Colour difference formula for photopic and mesopic vision incorporating cone and rod responses

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Abstract

The standard colour difference formulas, such as CIEDE2000, operate on colours defined by cone-fundamentals, which ignore the influence of rods on colour perception. In this work, we combine the rod intrusion model by Cao et al. with the popular CIEDE2000 colour difference formula and validate the accuracy of the new formula on three contrast sensitivity datasets. When compared with the standard CIEDE2000 formula, the new colour difference formula improves the perceptual uniformity of the space at low luminance levels.

Introduction

Max Schultze's seminal *duplicity theory of vision* dictates that rods and cones govern two independent visual mechanisms [1, 2], with the rods mediating only low luminance achromatic vision (scotopic vision), while the cones mediating high luminance vision with better spatial resolution (photopic vision) with some overlap in the respective luminance ranges (mesopic vision). Rods become active as luminance levels decrease and completely take over in scotopic luminance levels after complete dark adaptation. The assumption of different types of photoreceptors explains phenomenon like the well-known Purkinje effect where a brightness match obtained between two colours in a light-adapted state may no longer hold after dark adaptation [3]. This change in relative brightness between light and dark-adapted states could not be explained by earlier trichromatic theories of vision alone.

The role of rods is not just limited to coarse achromatic vision. The perception of colours in dim lights is also altered, including changes in brightness perception [4, 5], changes in gamut of perceived colours [6], desaturation of test colours [7]. These colour changes can be explained by the fact that rods and cones have different spectral responses in addition to different luminance operating ranges (Figure 2 (top-right)) [8, 9, 10, 11]. Rod response favours shorter wavelength stimuli while the photopic luminous efficiency function mediated by long and medium (L, M) cones peaks at higher wavelengths. This results in different wavelength ranges dominating the contribution to visual system for photopic, mesopic and scotopic ranges. The luminance-dependent shift in peak wavelength of spectral response results in changes in both chromatic and achromatic visual responses which indicates the role of rods in supplementing cone responses in low luminances.

The contribution of rods to human vision is not modelled by most colour spaces despite evidence of rods providing input to cones [12] and even providing stand-alone chromatic responses below small and medium cone thresholds [13]. The LMS colour spaces are based on the responses of long, medium, and short (L, M, and S) cones and only represents photopic vision [14]. Similarly, the XYZ colour spaces are linear transforms of the corresponding LMS cone spaces [15] and do not take into account the rod responses or the shifts in colour perception at low luminance levels. CIELAB colour difference error metrics ΔE_{76} [16], ΔE_{94}, [17] and ΔE_{2000} [18] also work for colour differences under photopic luminances only. Widely used colour appearance models such as CIECAM02 [19] and iCAM06 [20] are also limited to predict effects relevant to medium to high luminance image appearance only. Because most of the applications of these colour models are in higher light levels, modelling rod contributions has not been a priority. However, advances in high dynamic range imaging allow displaying content at both very high photopic levels and low mesopic and scotopic levels; consequently a faithful representation of low luminance scenes requires accurately capturing and relaying rod responses as well.

The aim of this work is to develop a colour difference formula that accounts for rod contribution to the colour perception. We propose three colour difference formulas that incorporate different stages of Cao et al. rod intrusion model [12] and test their perceptual uniformity in terms of STRESS and PF/3 metrics.

Related work

A number of researchers have developed algorithms and tone-mapping operators to predict image appearances in low light levels based on physiological and psychophysical data [21, 22, 23, 24, 25]. The scope of this work is to propose simple error metrics (comparable to CIELAB colour difference metrics) for L, M, S, and R (cones and rod) responses. So we will use some models that quantify rod responses to vision at receptoral and early post-receptoral levels.

Rods and cones feed into the same neural pathways [26] despite differences in anatomical structure, density distribution over the retina and different light activation ranges (scotopic, mesopic and photopic) [27]. That is, the light incident upon the retina is transduced by rods and the three different types of cones (long, medium, and short wavelength) then combined and converted into post-receptoral signals which are then passed on to the higher-order visual mechanisms. Rods contribution to cone responses has been modelled and quantified using perceptual matching where rod contrasts were matched with the equivalent cone contrasts [12]. [12] have determined the combined cone and rod responses through contrast matching psychophysical functions as follows:

$$L' = \frac{L+\alpha_1(l_{tr})R}{l_{max}}, \quad M' = \frac{M+\alpha_1(l_{tr})R}{m_{max}}, \quad S' = \frac{S+\alpha_2(l_{tr})R}{s_{max}} \tag{1}$$

where L, M, and S are the cone troland responses based on Smith and Pokorny cone fundamentals [10, 28] and R is the response from the CIE 1951 standard scotopic luminous efficiency function V' [11, p.259]. l_{max}, m_{max}, and s_{max} are the weights of the Smith and Pokorny cone fundamentals [10]. The strength of rod

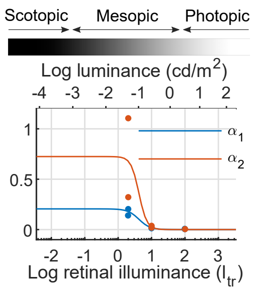

Figure 1. *Values of rod contribution parameters as functions of retinal illuminance in trolands. α_1 is the weight of rod contribution to L and M cones. α_2 is the weight of rod contribution to S cones. The solid lines are fitted logistic functions. Data points are from Fig. 3 in Cao et al. [12] for two observers. We used the mean of the values to fit the logistic function. We have also shown the equivalent luminance in cd/m² and the approximate range for scotopic, mesopic and photopic regions.*

input to the cone responses is modelled by the parameters α_1 and α_2 in Eq. Eq. (1). We used the mean of data points from Fig. 3 in Cao et al. [12] and fitted a logistic function to obtain equations of α_1 and α_2 as functions of retinal illuminance, l_{tr}:

$$\alpha_i(l_{tr}) = \frac{y_i}{1+e^{b_i(l_{tr}-0.62)}}, \quad i \in 1,2 \tag{2}$$

where, $y_1 = 0.2053$ and $y_2 = 0.7247$ represent the maximum values and $b_1 = 6.065$ and $b_2 = 8.465$ are the rates of change of rod weighting parameters α_1 and α_2 respectively. Note that the weight of rods' contribution to both L and M cones is equal to α_1 for any given retinal illuminance, while that to the S cones is α_2. Fig. 1 shows the original data points along with our obtained fits.

The cone responses undergo a sensitivity regulation before passing to the higher-order opponent colour mechanisms [29, 30, 24]. This gain parameter amplifies the low luminance responses and suppresses high luminance responses. The product of cone responses (with rod contributions) from Eq. (1) and the regulation parameter is:

$$G(P') = \frac{P'}{(1+k_1 P'_A)^{k_2}}, \quad P' \in L', M', S', \tag{3}$$
$$P'_A \in L'_A, M'_A, S'_A$$

where P' represents the cone responses of the stimuli with rod response added (Eq. (1)), and P'_A the corresponding coordinates (adapting luminance and chromaticity) for the background. Because we are interested in very small, just noticeable colour differences, we assume here $P'_A = P_A$. The commonly used values of k_1 is 0.33 (trolands) and for k_2 is 0.5 (unit-less) [29, 12]. We will use the modified LMS+R responses from Eqs. (1)-(3) for our proposed colour difference formulae as they represent the perceptual response of our human visual system.

One of the measures of performance of a perceptual colour difference metric is its consistency with equivalent visual data. Some of the commonly used metrics to measure correlation between colour difference formulae and perceptual error difference include the PF/3 [31, 32, 33], V_M [34], and STRESS [35]. We will use $PF/3$ and the $STRESS$ index to evaluate our LMSR error metric. The range of $PF/3$ is $[0,\infty]$, and that of $STRESS$ is $[0,1]$. For both uniformity metrics, a value of 0 indicates perfect agreement with visual performance. The values increase with increased deviation from perceptual data.

Experiment

Colour difference metrics

To illustrate the effect of rod contribution, we plotted in Fig. 2 the achromatic response ($L'+M'$ from Eq. (1)) for red and blue colours, corresponding to the two display primaries. The display primaries' spectral response was scaled to obtain photopic luminance from 0.01-100 cd/m^2, covering mesopic and photopic light levels. Each of the scaled spectral response was converted to the corresponding LMSR responses. The modified L', M', and S' responses were calculated using Eqs. (1)-(3) with the rods contribution taken into account. We also calculated the modified L', M', and S' responses while keeping $R = 0$ (assuming no rod response). The results are shown in Fig. 2 (b-c). Adding rod input did not make any difference to the output of the red primary. However, for the blue primary at low luminance, the responses with and without the rod input clearly deviated from each other. The responses with rod addition had higher luminance output for low light levels. The model clearly predicts the Purkinje shift, according to which the luminous efficiency curve shifts towards short wavelengths with decreasing luminance level. Consequently, short wavelength stimuli appear brighter in low luminance once the dark adaptation is complete. The model assumes full dark adaptation. We propose the following error metrics based on the model in Eqs. (1)-(3):

$$\Delta E_{LMSR} = \sqrt{(\Delta G(L'))^2 + (\Delta G(M'))^2 + (\Delta G(S'))^2} \tag{4}$$
$$\Delta E^{LMS}_{2000} = \Delta E_{2000}(f(L_1,M_1,S_1), f(L_2,M_2,S_2)) \tag{5}$$
$$\Delta E^{L'M'S'}_{2000} = \Delta E_{2000}(f(L'_1,M'_1,S'_1), f(L'_2,M'_2,S'_2)) \tag{6}$$
$$\Delta E^{G}_{2000} = \Delta E_{2000}(f(G(L'_1),G(M'_1),G(S'_1)), f(G(L'_2),G(M'_2),G(S'_2))) \tag{7}$$

where,

$$\Delta G(P') = G(P'_1) - G(P'_2), \quad P' \in L', M', S' \tag{8}$$
$$\text{and, } f(L,M,S) = M_{\mathrm{LMS}\rightarrow\mathrm{XYZ}}[L\ M\ S]^T = [X\ Y\ Z]^T \tag{9}$$

ΔE_{LMSR} in Eq. (4) is a RMSE metric for the regulated cone+rod responses in Eq. (3). Eqs. (5)-(7) use either an unmodified ΔE_{2000} colour difference formula (ΔE^{LMS}_{2000} in Eq. (5)), modified $L^*a^*b^*$ values transformed using Eq. (1) ($\Delta E^{L'M'S'}_{2000}$ in Eq. (6)), and modified regulated $L^*a^*b^*$ values transformed using Eq. (3) (ΔE^{G}_{2000} in Eq. (7)).

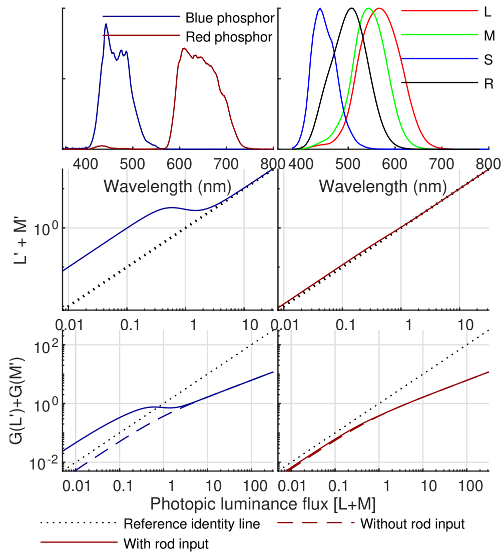

Figure 2. *Demonstration of Purkinje shift using the modified photoreceptor response model. The blue and red coloured lines correspond to responses for blue and red display primary respectively. Top row (left): Emission spectra of blue and red display primaries. Top row (right): Spectral sensitivities of cones and rods. Middle row: Luminance from modified long and medium cone response L'+M' from Eq.* (1)*. Bottom row: Luminance from regulated long and medium cone response G(L')+G(M') from Eq.* (3)*.*

Table 1. Summary of datasets

Dataset	Luminance (cd/m^2)	Spatial frequency (cpd)	Size (deg^2)	Backgrounds	Colour directions
HDR CSF [37]	0.002-10,000	0.125, 0.25, 0.5	0.78-50.26	1 (D65 grey)	3 (black-white, red-green, lime-violet)
HDRVDP CSF [38]	0.002-150	0.125, 0.25, 0.5	7.07	1 (D65 grey)	1 (black-white)
CC CSF [39]	8.8-72	0.06, 0.12, 0.24, 0.48	271.72	5 (grey, red, blue, green, yellow, blue)	6 (0, 40, 70, 100, 120, and 150 ° in $u'v'$ space)

Validation with contrast sensitivity datasets

Since traditional colour difference datasets based on patches [36], are missing samples at mesopic light levels, we test our metrics instead on three contrast sensitivity datasets. The three datasets are listed in Table 1 and number of data points at each luminance is presented in Figure 4.

Contrast sensitivity is the inverse of cone contrast threshold, which is the minimum required contrast for the stimuli to be just visible. The cone contrast is used to represent contrasts in cardinal colour directions (as opposed to only achromatic Michelson contrast). It is the magnitude of the vector of individual L, M, and S Michelson contrasts [37]. The stimuli used in the datasets are sine-wave gratings with a Gaussian window as shown in Fig. 3. We assume that the colour difference between the peak and the trough of a Gabor patch at the detection threshold forms a unit just noticeable difference (JND) in colour. For our validation purposes, we only use the low spatial frequency (Table 1) stimuli since we are interested in developing a colour metric for large stimuli. This is also important for comparison because we tested our metric against ΔE_{2000} which is a metric for larger stimuli.

We use the spectral response of the displays used in to measure each dataset to calculate the LMSR coordinates of the background colour, C_0, using Smith and Pokorny cone fundamentals and CIE V'. The threshold contrast is a function of the change in LMSR coordinates with respect to the background coordinates. We calculate the change in LMSR to obtain the LMSR coordinates of the peak and trough of the stimulus as follows:

$$\begin{aligned} C_1 &= [L_0+\Delta L, M_0+\Delta M, S_0+\Delta S, R_0+\Delta R], \\ C_2 &= [L_0-\Delta L, M_0-\Delta M, S_0-\Delta S, R_0-\Delta R] \end{aligned} \quad (10)$$

where, L_0, M_0, S_0, R_0 are the background coordinates, ΔL, ΔM, ΔS and ΔR are the differences in cone and rod responses corresponding to the threshold contrast, and C_1 and C_2 are pairs of LMSR coordinates with 1 JND perceptual difference.

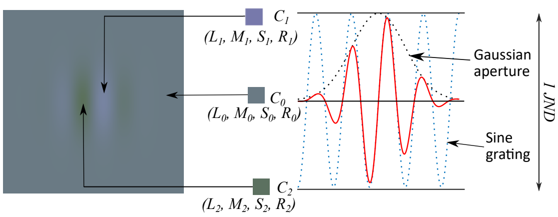

Figure 3. Left: An example CSF stimulus modulated in lime-violet direction against a grey background. Right: 1D representation of the stimulus on the left. The peaks of the red curve depict the contrast of blue and yellow areas of the grating against the background.

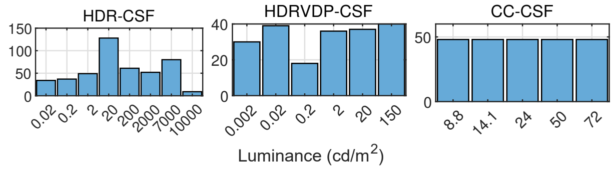

Figure 4. Number of data points across luminance levels in each dataset

Each pair of C_1 and C_2 in all three datasets is converted to the modified photoreceptor responses using Eqs. (1)-(3), transformed to the equivalent $L^*a^*b^*$ responses and then plugged in the error difference formulae in Eqs. (4)-(7) accordingly. We assumed that the background of the stimuli is shown at the mid-grey level of a display, which corresponds to the luma value of 0.5. In linear units the background luminance becomes $0.5^{2.2} = 0.2176$ which is about 5 times lower than the luminance value of the display white point. Thus, the white point of ΔE_{2000} formulae was set as a D65 white with 5 times higher luminance than the pairs tested. The $PF/3$ and $STRESS$ indices were calculated for the error results. The perceptual difference was assumed to be 1 unit for all colour pairs.

Results

We evaluated the performances of the colour difference metrics defined in Eqs. (4)-(7) using the three datasets described in Table 1.

Metric 1: Euclidean LMS distance (ΔE_{LMSR})

Figure 5 shows the error distributions across luminance levels from ΔE_{LMSR} for all the datasets. A well-performing colour difference metric should result in possibly similar colour difference predictions both within and across the luminance levels. The predicted ΔE_{LMSR} colour differences values vary greatly between low and high luminance levels. The errors and their spread increase with increasing luminance levels for all three datasets. For the high dynamic range datasets, HDR-CSF and HDRVDP-CSF, the differences between the low and high luminance stimuli are particularly large as shown in Figure 5. This suggests that ΔE_{LMSR} does not account for the effect of luminance on the colour discrimination.

Metric 2: CIEDE2000 (ΔE_{2000}^{LMS})

The colour differences from ΔE_{2000}^{LMS} metric are shown in Figure 6. Contrary to the previous metric ΔE_{LMSR}, the predicted colour differences are much larger for low luminance stimuli. The colour differences are much more consistent across pho-

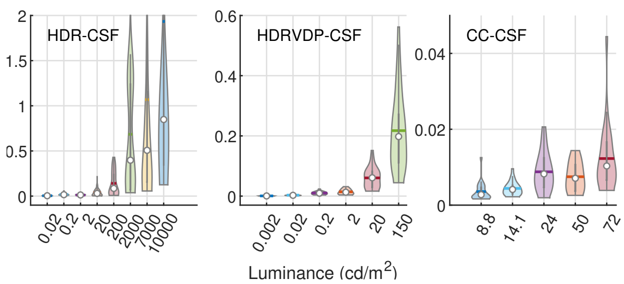

Figure 5. ΔE_{LMSR} error predictions for 1 JND colour difference data points from the three dataset. The violin shapes show the smoothed kernel density at each error value. The white circles are the median and the horizontal lines are the mean values. The vertical grey lines show the interquartile range of the errors. The same notation is used in the subsequent violin plots. A well-performing colour difference metric should result in similar error values across the luminance levels (the mean and median values would approximately lie on a straight line).

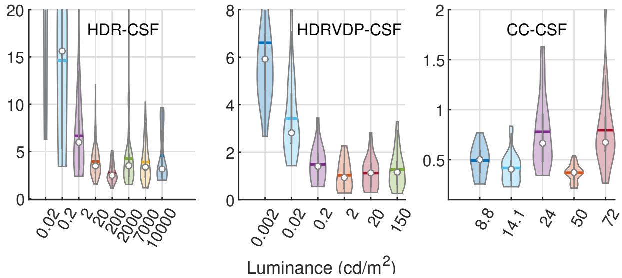

Figure 6. ΔE_{2000}^{LMS} *error predictions for 1 JND colour difference data points from the three dataset. Same notation as Figure 5*

topic and high mesopic luminance levels (≥ 2 cd/m^2). This is expected as ΔE_{2000} was fitted to mostly photopic data and was not meant to be used for low luminance colours. The colour difference values are the most consistent for the CC-CSF dataset, which contains only stimuli at the photopic luminance levels.

Metric 3: CIEDE200 + rod intrusion ($\Delta E_{2000}^{L'M'S'}$)

The $\Delta E_{2000}^{L'M'S'}$ metric differs with ΔE_{2000}^{LMS} in only the low luminance range. Therefore, the performance of $\Delta E_{2000}^{L'M'S'}$ at photopic levels (Figure 7) is the same as ΔE_{2000}^{LMS} (Figure 6) because the rod inputs in Eq. (1) only contribute to cone responses in scotopic range. The modified scotopic errors are much more consistent for this metric than for ΔE_{2000}^{LMS} showing that the modelled rod intrusion can much improve colour predictions in the scotopic range. This can be observed for low luminance level colour differences in both HDR-CSF and HDRVDP-CSF datasets. Their is no difference in performance between ΔE_{2000}^{LMS} and $\Delta E_{2000}^{L'M'S'}$ for CC-CSF dataset, since the dataset has photopic stimuli only and there is no rod contribution.

Metric 4: CIEDE2000 + rod intrusion + adaptation

ΔE_{2000}^{G} is based on the gain-regulated cone responses which are suppressed for high luminances and amplified for low luminances. Such suppression results in worse consistency across the luminance levels. The effect is clearly shown in Figure 8 for the datasets HDR-CSF and HDRVDP-CSF. This is most likely because ΔE_{2000} formula already accounts for the change in colour sensitivity across photopic luminance levels. Because the range of luminance of stimuli in CC-CSF dataset is relatively small, we do not observe a marked difference in performance for different luminance levels.

Perceptual uniformity

The perceptual uniformity of all four tested colour spaces is measured in terms of $PF/3$ and $STRESS$ indices and reported in Table 2. $\Delta E_{2000}^{L'M'S'}$ shows the best performance (smallest indices) across all the datasets both in terms of $PF/3$ and $STRESS$ values. This indicates that $\Delta E_{2000}^{L'M'S'}$ is the best predictor of 1 JND colour differences from the CSF datasets. For CC-CSF dataset,

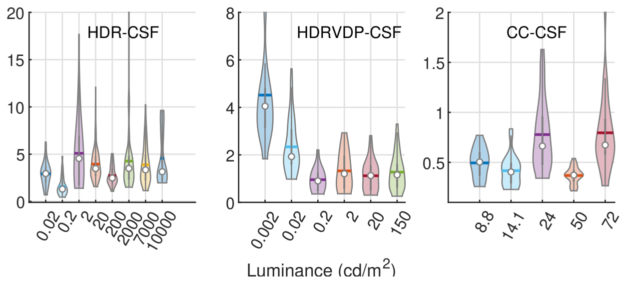

Figure 7. $\Delta E_{2000}^{L'M'S'}$ *error predictions for 1 JND colour difference data points from the three dataset. Same notation as Figure 5*

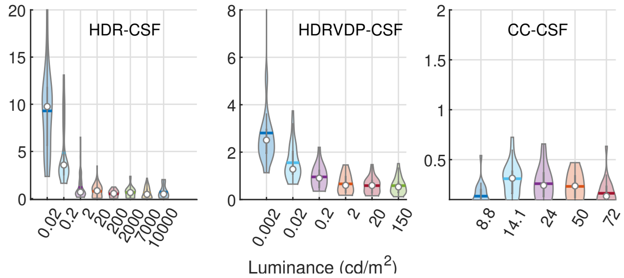

Figure 8. ΔE_{2000}^{G} *error predictions for 1 JND colour difference data points from the three dataset. Same notation as Figure 5*

ΔE_{2000}^{LMS} and $\Delta E_{2000}^{L'M'S'}$ show the exact same performance, which is expected because the dataset only consists of photopic stimuli, and the two metrics are effectively the same for high luminance levels. The metric ΔE_{LMSR} shows the worst performance in terms of perceptual uniformity almost across all conditions. This is a relatively simple metric based on RMSE of modified cone and rod responses and can not compete with more sophisticated ΔE_{2000} based colour difference metrics which take a number of human perceptual attributes into account.

Table 2. Performance comparison of the four metrics with respect to $PF/3$ and $STRESS$ uniformity measures. The values are compared for each column and the best and the worst performing metrics are highlighted with green and red colours respectively.

Colour difference metric	Datasets HDR-CSF		HDRVDP-CSF		CC-CSF	
	$PF/3$	$STRESS$	$PF/3$	$STRESS$	$PF/3$	$STRESS$
ΔE_{LMSR}	380.3	0.93	359	0.86	84.5	0.63
ΔE_{2000}^{LMS}	126.1	0.83	117	0.73	53.9	0.48
$\Delta E_{2000}^{L'M'S'}$	68.1	0.55	95.5	0.66	53.9	0.48
ΔE_{2000}^{G}	295.7	0.86	104	0.69	106	0.58

Conclusions

Our proposed modification of the standard ΔE_{2000} colour difference formula tackles its limitation in the low luminance levels. We propose the modification of L*a*b* coordinates for low luminance levels to include rod contributions. The rod contribution model by Cao et al. [12], based on colour matching data, shows also good predictions for threshold discrimination data from the CSF datasets. We demonstrated that the Purkinje shift and the change from photopic to mesopic and scotopic luminous efficiency curves is well-captured by the model. The $\Delta E_{2000}^{L'M'S'}$ colour difference formula shows improved colour difference predictions in mesopic and scotopic luminance ranges.

The metric currently does not account for chromatic adaptation, which is a potential future direction for this work. It also does not take time-scale of adaptation into account either which could be interesting given the different adaptation speeds of rods and cones. $\Delta E_{2000}^{L'M'S'}$ could be used as a custom loss function for task that require matching or reproduction of low luminance colours. It is also a good tool to evaluate the colour reproduction at low luminance levels, for example, in case of HDR displays.

Acknowledgements

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