Names and Symmetry in Computer Science

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LICS 2015 Tutorial

An introduction to
nominal techniques
motivated by

Programming language semantics

Automata theory

Constructive type theory

Compositionality in semantics

> The meaning of a compound phrase should be a well-defined function of the meanings of its constituent subphrases.

Although one can define behaviour of whole

C/Java/OCaml/Haskell... <u>programs</u>

(e.g. via an abstract machine) it's often more illuminating, but harder, to give a semantics for individual

C/Java/OCaml/Haskell... <u>statement-forming constructs</u>

as operations for composing suitable mathematical structures.

Compositionality in semantics

The meaning of a compound phrase should be a well-defined function of the meanings of its constituent subphrases.

Denotational semantics are compositional:

- Each program phrase P is given a denotation $[\![P]\!]$—a mathematical object representing the contribution of P to the meaning of *any* complete program in which it occurs.
- The denotation of a phrase is a function of the denotations of its subphrases.

Question: are these OCaml expressions behaviourally equivalent?

$$\begin{array}{l}\texttt{let}\,a = \texttt{ref}\,42\ \texttt{in}\\ \texttt{fun}\ x \to a := (!a + x)\,;\\ \qquad\qquad !a\end{array}$$

$$\begin{array}{l}\texttt{let}\,b = \texttt{ref}(-42)\ \texttt{in}\\ \texttt{fun}\ y \to b := (!b - y)\,;\\ \qquad\qquad -(!b)\end{array}$$

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Answer: yes, but devising denotational semantics that exactly match the execution behaviour of

> higher-order functions + locally scoped names (of storage locations, of exceptions, of ...)

has proved hard.

Current state-of-the-art: nominal game semantics (see forthcoming tutorial article by Murawski & Tzevelekos in *Foundations & Trends in Programming Languages*).

Dependence & Symmetry

What does it mean for mathematical structures [needed for denotational semantics] to

$$\begin{cases} \text{depend upon some names?} \\ \text{be independent of some names?} \end{cases}$$

- Conventional answer: parameterization (explicit dependence via functions).
  Can lead to 'weakening hell'.

Dependence & Symmetry

What does it mean for mathematical structures [needed for denotational semantics] to

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- Conventional answer: parameterization (explicit dependence via functions).
  Can lead to 'weakening hell'.
- *Nominal techniques* answer: independence via invariance properties of symmetries.

Name permutations

- $\mathbb{A}$ = fixed countably infinite set of atomic names (a,b,...)
- $\mathbf{S}_{\mathbb{A}}$ = group of all (finite) permutations of $\mathbb{A}$

Typical elements:

$$\begin{tikzcd} a \arrow[r, bend left] & b \arrow[l, bend left] & c \arrow[dr, bend right] & & e \arrow[ll, bend right] & \cdots \\ & & & d \arrow[ur, bend right] & & \end{tikzcd}$$

(but NOT $\begin{tikzcd} f \arrow[dr, bend right] & & g \arrow[dl, bend left] \\ & h & \end{tikzcd}$)

Name permutations

- $\mathbb{A}$ = fixed countably infinite set of atomic names (a,b,...)
- $\mathbf{S}_{\mathbb{A}}$ = group of all (finite) permutations of $\mathbb{A}$
  - each π is a bijection $\mathbb{A} \cong \mathbb{A}$ (injective and surjective function)
  - π finite means: $\{a \in \mathbb{A} \mid \pi(a) \neq a\}$ is finite.
  - group: multiplication is composition of functions $\pi' \circ \pi$; identity is identity function $\mathbf{id}$; inverses are inverse functions π^{-1}.

Actions

A $\mathbf{S}_{\mathbb{A}}$-action on a set X is a function

$$\pi \in \mathbf{S}_{\mathbb{A}},\ x \in X \ \mapsto\ \pi \cdot x \in X$$

satisfying

- $\pi' \cdot (\pi \cdot x) = (\pi' \circ \pi) \cdot x$
- $\mathbf{id} \cdot x = x$

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Simple example: $\mathbf{S}_{\mathbb{A}}$ acts on sets of names $A \subseteq \mathbb{A}$ via

$$\pi \cdot A = \{\pi(a) \mid a \in A\}$$

E.g.

$$\big(\, a \quad b \,\big) \cdot \{c \mid c \neq a\} = \{c \mid c \neq b\}$$

Support – the key definition

Suppose $\mathbf{S}_{\mathbb{A}}$ acts on a set X and $x \in X$.

A set of names $A \subseteq \mathbb{A}$ supports x if for all $\pi \in \mathbf{S}_{\mathbb{A}}$

$$(\forall a \in A.\, \pi(a) = a) \;\Rightarrow\; \pi \cdot x = x$$

X is a nominal set if every $x \in X$ has a finite support.
[AMP-Gabbay, LICS 1999]

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[AMP-Gabbay, LICS 1999]

E.g. for $X = \{A \mid A \subseteq \mathbb{A}\}$,

- $\{b \mid b \neq a\}$ is infinite, but is supported by $\{a\}$
  because if $\pi(a) = a$, then π is a permutation of $\mathbb{A} - \{a\}$

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E.g. for $X = \{A \mid A \subseteq \mathbb{A}\}$,

- $\{b \mid b \neq a\}$ is infinite, but is supported by $\{a\}$
- no finite set of names supports $\{a_0, a_2, a_4, \ldots\}$ (supposing $a_0, a_1, a_2, a_3, \ldots$ enumerates $\mathbb{A}$), so $\{A \mid A \subseteq \mathbb{A}\}$ is not a nominal set
- $\{A \subseteq \mathbb{A} \mid A \text{ finite, or } \mathbb{A} - A \text{ finite}\}$ is a nominal set

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E.g. for λ-terms with (free & bound) variables from $\mathbb{A}$

$$t ::= a \mid t\,t \mid \lambda a.t \quad \text{modulo } \alpha\text{-equivalence}$$

with $\mathbf{S}_{\mathbb{A}}$-action: $\begin{cases} \pi \cdot a &= \pi(a) \\ \pi \cdot (t\,t') &= (\pi \cdot t)(\pi \cdot t') \\ \pi \cdot (\lambda a.t) &= \lambda \pi(a).(\pi \cdot t) \end{cases}$

FACT: each t is supported by its finite set of *free variables*.

Category of nominal sets, **Nom**

objects are nominal sets

= sets equipped with an action of all (finite) permutations of $\mathbb{A}$, all of whose elements have finite support

morphisms are equivariant functions

= functions preserving the permutation action.

identities and **composition**

= as usual for functions

Why use category theory?

- **equivalence of categories** from different mathematical realms (or even just functors between them) can tell us a lot.
  For example, the following are all equivalent:
  - **Nom**
  - the Schanuel topos
    (from Grothendieck's generalized Galois theory)
  - the category of named sets
    (from the work of Montanari *et al* on model-checking π-calculus)
- **universal properties** (adjoint functors) can characterize a mathematical construction uniquely up to isomorphism and help predict its properties.
  For example…

Nominal exponentials

$$\boxed{X \to_{\mathbf{fs}} Y} \triangleq$$
$$\left\{ f \in Y^X \,\middle|\, \begin{array}{l} f \text{ is finitely supported w.r.t. the action} \\ \pi \cdot f = \lambda x \to \pi \cdot (f(\pi^{-1} \cdot x)) \end{array} \right\}$$

This is characterised uniquely up to isomorphism by the fact that it give the right adoint to $(\_) \times X : \mathbf{Nom} \to \mathbf{Nom}$:

$$\frac{Z \times X \to Y}{Z \to (X \to_{\mathbf{fs}} Y)}$$

(Products in $\mathbf{Nom}$ are created by the forgetful functor to the category of sets.)

Nominal exponentials

$\boxed{X \to_{\mathbf{fs}} Y} \triangleq$

N.B. permutations have inverses

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Name abstraction

Each $X \in \mathbf{Nom}$ yields a nominal set $\boxed{[\mathbb{A}]X}$ of

name-abstractions $\langle a \rangle x$ are $\sim$-equivalence classes of pairs $(a, x) \in \mathbb{A} \times X$, where

$$
\begin{aligned}
(a, x) \sim (a', x') \;\Leftrightarrow\; & \exists\, b \,\#\, (a, x, a', x') \\
& (b\ a) \cdot x = (b\ a') \cdot x'
\end{aligned}
$$

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Freshness relation: $a \# x$ means $a \notin A$ for some finite support A for x

The freshness relation gives a
well-behaved, syntax-independent
notion of freeness, or non-occurrence.

Name-abstraction

Name abstraction $[\mathbb{A}](\_) : \mathbf{Nom} \to \mathbf{Nom}$ is right adjoint to adjoint 'separated tensor' $\_ * \mathbb{A}$

$$X * \mathbb{A} \triangleq \{(x, a) \in X \times \mathbb{A} \mid a \# x\}$$

$$\frac{X * \mathbb{A} \to Y}{X \to [\mathbb{A}]Y}$$

So $[\mathbb{A}]X$ is a kind of (affine) linear function space from $\mathbb{A}$ to X, but with great properties – e.g. it too has a right adjoint

$$\frac{([\mathbb{A}]X) \to Y}{X \to \{f \in Y^{\mathbb{A}} \mid (\forall a \in \mathbb{A})\ a \# f(a)\}}$$

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$\rightsquigarrow$ an initial algebra semantics for syntax with binders using $[\mathbb{A}](\_)$ to model binding operations with user-friendly inductive/recursive properties. . .

Initial algebras

- $[\mathbb{A}](\_)$ can be combined with $\_ \times \_$ and $\_ + \_$ to give functors $\mathbf{T} : \mathbf{Nom} \to \mathbf{Nom}$ that have initial algebras $I : \mathbf{T}\,D \to D$

$$
\begin{tikzcd}
\mathbf{T}\,D \arrow[d, "I"'] & & \mathbf{T}\,X \arrow[d, "F", "\text{for all}"'] \\
D & & X
\end{tikzcd}
$$

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$$
\begin{tikzcd}
\mathbf{T}\,D \arrow[r, dashed, "\mathbf{T}\,\hat{F}"] \arrow[d, "I"'] & \mathbf{T}\,X \arrow[d, "F"] \\
D \arrow[r, dashed, "\text{exists unique}", "\hat{F}"'] & X
\end{tikzcd}
$$

Initial algebras

- $[\mathbb{A}](\_)$ can be combined with $\_ \times \_$ and $\_ + \_$ to give functors $\mathbf{T} : \mathbf{Nom} \to \mathbf{Nom}$ that have initial algebras $I : \mathbf{T}\, D \to D$
- For a wide class of such functors ('nominal algebraic functors') the initial algebra D coincides with sets of abstract syntax trees modulo α-equivalence.

  E.g. the initial algebra for

  $$\mathbf{T}(\_) \triangleq \mathbb{A} + (\_ \times \_) + [\mathbb{A}](\_)$$

  is the usual set of untyped λ-terms and the initial-algebra universal property yields. . . .

α-Structural recursion

Theorem.
Given any $X \in \mathbf{Nom}$ and $\begin{cases} f_1 & \in & \mathbb{A} \to_{\mathrm{fs}} X \\ f_2 & \in & X \times X \to_{\mathrm{fs}} X \\ f_3 & \in & [\mathbb{A}]X \to_{\mathrm{fs}} X \end{cases}$

$\exists!\ \hat{f} \in \Lambda \to_{\mathrm{fs}} X$

$$\begin{cases} \hat{f}\,a = f_1\,a \\ \hat{f}\,(t_1\,t_2) = f_2(\hat{f}\,t_1, \hat{f}\,t_2) \\ \hat{f}(\lambda a.t) = f_3(\langle a\rangle(\hat{f}\,t)) & \text{if } a \,\#\, (f_1, f_2, f_3) \end{cases}$$

untyped λ-terms:
$t ::= a \mid t\,t \mid \lambda a.t$

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$$(\forall a)\ a \# (f_1, f_2, f_3) \Rightarrow (\forall x)\ a \# f_3(a, x) \qquad \text{(FCB)}$$

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E.g. capture-avoiding substitution $(\_)[t'/a']$ is the $\hat{f}$ for

$$\begin{aligned} f_1\, a &\triangleq \textbf{if } a = a' \textbf{ then } t' \textbf{ else } a \\ f_2(t_1, t_2) &\triangleq t_1\, t_2 \\ f_3(a, t) &\triangleq \lambda a.t \end{aligned}$$

for which (FCB) holds, since $a \mathbin{\#} \lambda a.t$

Nominal System T

The notion of 'α-structural recursion' generalizes smoothly from λ-terms to any nominal algebraic signature, giving a version of Gödel's System T for nominal data types [J ACM 53(2006)459–506].

Urban & Berghofer's Nominal package for Isabelle/HOL (interactive theorem prover for classical higher-order logic) implements this, and more.

Seems to capture informal practice quite well.

Applications: PL semantics

- operational semantics of ‘nominal’ calculi
- game semantics; domain theory
- equational logic & rewriting modulo α-equivalence
- logic programming (Cheney-Urban: αProlog)
- functional metaprogramming (AMP-Shinwell: Fresh Ocaml)

Take-home message: permutation comes before substitution when dealing with the meta-theory of names and binding operations in syntactical structures.

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X is a nominal set if every $x \in X$ has a finite support. [AMP-Gabbay, LICS 1999]

Consider replacing $\mathbf{S}_{\mathbb{A}}$ by a subgroup G
i.e. use a more restrictive notion of symmetry

Different symmetries

Three interesting examples:

1. **Equality**: $G =$ all (finite) permutations.
2. **Linear order**: $\mathbb{A} = \mathbb{Q}$ and $G =$ order-preserving perms.
3. **Graphs**: $\mathbb{A} =$ vertices of the Rado graph and $G =$ graph automorphisms.

Different symmetries

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1. **Equality**: $G =$ all (finite) permutations.
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3. **Graphs**: $\mathbb{A} =$ vertices of the Rado graph and $G =$ graph automorphisms.

In general [Bojańczyk, Klin, Lasota]:

$\mathbb{A} =$ Carrier of the universal homogeneous structure (Fraïssé limit) for a finite relational signature

$G =$ automorphisms w.r.t. the signature

yields a universe $\mathcal{U} = \mathbf{P}_{\mathbf{fs}}\,(\mathbb{A} + \mathcal{U})$ with interesting applications for nominal computation.

Applications: automata theory

Automata over infinite alphabets:

- HD-automata: π-calculus verification [Montanari el al]
- fresh-register automata [Tzevelekos] (extending finite-memory automata of Kaminski & Francez): verifying programs with local names
- automata & Turing machines in sets with atoms [Bojańczyk et al]; CSP with atoms [Klin et al, LICS 2015]; ...

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General approach: try to do 'ordinary' computation theory inside the universe/category of nominal sets (for a Fraïssé symmetry), but replace finite-state notions by orbit-finite ones...

Orbit finiteness

$=$ having finitely many equivalence classes for

$$x \sim y \triangleq \exists \pi .\ \pi \cdot x = y$$

E.g. for the equality symmetry, $\mathbb{A}^n$ is orbit-finite, $\mathbb{A}^*$ is not.

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Orbit-finite elements of $\mathcal{U}$ have good closure properties, except for powerset. Nominal automata (NA) satisfy:

- deterministic (D) $\neq$ non-deterministic (ND)
- emptiness for ND-NA is decidable
- equivalence for D-NA is decidbale
- D-NAs can be minimized

See [Bojańczyk-Klin-Lasota, LMCS 10(3) 2014].

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Questions:

- Martin-Löf Type Theory generalizes Gödel's System T to inductively defined families of dependent types.
  What is the right version of the notions of *nominal set*, *freshness* and *name abstraction* within constructive type theory?

  Stark-Schöpp CSL 2004 [extensional]
  Cheney LMCS 2012 [missing locally scoped names]
  AMP-Matthiesen-Derikx LSFA 2014 [locally scoped, judgementally fresh names]

Applications: Homotopy Type Theory

Cubical sets [Bezem-Coquand-Huber] model of Voevodsky's axiom of univalence can be described using nominal sets equipped with an operation of substitution $x \mapsto x(i/a)$ where $i \in \{0, 1\}$.

- names are names of directions (cartesian axes)

  (so e.g., if an object has support $\{a, b, c\}$ it is 3-dimensional)
- freshness ($a \# x$) = degeneracy ($x(i/a) = x$)
- identity types are modelled by name-abstraction: $\langle a \rangle x$ is a proof that $x(0/a)$ is equal to $x(1/a)$.

HoTT is about (proof-relevant) *mathematical foundations* (a topic no longer very popular with mathematicians). That's where the mathematics of nominal sets came from...

Impact can take a long time

The mathematics behind nominal sets goes back a long way...

Abraham Fraenkel, *Der Begriff "definit" und die Unabhängigkeit des Auswahlsaxioms*, Sitzungsberichte der Preussischen Akademie der Wissenschaften, Physikalisch-mathematische Klasse (1922), 253–257.

Andrzej Mostowski, *Uber die Unabhängigkeit des Wohlordnungssatzes vom Ordnungsprinzip*, Fundamenta Mathematicae 32 (1939), 201–252.

Impact can take a long time

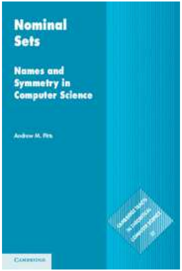
The mathematics behind nominal sets goes back a long way...

...and it's still too early to tell what will be the impact of the applications of it to CS developed over the last 15 years.

Take-home messages:

- Computation modulo symmetry deserves further exploration.
- Permutation comes before substitution and (hence) name-abstraction before lambda-abstraction...but it seems that constructive type theory and nominal techniques can coexist (wts).

Homework



Nominal Sets

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Cambridge Tracts in Theoretical Computer Science, Vol. 57 (CUP, 2013)