

Automatic certification and interactive theorem proving: An impossible combination ?

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Be provocative - Disclaimer

- Provocative statements - take them with a pinch of salt



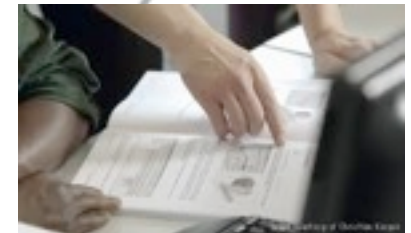
About the speaker

- Limited knowledge about certification
- Interactive theorem proving systems
 - mainly ACL2
 - Isabelle on one project (1 year)
 - sharing office with Coq user
- Application domains
 - mainly on-chip interconnects
 - time-triggered hardware
- Other research projects
 - model-based testing with (Timed) LTS
 - real-time model-checking using UPPAAL
 - application to Wireless Sensor Networks



Automatic certification

- Automatic
 - tools - easy to use and efficient
 - no human interaction - scalability
- Certification
 - high-quality **design process**
 - less bugs at the end



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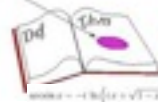
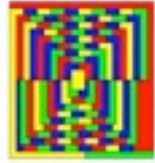
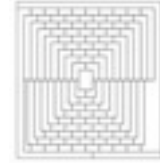
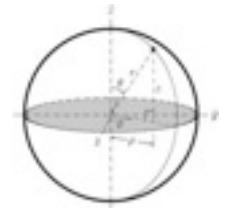


Interactive Theorem Proving

- Interactive
 - hard thinking
 - complex tools
- Theorem Proving
 - complex, tedious proofs
 - bug free but expensive
 - deep insight in products
 - true correctness

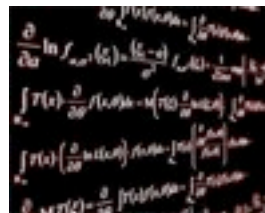
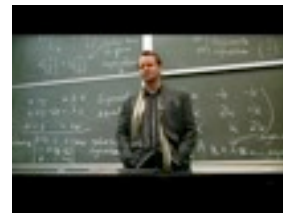
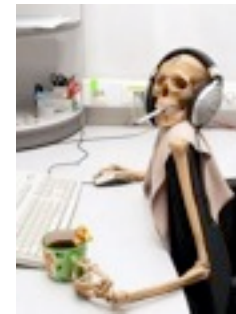


Why Logic?
 ICD must be applicable to any (intellectual) problem:
 1. Theorem proving
 2. Universality
 3. Natural language
 If this sounds too easy to be true, you're right!



$$\begin{aligned}
 & \text{lemma } \rightarrow \ln(x + \sqrt{x^2 - 1}) \\
 & \text{lemma } \rightarrow \ln(x + \sqrt{x^2 - 1}) - \frac{1}{2} + \ln(x + \sqrt{x^2 - 1}) - \frac{1}{2} - \text{lemma } \\
 & \text{lemma } \rightarrow \frac{1}{2}(\ln(x + \sqrt{x^2 - 1}) - \ln(x - \sqrt{x^2 - 1})) \\
 & \text{lemma } \rightarrow \ln\left(\frac{x + \sqrt{x^2 - 1}}{x - \sqrt{x^2 - 1}}\right) - \ln\left(\frac{x + \sqrt{x^2 - 1}}{x - \sqrt{x^2 - 1}}\right) \\
 & \text{lemma } \rightarrow \ln\left(\frac{x + \sqrt{x^2 - 1}}{x - \sqrt{x^2 - 1}}\right) - \ln\left(\frac{x + \sqrt{x^2 - 1}}{x - \sqrt{x^2 - 1}}\right) + \frac{1}{2} - \frac{1}{2} - \text{lemma } \\
 & \text{lemma } \rightarrow \ln\left(\frac{x + \sqrt{x^2 - 1}}{x - \sqrt{x^2 - 1}}\right)
 \end{aligned}$$

$$\frac{\bigvee_{i \in A} L_i \quad \bigvee_{i \in B} L_i}{\bigvee_{i \in C} \text{Subst}(\theta, L_i)}$$



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Certification vs. Theorem Proving

- Automatic Certification
 - scalability, ease of use
 - stamp about system quality
 - bug removal by good design process
 - low injection + good hunting
- Interactive Theorem proving
 - tedious proofs, complex tools, "intelligence required"
 - proof of (total) correctness
 - about systems not their design process
 - can prove tools correct
 - tools with insight and true correctness

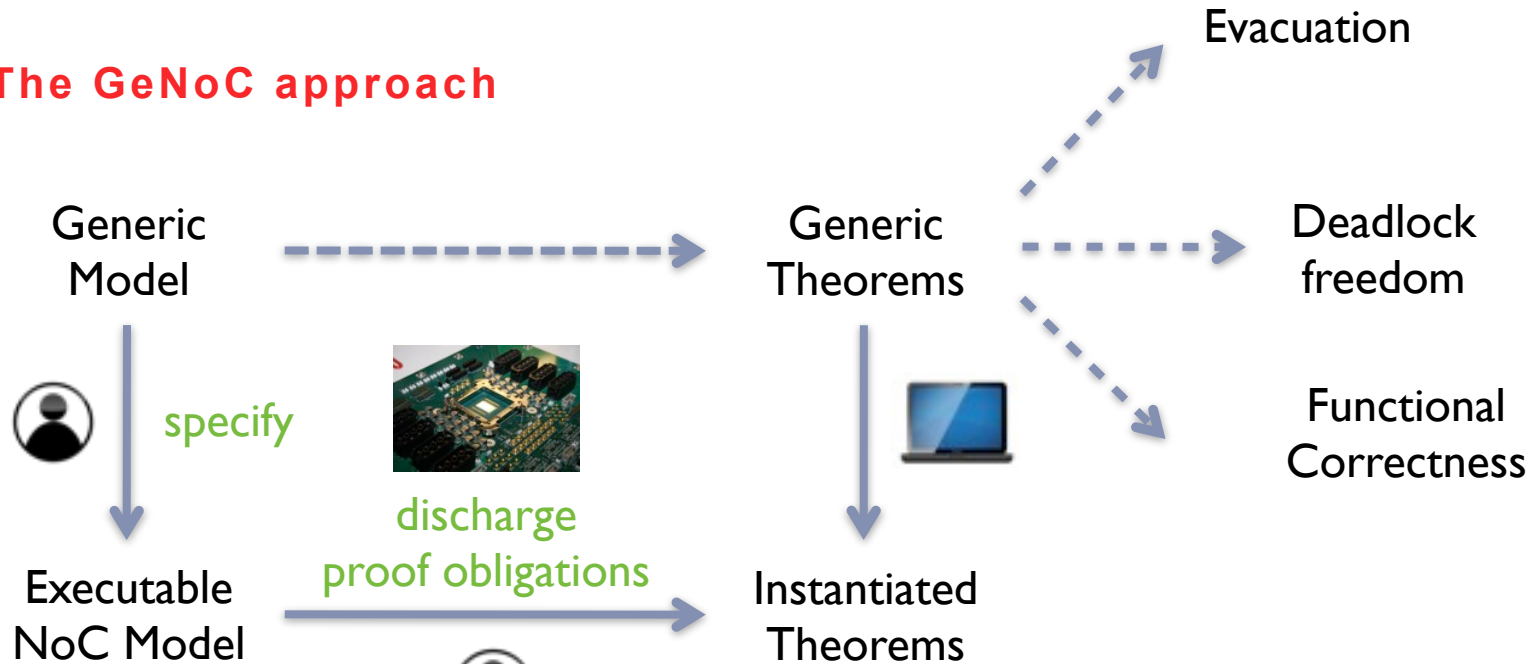


Bugs and NoCs

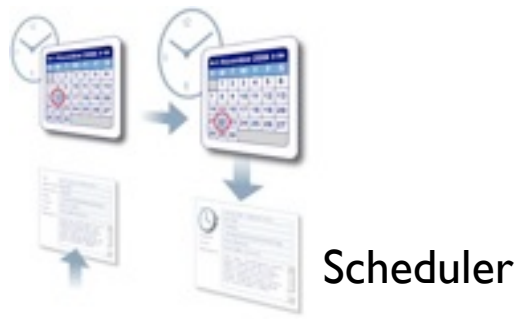
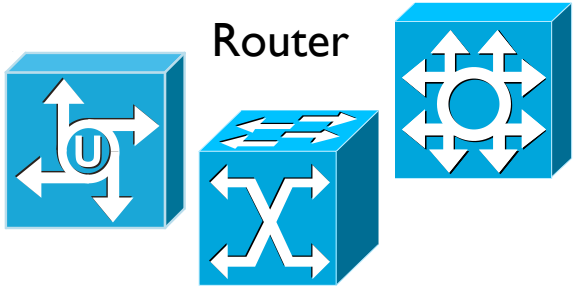
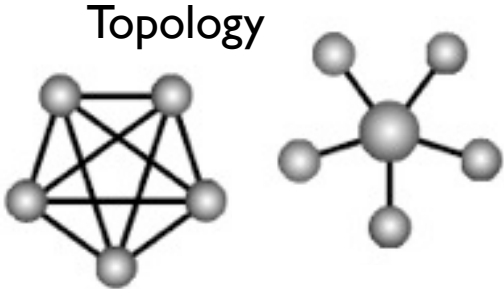
- Bug hunting - Model Checking&Co
 - algorithmic technique - automation
 - routine in HW industry
 - find subtle bugs
 - state-explosion problem - small, fixed size systems
- A mosquito-net for NoCs - The GeNoC approach
 - a *generic* model for reasoning about NoCs
 - highly *parametric*
 - generic definition of *correctness theorems*
 - identify *constraints* sufficient to prove the theorems
 - only need to check constraints on *particular instances*



The GeNoC approach



The Generic Model: Constituents



Formal model of network architectures

Let σ be a configuration containing a state and messages
Let M be a set of messages to be sent over the NoC

σ iff $\sigma.M = \emptyset$ // empty list of messages

GeNoC (σ) = σ iff **deadlocked**(Routing(Injection(σ)))

GeNoC(**Scheduling**(**Routing**(**Injection**(σ))))

Advance of
one hop if possible

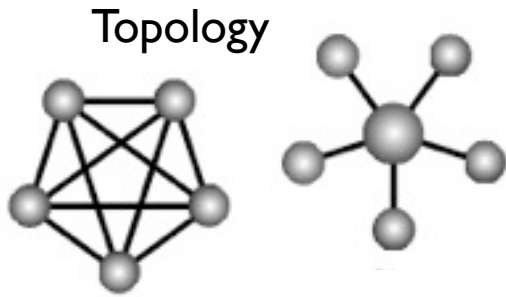
Routes
from current to destination

inject messages



The Generic Model: Proof obligations (or constraints)

Local constraints sufficient to prove *global* generic theorems.

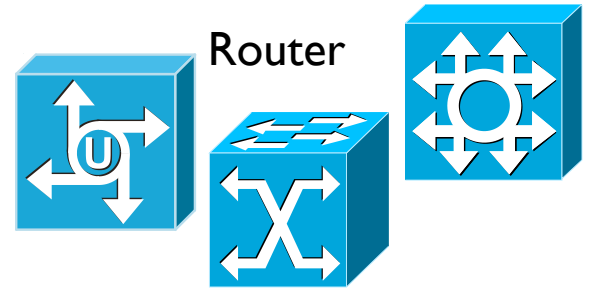


“Sinks have no outgoing edges”



Scheduler

“A message moves, unless it is stuck”



Type: “ $R : P \times P \rightarrow P$ ”

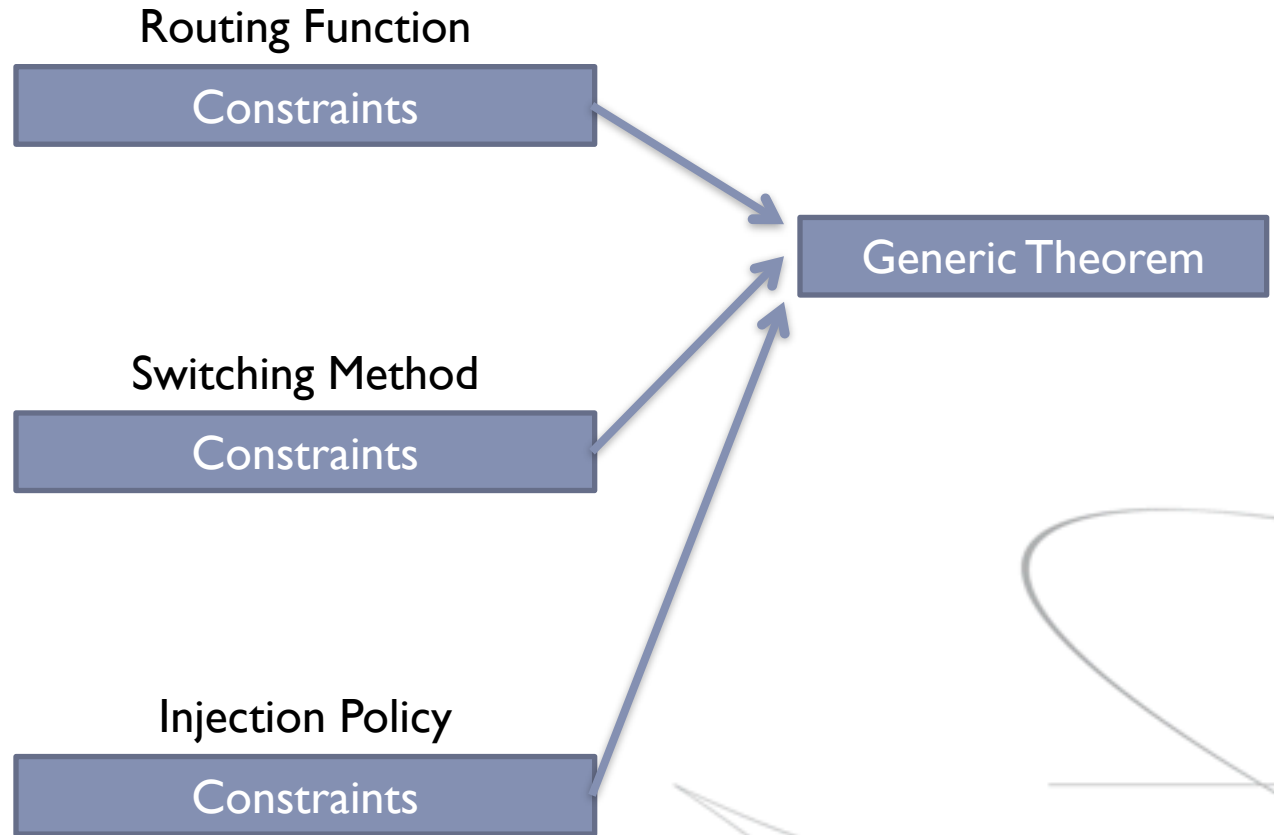
Acyclic dependency graph

Inject if network is empty

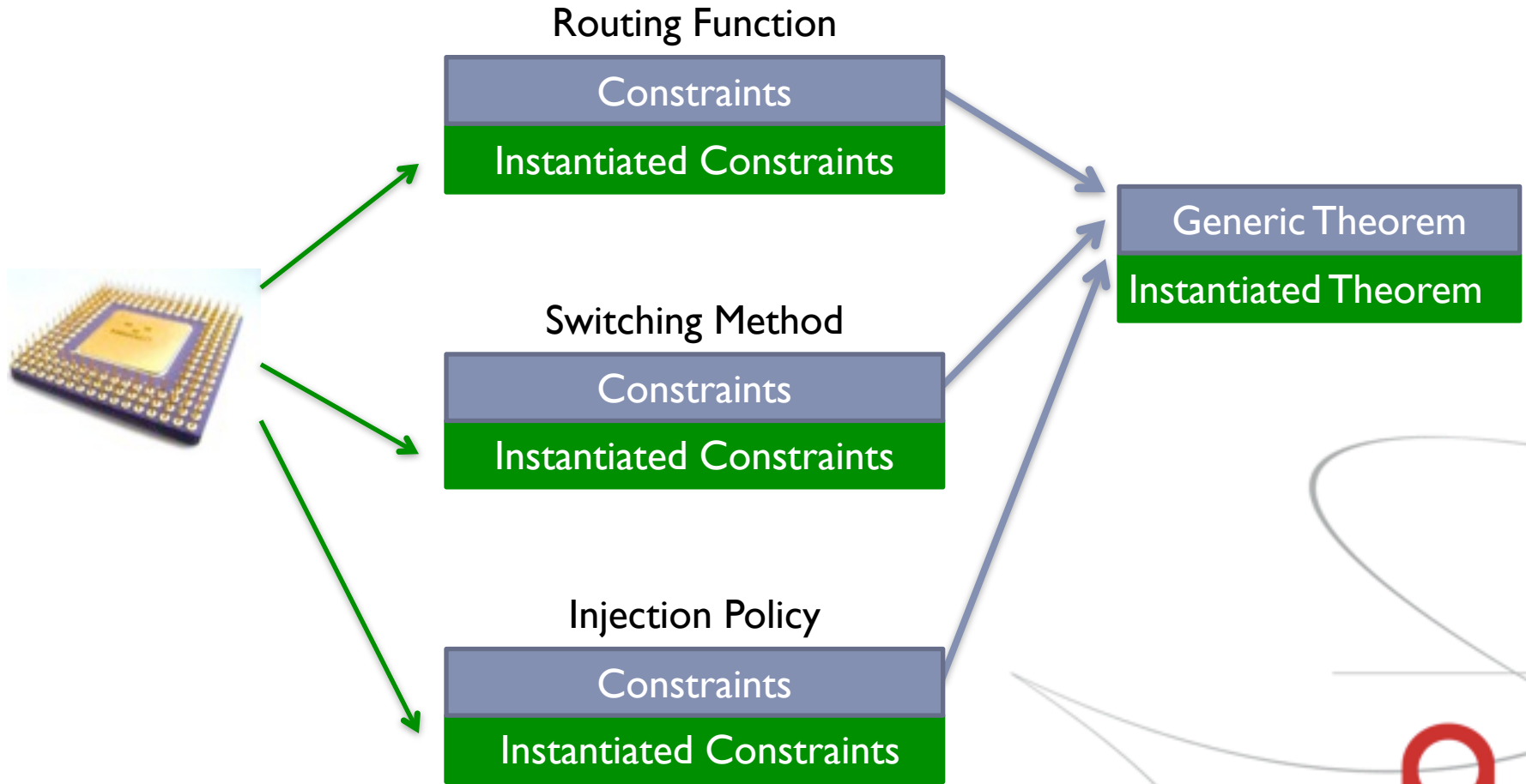
Injection



The Generic Model: Generic theorems



The Generic Model: Generic theorems



GeNoC Theorem (1): Functional correctness

- Functional correctness
 - *if a message reaches a destination, it reaches its expected destination without modification of its content*
 - Note: trivially holds if no message reach a destination
- Main proof obligations on routing
 - last of route from s to d is d
 - route computation terminates
 - length of routes (opt)
- Proof obligation on scheduling
 - mutual exclusion of scheduled and delayed messages
 - union of scheduled and delay contains exactly all messages (no spontaneous generation of new messages)



GeNoC Theorem (2): Deadlock freedom

- A network is deadlock-free iff
 - *there is no reachable deadlocked configuration*
 - deadlocked configuration = configuration where all messages are stuck
- Main proof obligations on routing
 - acyclic resource dependency graph (deterministic)
 - escape for all cycles (adaptive)
 - consistency between dependency graph and routing function
- Main proof obligation on scheduling
 - next-hop based scheduling policy



GeNoC Theorem (3): Evacuation

σ iff $\sigma.M = \emptyset$ // empty list of messages

GeNoC (σ) =

σ iff **deadlocked**(Routing(Injection(σ)))

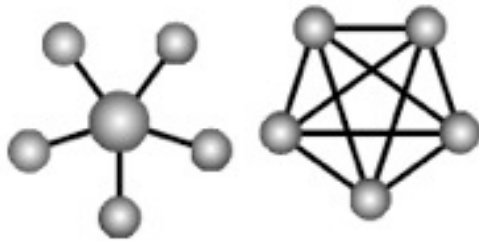
GeNoC(**Scheduling**(Routing(**Injection**(σ))))

- Evacuation theorem
 - *all messages eventually leave the network*
- Main proof obligations on function GeNoC
 - **function GeNoC terminates**
 - generic termination measure
- Main proof obligation on scheduling
 - **decreases measure if no deadlock**
- Main proof obligation on routing
 - **deadlock-free routing**
- Main proof obligation on injection
 - **decreases measure if when network is empty**



Overview of applications of GeNoC

Topology

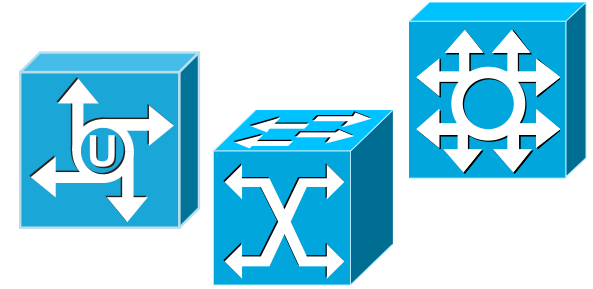


2D-mesh

Octagon

Router

- xy routing
- double Y routing
- Octagon routing
- Hot potato routing



“R : P x P → P”

Injection

- time dependent
- immediate



Scheduler

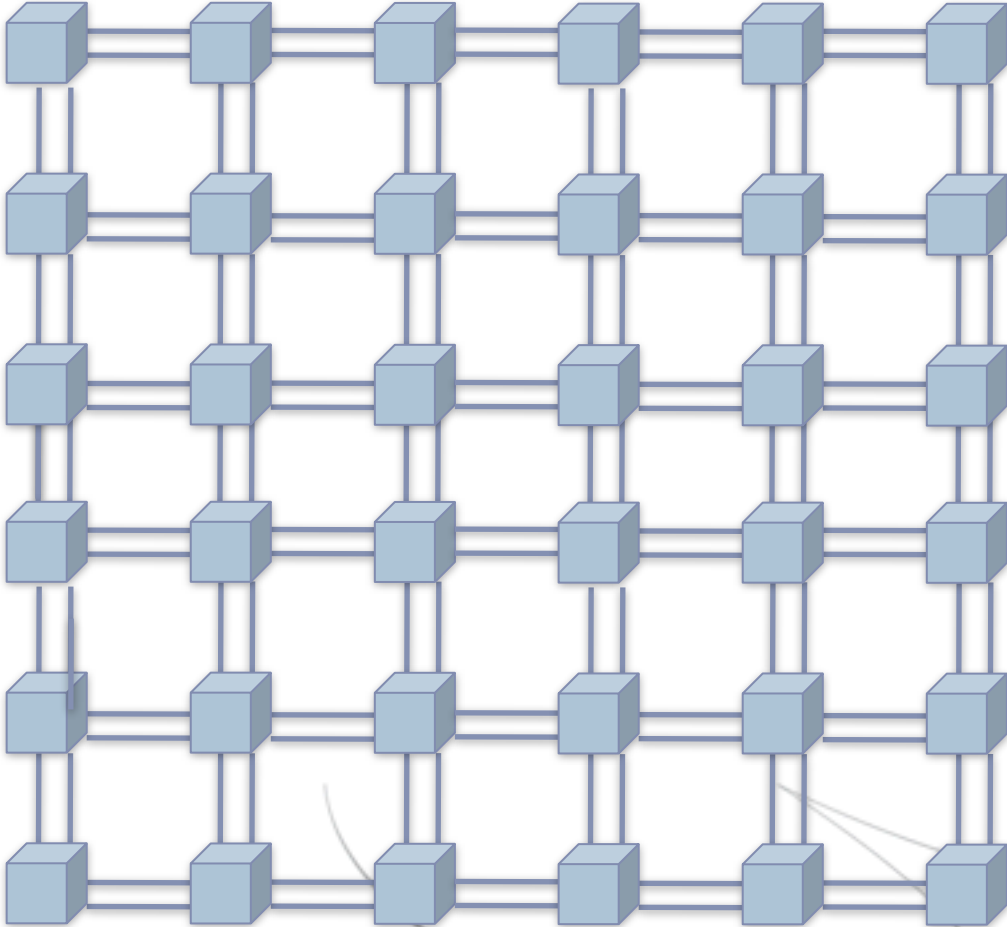
- circuit switching
- packet switching
- wormhole switching
- bus arbitration



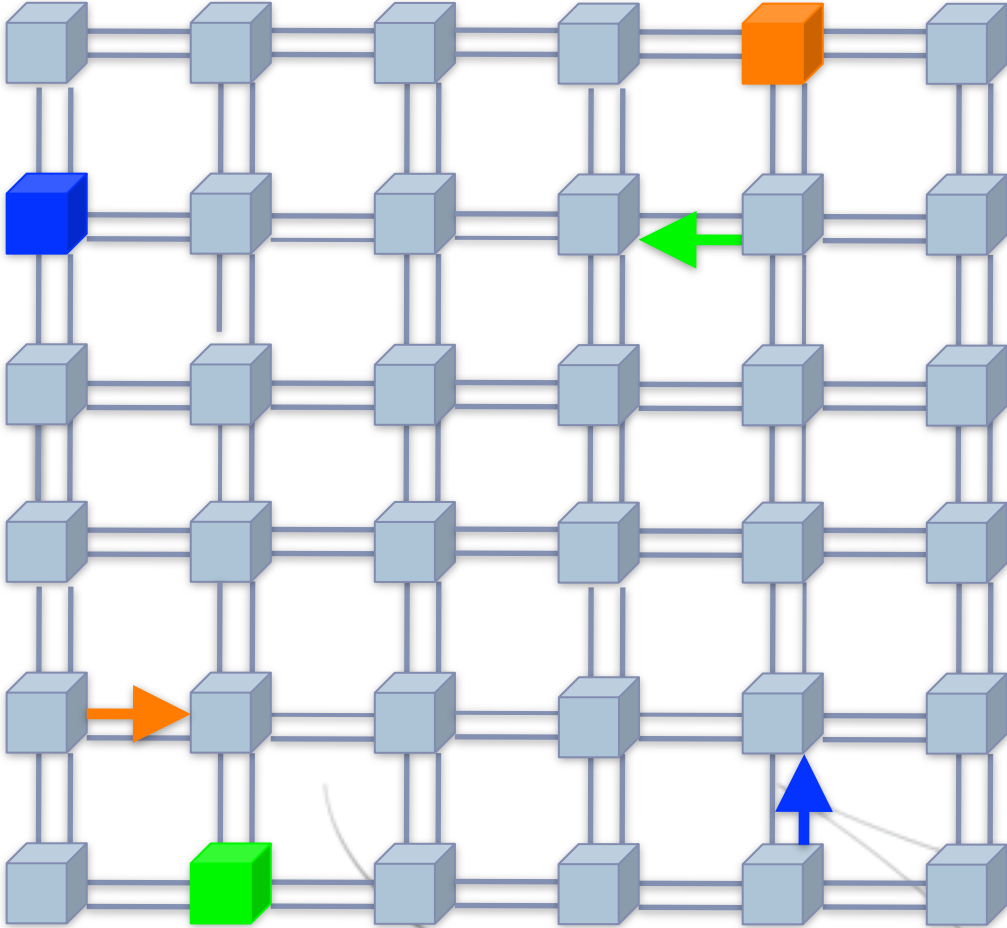
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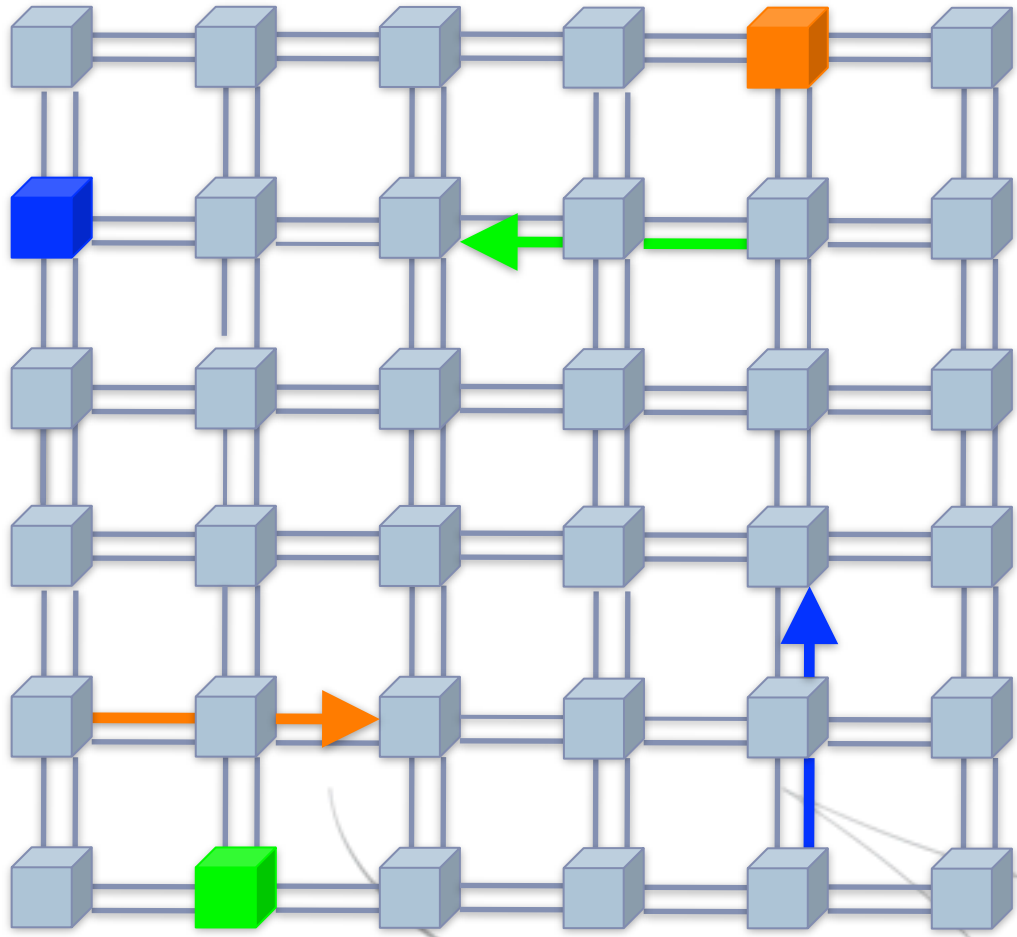
Deadlock



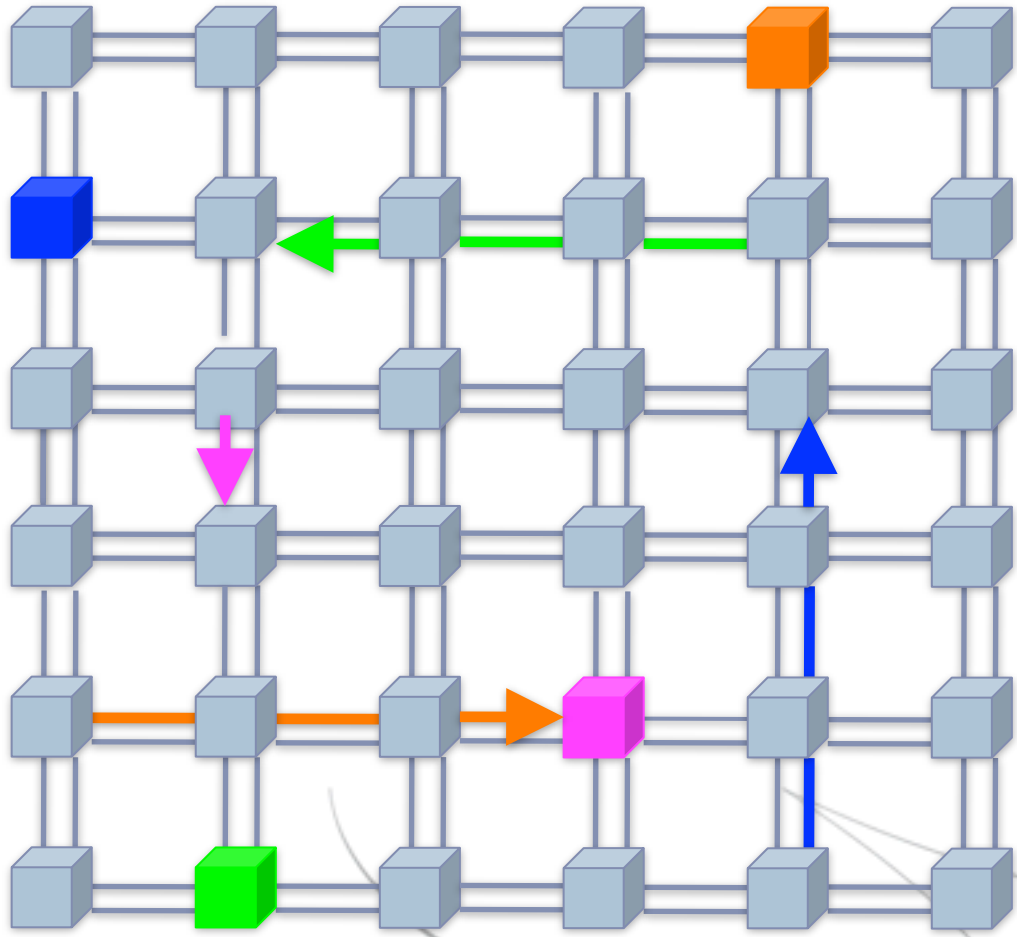
Deadlock



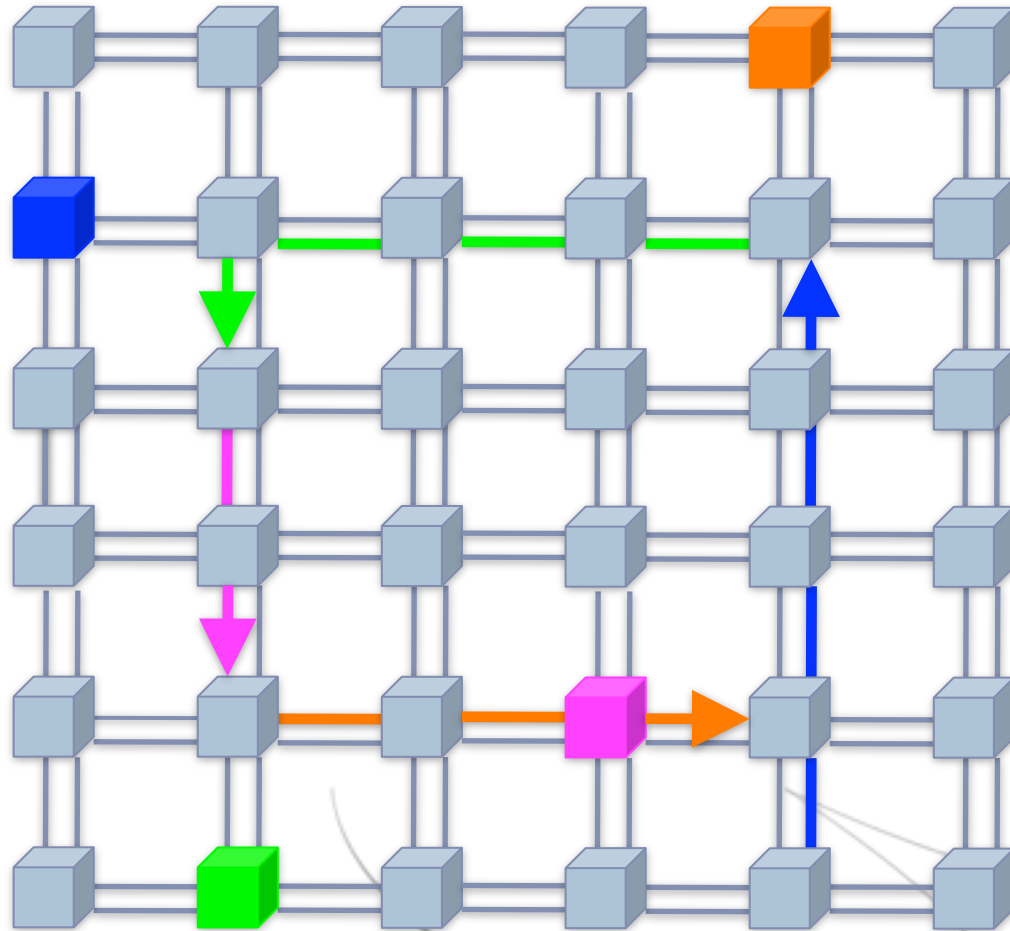
Deadlock



Deadlock



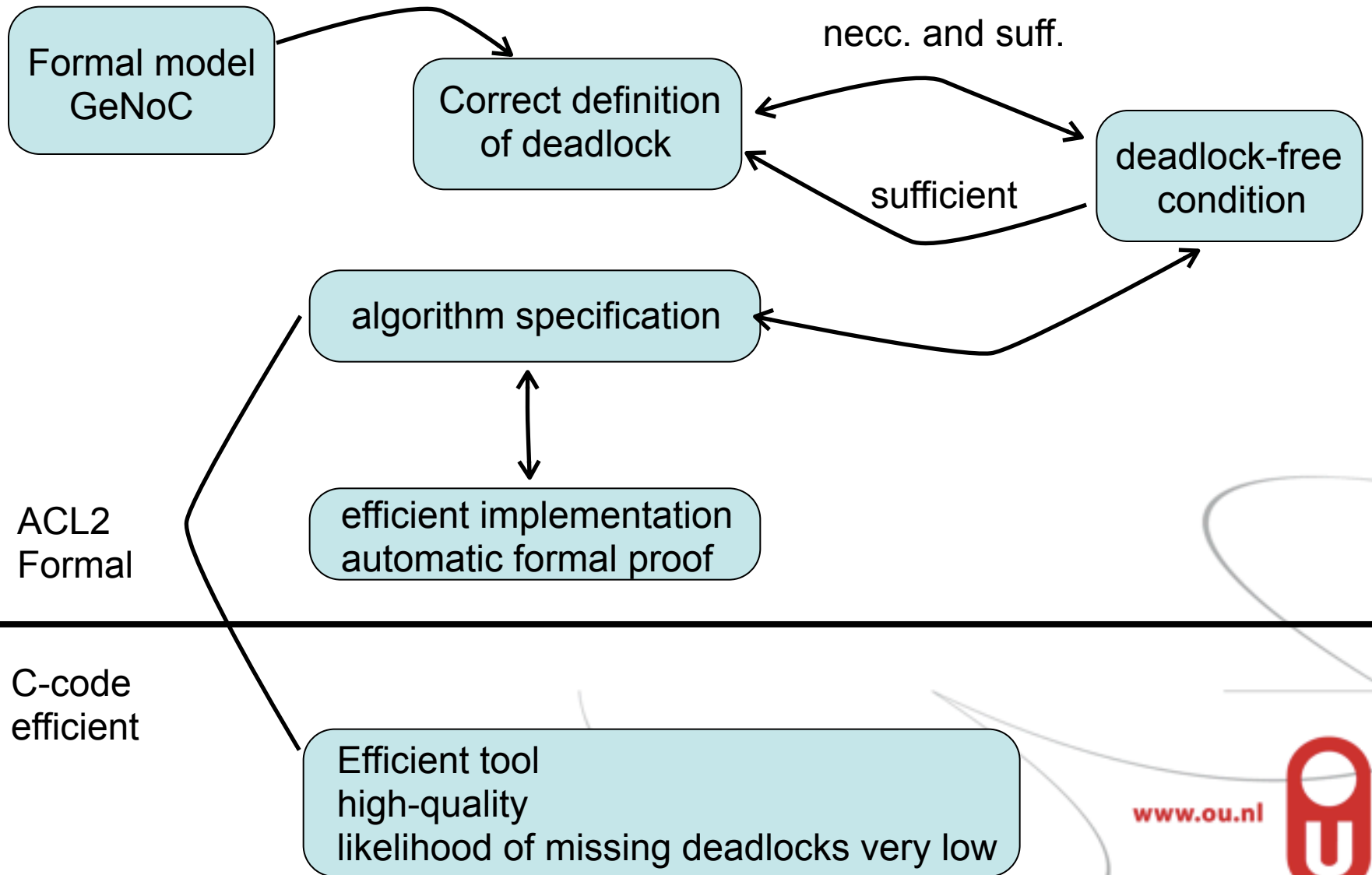
Deadlock



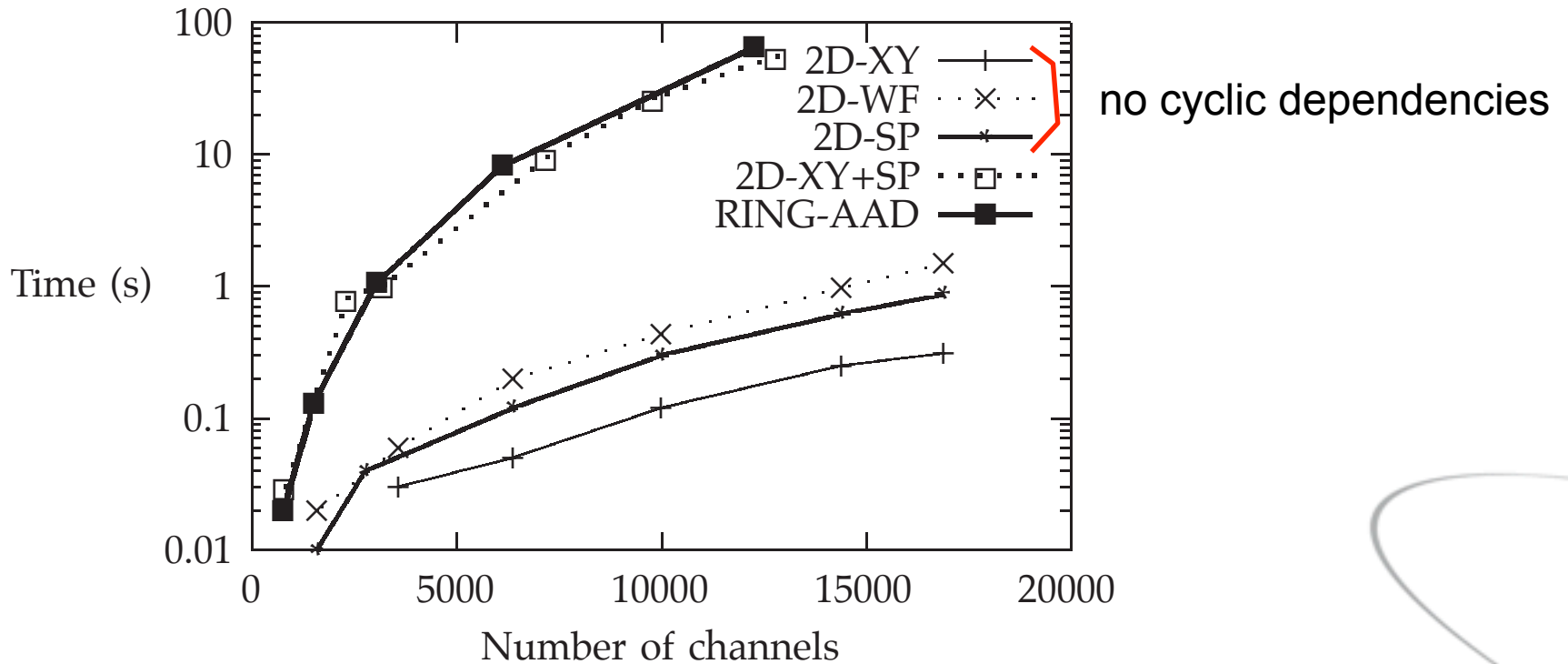
Deadlock is an emerging property



Deadlock verification - The big picture



Automatically checking sufficient condition (C-code)



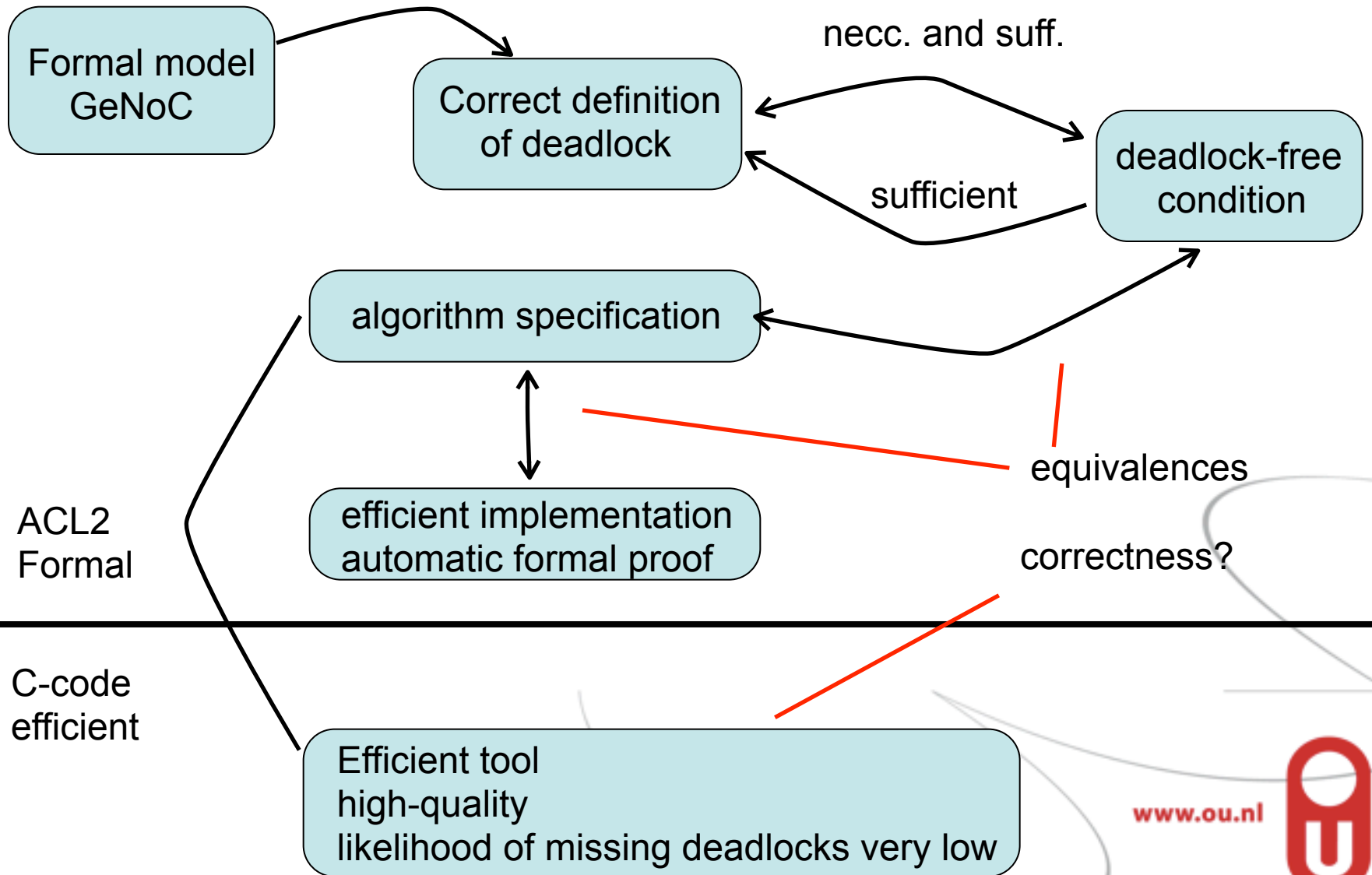
Condition as strong as Duato's one and its variations

10000s of channels in 100 seconds

Time complexity in $O(N^3)$ where N = number of nodes



Deadlock verification - Unintended functionality



Our approach

- Develop formal theory of the domain (e.g. NoCs)
 - identify components and their interactions
- Prove general theorems in this theory
 - what are the interesting global properties (no deadlock)
- Extract proof obligations on the components
 - what is important to know about each component
- Develop verified algorithms checking the POs
- Implement these algorithms
 - within the logic of an ITP (e.g. ACL2)
 - every run of the algorithm is a **formal** proof
 - in standard languages (e.g. C)
 - high-quality "bug hunter"



Reflection and side effects of formal efforts

- Found a (small) flaw in seminal paper of Duato
 - work was a breakthrough
 - paper 250 cites on GS
 - flaw in other paper with 630 cites and book with 1450 cites
 - flaw in many papers inspired by Duato's work
- Correcting the flaw makes problem co-NP-complete
 - previous work claimed polynomial solution
 - made same mistake as Duato
- Theorem proving ensure correctness of algorithms
 - lots of corner cases
 - hard to debug when 1 single incorrect trace
- *In-depth understanding* of the issue



Conclusion

- Verified certifiers
- ITP is used to develop general theories and verified algorithms
- Verified algorithms implemented as high-quality "bug hunters"
 - likelihood of bugs after running the certifier
 - formal proof when running verified code (ACL2)
- Domain specific
 - static (on-chip) interconnection networks
- Very efficient
 - proven correct (sound)
 - linear or polynomial when possible
 - boundary to co-NP-complete well-defined

