Exercises: sheet 2

1. Devise a λ-expression **Diff** such that

$$\begin{aligned} \mathbf{Diff}(\mathbf{true})(\mathbf{true}) &= \mathbf{false} \\ \mathbf{Diff}(\mathbf{true})(\mathbf{false}) &= \mathbf{true} \\ \mathbf{Diff}(\mathbf{false})(\mathbf{true}) &= \mathbf{true} \\ \mathbf{Diff}(\mathbf{false})(\mathbf{false}) &= \mathbf{false} \end{aligned}$$

2. Show that

$$\begin{aligned} \mathbf{suc}(\underline{5}) &= \underline{6} \\ \mathbf{iszero}\ (\mathbf{suc}(\underline{n})) &= \mathbf{false} \\ \mathbf{add}(\underline{m})(\underline{n}) &= \underline{m+n} \\ \mathbf{pre}(\mathbf{suc}(\underline{n})) &= \underline{n} \\ \mathbf{pre}(\underline{0}) &= \underline{0} \end{aligned}$$

3. Give a λ-term **Mult** which uses the Y combinator and satisfies the equation $\mathbf{Mult}(\underline{m})(\underline{n}) = \underline{m+n}$.

4. Give a λ-term **reverse** which reverses a list.

5. Consider the function listadd defined inductively on the structure of lists of natural numbers by

$$\begin{aligned} \mathsf{listadd}(m, [\,]) &= [\,] \\ \mathsf{listadd}(m, [n, L]) &= [m+n, \mathsf{listadd}(m, L)] \end{aligned}$$

   Give a λ-term which encodes this function listadd, and justify your answer.

6. Give the λ-term for the infinite list $[0, 1, 0, 1, 0, 1, \ldots]$.

7. Illustrate that the λ-terms representing the total recursive functions are correct.