\ \mathtt{x}, \mathtt{y}, \mathtt{z}\ \} \\
 & \boxed{\text{ref } \mathtt{y}} \arrow[d] \arrow[uu, bend left] & \{\ \mathtt{y}, \mathtt{z}\ \} \\
 & \boxed{\text{ref } \mathtt{z}} & \{\ \mathtt{z}\ \}
\end{tikzcd}$$

Algorithm



Algorithm

```
for i = 1 to n do live[i] := {}
while (live[] *changes*) do
  for i = 1 to n do
    live[i] := $\left(\left(\bigcup_{s \in succ(\texttt{i})} \texttt{live}[s]\right) \setminus def(\texttt{i})\right) \cup ref(\texttt{i})$
```

Algorithm

This algorithm is guaranteed to terminate since there are a *finite* number of variables in each program and the effect of one iteration is *monotonic*.

Furthermore, although any solution to the data-flow equations is safe, this algorithm is guaranteed to give the *smallest* (and therefore most precise) solution.

(See the *Knaster-Tarski theorem* if you're interested.)

Algorithm

Implementation notes:

- If the program has *n* variables, we can implement each element of `live[]` as an *n*-bit value, with each bit representing the liveness of one variable.
- We can store liveness once per basic block and recompute inside a block when necessary. In this case, given a basic block *n* of instructions $i_1, \ldots, i_k$:

$$live(n) = \left(\bigcup_{s \in succ(n)} live(s) \right) \setminus def(i_k) \cup ref(i_k) \cdots \setminus def(i_1) \cup ref(i_1)$$

Safety of analysis

- Syntactic liveness safely overapproximates semantic liveness.
- The usual problem occurs in the presence of address-taken variables (cf. labels, procedures): *ambiguous* definitions and references. For safety we must
  - overestimate ambiguous references (assume all address-taken variables are referenced) and
  - underestimate ambiguous definitions (assume no variables are defined); this increases the size of the smallest solution.

Safety of analysis

```
   MOV  x,#1
   MOV  y,#2
   MOV  z,#3
   MOV  t32,#&x
   MOV  t33,#&y
   MOV  t34,#&z
    ⋮
m: STI  t35,#7
    ⋮
n: LDI  t36,t37
```

def(*m*) = { }
ref(*m*) = { `t35` }

def(*n*) = { `t36` }
ref(*n*) = { `t37`, `x`, `y`, `z` }

Summary

- Data-flow analysis collects information about how data moves through a program
- Variable liveness is a data-flow property
- Live variable analysis (LVA) is a backwards data-flow analysis for determining variable liveness
- LVA may be expressed as a pair of complementary data-flow equations, which can be combined
- A simple iterative algorithm can be used to find the smallest solution to the LVA data-flow equations